Supply Chain Constraints and Inflation*

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Abstract

We develop a New Keynesian framework to evaluate how potentially binding capacity constraints, and shocks to them, shape inflation. We show that binding constraints for domestic and foreign producers shift domestic and import price Phillips Curves up. Further, data on prices and quantities together identify whether constraints bind due to increased demand or reductions in capacity. Applying the model to interpret recent US data, we find that binding constraints in the goods sector explain half of the increase in inflation during 2021-2022. In particular, tight capacity served to amplify the impact of loose monetary policy in 2021, fueling the inflation takeoff.

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In the later half of 2021 and into 2022, the United States (like many other countries) experienced a burst of inflation as it emerged from the COVID-19 pandemic, led by a large increase in goods price inflation. Many standard explanations been advanced to account for the inflation surge, including expansionary fiscal and monetary policy, non-policy demand shocks, disruptions in labor supply, and energy market shocks. In addition to these usual suspects, policymakers and journalists fingered a new culprit: supply chain constraints. Disruptions in both domestic and foreign segments of the supply chain were widely blamed for restraining the supply of goods and fueling inflation in the face of strong aggregate demand.[1]

Though this supply chain narrative is both ubiquitous and plausible, it has thus far been difficult to parse out the quantitative impact of supply chain constraints on inflation. One challenge is that standard New Keynesian models do not include the type of constraints faced by producers that seem most relevant in the post-pandemic period. These included constraints on access to imported inputs, as well as the myriad of constraints that impinged on the ability of domestic firms to combine inputs and labor to produce (e.g., social distancing, factory shutdowns, missing critical inputs, etc.). A second challenge is that the existence of these kind of constraints may alter the impact of other macroeconomic shocks. For example, the impact of monetary and fiscal shocks could be larger in the presence of supply chain constraints than in normal times. Taking these interactions into account is particularly important for interpreting recent data, since there was surely a panoply of shocks. At the same time, the non-linear interaction of shocks and constraints raises technical challenges for model analysis, which need to be addressed.

In this paper, we develop a quantitative framework that addresses these challenges and deploy it to analyze recent US inflation outcomes. The baseline model appeals to occasionally binding, output-based capacity constraints to capture the role of domestic and foreign supply chain disruptions. Specifically, we assume that domestic and foreign producers are able to supply output at constant marginal costs up to a predetermined level, at which point production is quantity-constrained. The foreign constraint stands in for disruptions in the supply of imports; motivated by data and anecdotes, we devote particular attention to disruptions in the supply of imported inputs.[2] The domestic constraints then limit domestic production, even when imported inputs and domestic factors are plentiful. Further, we allow the level of these capacity constraints to vary over time, motivated by the idea that effective production capacities may have been lower than normal in the post-COVID

[1] In International Monetary Fund (2021), Gita Gopinath writes: "Pandemic outbreaks in critical links of global supply chains have resulted in longer-than-expected supply disruptions, further feeding inflation in many countries." Smialek and Nelson (2021) characterize the views of the US Federal Reserve chair with: "[Jerome Powell] noted that while demand was strong in the United States, factory shutdowns and shipping problems were holding back supply, weighing on the economy and pushing inflation above the Fed's goal." See also Lane (2022) for views at the European Central Bank. The Economist (2021) and Goodman (2021) are examples of journalistic coverage of this narrative.

[2] In a companion note [Comin et al. (2024)], we analyze constraints on import logistics capacity (e.g., fixed port infrastructure), which restrict the availability of imports, but not foreign production.

period. In an extension of the baseline framework, we introduce (novel) time-varying constraints on household labor supply as well, which proxy for COVID mitigation measures and school shutdowns that constrained labor supply.

We embed these potentially-binding constraints into a multisector, open economy, New Keynesian (NK) model. Solving for the model's non-linear equilibrium dynamics via piecewise linear approximations, we develop a Bayesian maximum likelihood procedure to estimate key parameters and infer when constraints bind. Applying the estimated model to filter shocks from data and conduct counterfactuals, we find that binding constraints account for about half (two percentage points) of the increase in inflation during 2021-2022. Interestingly, no single set of shocks can explain the inflation takeoff. Rather, shocks that tightened capacity set the stage for demand shocks – most importantly, monetary policy shocks – to trigger binding constraints and accelerate inflation in 2021. Relaxation of the constraints, in part due to monetary tightening, then also explains the rapid decline in goods price inflation since the latter half of 2022.

One nice feature of the model we develop is that it features a distinction between supply-side versus demand-side explanations for binding constraints, with potentially important implications for policy. On the supply side, we assume the levels of the capacity constraints are subject to stochastic shocks.[3] On the demand side, an increase in demand may also exhaust excess capacity and induce capacity constraints to bind. This mechanism is salient, because the abrupt recovery of demand in 2021 seemed to stress existing supply chain capacity. Separating these two mechanisms – that binding constraints may be the result of strong demand, or disruptions to capacity – represents a key quantitative challenge. Breaking the challenge into two pieces, we must ascertain whether constraints bind, while also identifying why they bind.

To shed light on how binding constraints may be detected, we note that binding constraints impact pricing decisions. In the model, constraints are internalized by each firm as it sets its price, such that the firm's optimal markup differs depending on whether the constraint is binding. Assuming that both exports and imports are invoiced in US Dollars, and prices are subject to adjustment frictions, then domestic and import price inflation satisfy Phillips Curve type relationships. When the domestic constraint binds, we show that there is an additional term in sector-level, domestic price Phillips Curves that resembles a markup (equivalently, cost-push) shock. Similarly, there is a quasi-markup shock in the import price Phillips Curve when the import constraint binds. Thus, our framework provides a structural interpretation for reduced-form markup/cost-push shocks, based on binding constraints.

This "markup shock" interpretation of the role of binding constrains dovetails well with related

[3]There are various plausible sources of these shocks during the COVID period, including pandemic-related factory shutdowns in the US, China, Vietnam and elsewhere, as well as other disruptions to global supply relationships (e.g., cancellation of supply contracts by US auto producers, which led to shortages of foreign-supplied semiconductors). Other historical shocks, such as the 2011 Tōhoku earthquake/tsunami, are also plausibly thought of as capacity shocks.

work by Bernanke and Blanchard (2023), which uses an empirical model to argue that product market shocks (which raise prices given wages) explain a large share of recent US inflation. Importantly, our work investigates the structural origins of these empirically plausible shocks.[4] The markup shock interpretation also highlights the contrast between binding constraints and other competing mechanisms that work through marginal costs, such as factor reallocation frictions [Ferrante et al. (2023)], labor shortages [Amiti et al. (2023)], or fixed factors in production [Lorenzoni and Werning (2023)]. Finally, the markup shock interpretation is also prima facie consistent with the fact that US profit margins increased as inflation took off in 2021.

Turning to the second challenge, data on quantities and prices together serve to identify the reasons why constraints bind – i.e., to disentangle whether demand shocks or supply-side constraint shocks lead constraints to bind. While either a positive demand shock or negative constraint shock may trigger binding constraints and thus lead inflation to rise, these shocks have distinct implications for quantities. A positive demand shock pushes both inflation and output quantity up, while a negative constraint shock raises inflation whilst lowering output. Implicitly, we use these responses to identify demand versus constraint shocks when applying the model to filter data. Further, because employ a rich panel of macroeconomic data, we admit many shocks (see the next paragraph); thus, we can distinguish between different types of demand shocks that trigger constraints, as well as effectively control for confounding (non-capacity) supply-side shocks.

To lay out the structure of the paper, we start by collecting stylized facts in Section 1, which both motivate elements of the framework and serve as inputs into quantification.[5] In Section 2, we develop a model to organize our interpretation of these facts, in which we study the impact of constraints for domestic goods producers and foreign goods input suppliers. In Section 3, we then apply the model to filter shocks from US national accounts data. To capture the rich data dynamics, we allow for a number of different shocks, including shocks to aggregate demand (time preference), demand for goods (preferences for goods versus services), monetary policy, capacity levels at home and abroad, sector-specific productivity, and foreign production costs. In one extension, we also incorporate fiscal shocks via deficit-financed transfers to hand-to-mouth households. In another, we allow for both labor supply shocks (disutility of labor) and stochastic constraints on labor supply.[6]

As an intermediate step, we develop a Bayesian Maximum Likelihood estimation procedure to

[4]In a blog post, Del Negro et al. (2022) also argue that markup shocks are important, based on analyzing US data through the lens of a closed economy model without capacity constraints (the NYFed model).

[5]To summarize, headline consumer price inflation rose a lot, more for goods than services. And consumer expenditure shifted from services to goods, driving real goods expenditures above trend. On the import side, prices for imported industrial materials (inputs) rose rapidly in 2021, while prices for imported consumer goods were essentially flat. As for quantities, production of goods has recovered from its temporary pandemic downturn, but it has not increased in response to the surge in consumer demand for goods.

[6]In the Online Appendix, we add exogenous markup shocks as well, which demonstrates that binding constraints may be identified separately from conventional reduced-form markups shocks.

infer when constraints are binding and estimate structural parameters.[7] Our model presents several challenges for estimation. One challenge is that it features capacity shocks, and capacity is a latent variable that has no first order impact on other potentially observable equilibrium variables when constraints are slack. As a result, prior estimation routines (e.g., Guerrieri and Iacoviello (2017)) that use inversion filters to construct the likelihood function are not applicable in our context. Instead, our estimation procedure builds on prior work by Kulish et al. (2017), Kulish and Pagan (2017), and Jones et al. (2022a), which treats the duration of binding constraints as a parameter to be estimated. In this, a second challenge is that the duration of binding constraints is an equilibrium outcome in our model, unlike prior applications of the duration-based estimation approach. Therefore, we impose constraints on admissible duration parameter draws.[8] We validate the estimation procedure via simulation exercises, and we anticipate this contribution will be useful in other settings with occasionally binding constraints.

Overall, our estimated model fits the data well; most importantly, it captures the evolution of inflation for goods, services, and imports during the post-2020 period, making it a useful laboratory for analysis. Smoothed values for multipliers on the constraints imply that constraints bind during most of 2021-2022, and how tight they are fluctuates over time. Further, we detect slackening in both foreign and domestic constraints in 2023, coincident with the decline in US inflation. The path of these multipliers also tracks with external, atheoretical measures of supply chain disruptions (such as the New York Fed's GSCPI index).

With the model and estimates in hand, we evaluate the role of binding constraints in explaining the evolution of inflation through a sequence of counterfactual exercises. The first counterfactual allows all shocks to be active, but exogenously relaxes the capacity constraints in all periods. Comparing this counterfactual to the data, we find that binding constraints explain about half of the increase in inflation in 2021-2022, about two percentage points of the four percentage point increase in overall inflation. Further, subsequent easing of constraints helps explain recent declines in goods and import price inflation.

To evaluate the role of individual shocks, we run a series of counterfactuals in which we introduce shocks one at a time and in combination. We find that tight capacity, in part due to negative capacity shocks, set the stage for monetary policy shocks – looser policy than suggested by an extended Taylor rule – to ignite inflation in 2021. By implication, neither aggregate nor goods-biased consumer demand shocks play an important role in 2021, though they do account for inflation dy-

[7]The structural parameters we estimate are substitution elasticities between home and foreign inputs, coefficients in the monetary policy rule, the mean level of capacity, and the stochastic processes for shocks.

[8]In Kulish et al. (2017) and Jones et al. (2022a), the binding constraint is the zero lower bound on interest rates, so the duration to be estimated reflects beliefs about how long the central bank will hold the interest rate at zero. Because this is a free policy variable, these papers treat durations as unconstrained in the estimation. In our application, the anticipated duration of binding capacity constraints is determined by the realized shock today and the state of the economy. Thus, we adapt the estimation procedure to this new environment.

namics in 2020. Once monetary policy was tightened, inflation falls rapidly from the 2022 through 2023, despite underlying positive shocks to consumer demand during this period.

Probing the robustness of these results, we show that these results are not spuriously driven by fluctuations in energy prices, by re-estimating and simulating the model using inflation data that excludes energy. In a two-agent, non-Ricardian New Keynesian model, we also demonstrate that incorporating fiscal policy shocks into the framework leaves the key results unchanged. While fiscal shocks pay an important role in preventing deflation in 2020, their impact tapers rapidly as fiscal policy is normalized in 2021. Thus, the interaction between monetary policy and capacity constraints remains the driving force behind the inflation take-off. We also investigate the role for labor market shocks. To do so, we enrich the labor market to introduce wage rigidity, labor supply shocks, and potentially binding constraints on labor supply. While these additional features improve the model's ability to fit labor market data (labor quantities and real wages), and serve to explain the absence of deinflation in 2020, binding capacity constraints continue to play an important role in explaining inflation dynamics in 2021-2022.

In addition to work cited above, our paper is related to (at least) three strands of research. First, output-based capacity constraints have appeared in prior papers by Fagnart et al. (1999), Álvarez-Lois (2006), Murphy (2017), and Boehm and Pandalai-Nayar (2022). We will discuss this work in Section 2.6, so highlight only a few points here. First, this approach to capacity is distinct from models of capital utilization and fixed factors in production. Second, while output-based constraints may be micro-founded via putty-clay technologies [Fagnart et al. (1999)], we take the constraints as given and allow them to evolve stochastically over time, inferring their realized values from data. A strength of our approach is that it allows for capacity shocks, which seem important for explaining recent data. Third, in contrast to models that feature heterogeneous firms [e.g., Boehm and Pandalai-Nayar (2022)], we adopt a homogeneous firms framework. One pedagogical advantage to our approach is that our model is easily comparable to standard log-linear New Keynesian models, even though it features state-dependent responses to shocks. A second is that we can use piecewise linear solution techniques to capture non-linearities, which in turn enables us to employ "fast" filtering and estimation routines, which exploit the Kalman Filter.

Second, our paper is related to a rapidly growing literature on how global value chains transmitted shocks during the pandemic crisis.[9] Several contributions specifically study the impact of supply chain disruptions on prices during the pandemic period. For the United States, Amiti et al. (2021) and Santacreu and LaBelle (2022) find that output price changes across industries are related to their exposure to input price shocks and/or supply chain disruptions. Relatedly, Benigno et al.

[9] In addition to references in the main text, see also Bonadio et al. (2021), Gourinchas et al. (2021), Celasun et al. (2022), and Lafrogne-Joussier et al. (2023). Taking a longer view, Comin and Johnson (2020) study how rising trade impacted US inflation over the past several decades.

(2022) develop an index of global supply chain pressures, and they find it has predictive power for inflation. Focusing disruptions in the shipping sector (e.g., port blockages), Bai et al. (2023) and Finck and Tillmann (2023) also find that disruptions raise inflation in vector auto-regressive models. Lastly, Alessandria et al. (2023) study how international shipping delays that lead to inventory depletion induce firms to raise prices in a flexible price model.

Several related contributions dedicate attention to multisector models. Amiti et al. (2023) study how the combination of domestic labor market shocks and import disruptions contribute to inflation across sectors. di Giovanni et al. (2022) examine the role of labor shortages and supply chain disruptions on inflation during the pandemic in a closed economy, while di Giovanni et al. (2024) study cross country spillovers in a multi-country setting, both building on the multi-sector model in Baqaee and Fahri (2022). While the framework we lay out includes multiple sectors, and could be extended to many countries, our empirical application focuses on two sectors in a small open economy.[10] As will be evident in Section 1, two sectors are sufficient to capture the most important dimensions of the post-pandemic experience (e.g., sharp differences in quantities and prices for goods versus services). At the same time, limiting the number of sectors reduces the size of the underlying parameter space, which facilitates estimation. Whereas Baqaee and Fahri (2022) adopt a two period structure with quasi-perfect foresight, we take a complete DSGE model to the data.[11]

Our paper is the first (to our knowledge) to analyze occasionally binding capacity constraints in the supply chain within a complete, estimated DSGE model. This means that we can parse the impact of these constraints relative to and in conjunction with a myriad of competing alternative drivers of inflation, within a workhorse model of inflation dynamics. In this, our paper extends the new literature on monetary policy in economies with production networks [e.g., La'o and Tahbaz-Salehi (2022); Rubbo (2023)] to accommodate occasionally binding constraints. Thus, it opens the door to further study of the implications of bottleneck constraints for the conduct of policy.

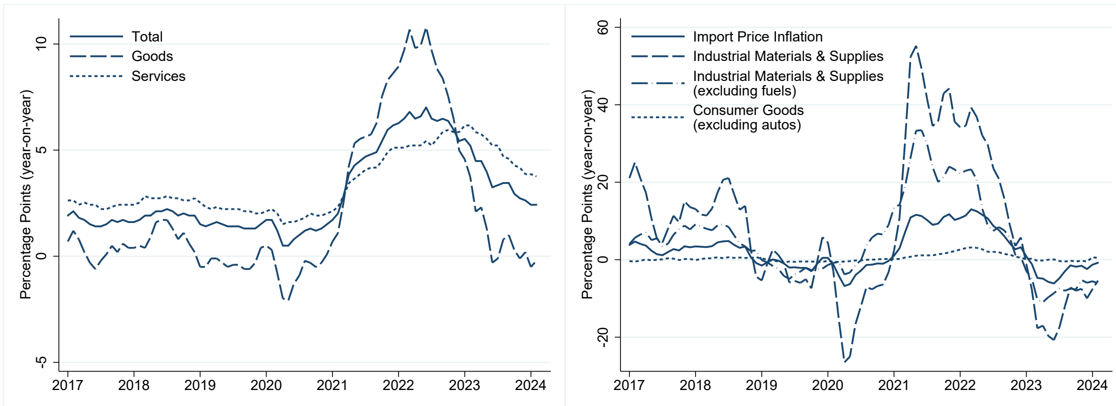
Finally, our paper is also related to a broader literature on drivers of inflation during the pandemic recovery, other than supply chain forces. Gagliardone and Gertler (2024) study the impact of oil shocks on inflation, and we address energy prices in Section 4.1. Other contributions focus on the impacts of fiscal policy, including di Giovanni et al. (2023), de Soyres et al. (2023), Bianchi et al. (2023), and we address fiscal policy directly in Section 4.2. In contrast to Bianchi et al. (2023), which emphasizes the fiscal theory of the price level, we study a standard New Keynesian environment with monetary dominance. Lastly, we cite several papers on labor market disruptions above,

[10] Regarding the small open economy assumption, we allow for foreign shocks, but abstract from the possibility that domestic shocks spillover to the rest of the world and then blow back via imports. Spillovers are unlikely to be quantitatively important for interpreting US data, not least because the US is relatively closed.

[11] Baqaee and Fahri (2022) also assume downward nominal wage rigidity is the sole pricing friction. While this is useful to explain unemployment given a negative demand shock, it is prima facie less useful to rationalize high inflation following positive demand shocks.

Figure 1: Consumer and Import Price Inflation

(a) Consumer Price Inflation (b) Import Price Inflation by End Use



Note: Consumer price indexes are from the US Bureau of Economic Analysis, corresponding to the Personal Consumption Expenditure (PCE) price index and components (series identifiers: DPCERGM, DGDSRGM, and DSERRGM). Import price indexes are obtained from the US Bureau of Labor Statistics (series identifiers: IR for total imports, EIUIR1 for industrial materials, EUIIR1EXFUEL for industrial materials excluding fuels, and EIUIR4 for consumer goods).

and we return to address labor market shocks in Section 4.3.

1 Collecting Facts

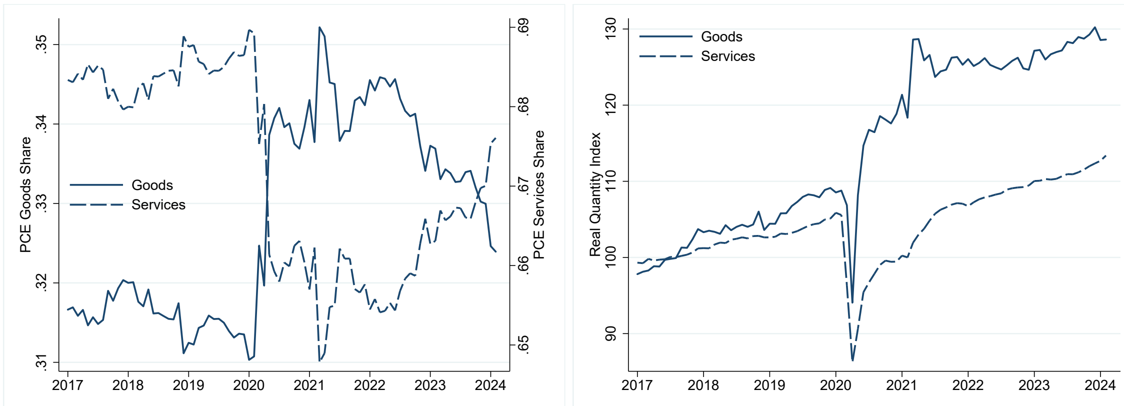
We begin by collecting several key facts about recent inflation, consumer expenditure, production, and imports that motivate various elements of the framework we construct. The first facts about consumer price inflation are well known: consumer price inflation rose substantially in 2021, led by inflation for goods. In Figure 1, we plot year-on-year growth in the price deflator for US personal consumption expenditure (PCE), as well as separate series for goods and services. The rise in headline inflation – from roughly 2 percent in 2021 to 7 percent as of early 2022 – is obviously startling. Importantly, this rise in inflation was led by goods price inflation, which rose from near zero to 10 percent in 2021 and then plummeted in the second half of 2022.

A second set of facts concerns import price inflation: prices for imported inputs rose dramatically in 2021, while price changes for imported consumer goods were modest. Plotting import price inflation by end use in Figure 1b, we see that inflation for imported industrial materials rose substantially in 2021, peaking at 50% year on year.[12] While the price of oil and derivative fuels

[12] This data is from the International Price Program of the Bureau of Labor Statistics. The source data consist primarily of free on board (FOB) prices (i.e., prices received by foreign producers at foreign dock). During 2021-2022, transport costs also increased dramatically, which then would be added to these FOB prices to arrive at CIF prices (inclusive of cost, insurance, and freight) paid by the importer. We abstract from these additional transport

Figure 2: Consumption by Sector

(a) Sector Shares in Expenditure (b) Real Quantities Consumed by Sector



Note: Personal Consumption Expenditure shares and real quantity indexes by sector are obtained from the US Bureau of Economic Analysis (series identifiers: DPCERC, PCES, DGDSRA3, and DSERRA3).

doubled during this period, the price of industrial materials excluding fuels also rose over 30% in 2021. In contrast, inflation for imported consumer goods was subdued. This large difference between import price inflation for inputs versus consumer goods motivates our ensuing focus on disruptions impacting markets for imported inputs, rather than consumer goods.[13] In 2022, imported input price inflation dissipates rapidly, even excluding volatile fuels prices.

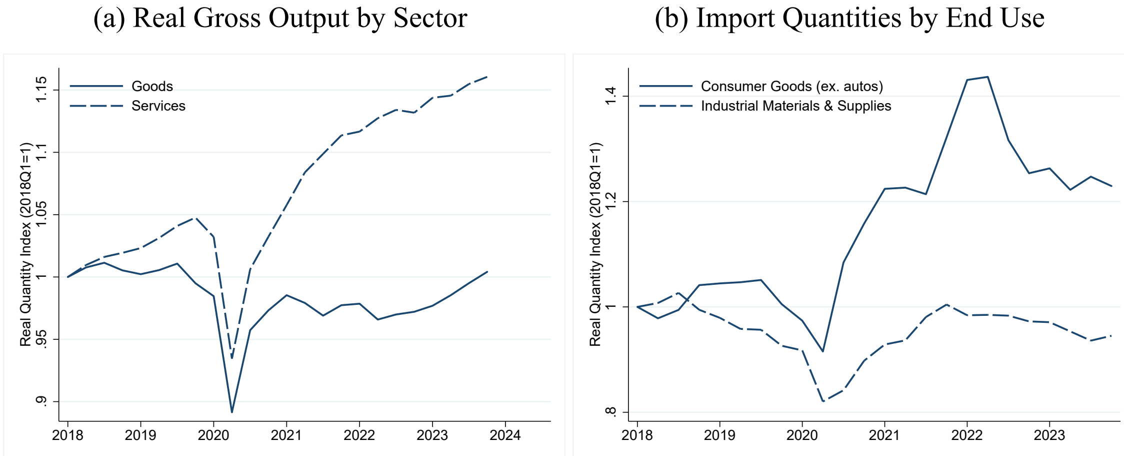
Tying the first and second set of facts together, goods production relies heavily on imported materials, relative production of services. Thus, the large increase in imported materials prices may play a role in explaining the surge of inflation in the goods sector discussed above. Our model framework will include this potential mechanism, alongside other competing drivers of inflation.

The third set of facts relate to consumer expenditures. While consumer expenditure collapsed during the lockdown phase of the pandemic, it returned to trend by the end of 2021. At the same time, the sector composition of consumer expenditures changed dramatically, as consumers reallocated away from services toward goods. This is illustrated in terms of nominal expenditure shares in Figure 2a, and in terms of real quantities consumed for goods and services in Figure 2b. Further, note that the change in composition has proven remarkably persistent: real consumption of goods (correspondingly, the goods share in expenditure) remains high relative to pre-pandemic levels through 2023.

margins, in order to focus on changes in supply prices.

[13] We have omitted several categories of imports from the figure for clarity, including capital goods imports (IR2), imports of automotive vehicles, parts, and engines (IR3), and foods, feeds, and beverages (IR0). To verbally summarize, inflation for capital goods imports was generally low, similar to imported consumer goods. Inflation for the automotive sector was also very low, and inflation for foods tracked total import price inflation closely. Thus, the behavior of imported materials prices stands out.

Figure 3: Production and Import Quantities



Note: Real gross output is constructed using data from the US Bureau of Economic Analysis (GDP by Industry, Table 17). Real quantity indexes for imports are obtained from the US Bureau of Economic Analysis (series identifiers: IB0000043 and B652RA3).

The final set of facts point to potential supply-side constraints. In Figure 3a, we plot real US gross output by broad sector. The key fact is that real production of goods (already stagnant before the pandemic) only just recovered and then trended slightly down in 2021-2022, which contrasts sharply with services output. Stagnant goods production in the face of high domestic demand for goods immediately suggests that US producers may have faced binding constraints. Correspondingly, consumer demand for goods was filled by imports: in Figure 3b, imported quantities for consumer goods (excluding autos) surge. In contrast, imports of industrial materials are flat, recovering only to its 2017 levels by the end of 2021 and plateauing there.

Deficient US goods production and stagnant imports of industrial materials are naturally connected, though the direction of causality is not immediately clear. Limited supplies of imported materials may have constrained domestic production, or distinct binding constraints of domestic origin may have curtailed production and indirectly depressed demand for imported inputs. Quantity and price data together will distinguish between binding domestic versus foreign supply constraints in our model. With this background in mind, we turn to details of the model.

2 Model

This section presents a small open economy model with sectors $s \in \{1, \ldots, S\}$, which are connected through input-output linkages. Within each sector, there is a continuum of monopolistically competitive firms, who set prices subject to Rotemberg-type adjustment costs. As in Gopinath et al. (2020), we assume that both exports and imports for the Home country are denominated in Home currency (i.e., US Dollars). Motivated by the data, we also allow import prices to differ for final goods and inputs.

The principal new features of the model are the output capacity constraints for foreign and domestic firms.[14] These constraints capture the reduced-form impact of production disruptions, which may take various underlying forms, but ultimately limit the amount of output the firm can produce (see Section Section 2.6.2 for further discussion). Because we embed domestic producers within an open economy with input-output linkages, the model parsimoniously captures both restrictions on the upstream supply of domestic and foreign inputs, as well the capacity of downstream producers to combine those inputs into output. We assume that the constraints are exogenously determined and (potentially) time varying, subject to stochastic shocks. This sets up a framework in which constraints may bind either due to negative shocks to capacity, or because other shocks lead firms to exhaust their excess capacity.

2.1 Consumers

There is a representative Home consumer, with preferences over labor supply L_t and consumption of sector composite goods $\{C_t(s)\}_{s\in S}$ represented by:

$$U\left(\{C_t, L_t\}_{t=0}^{\infty}\right) = \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \Theta_t \left[\frac{C_t^{1-\rho}}{1-\rho} - \frac{L_t^{1+\psi}}{1+\psi}\right] \tag{1}$$

$$\text{with} \quad C_t = \left(\sum_s \zeta_t(s)^{1/\vartheta} C_t(s)^{(\vartheta-1)/\vartheta}\right)^{\vartheta/(\vartheta-1)}. \tag{2}$$

The parameter $\beta < 1$ is the usual time discount rate, $\rho \geq 0$ controls intertemporal substitution, $\psi > 0$ governs the elasticity of labor supply, and $\vartheta \geq 0$ is the elasticity of substitution across sectors. The parameter Θ_t is an aggregate preference (discount rate) shock at date t. The sectoral composite good $C_t(s)$ is comprised of domestic ($C_{Ht}(s)$) and foreign ($C_{Ft}(s)$) composite goods:

$$C_t(s) = \left(\sum_s \gamma(s)^{1/\epsilon(s)} C_{Ht}(s)^{(\epsilon(s)-1)/\epsilon(s)} + (1-\gamma(s))^{1/\epsilon(s)} C_{Ft}(s)^{(\epsilon(s)-1)/\epsilon(s)}\right)^{\epsilon(s)/(\epsilon(s)-1)}, \tag{3}$$

where $\epsilon(s) \geq 0$ is the elasticity of substitution between home and foreign composites. The parameter $\zeta_t(s)$ is a time-varying parameter that controls tastes for goods from sector s, and we require that $\sum_s \zeta_t(s) = 1$, so $\zeta_t(s)$ should be interpreted as a relative sectoral demand shock.

[14] We write the model in a general way here, allowing the constraints to be potentially binding in all domestic sectors and for both foreign final goods and input producing firms. We will then restrict attention to the particular constraints that are most empirically relevant as we estimate the model and conduct counterfactuals.

Financial markets are complete, and the agent's budget constraint is given by:

$$P_t C_t + \mathbb{E}_t \left[S_{t,t+1} B_{t+1} \right] \leq B_t + W_t L_t, \tag{4}$$

where $P_t C_t = \sum_s P_t(s) C_t(s)$, P_t is the price for one unit of the composite consumption good, $P_t(s)$ is the price of the sector composite good, and W_t is the wage.[15] B_t denotes the portfolio of Arrow-Debreu securities that pay off in domestic currency, and $S_{t,t+1}$ is the Home consumer's stochastic discount factor. Further, sectoral consumption expenditure is $P_t(s)C_t(s) = P_{Ht}(s)C_{Ht}(s) + P_{Ft}(s)C_{Ft}(s)$, where $P_{Ht}(s)$ and $P_{Ft}(s)$ are the prices of the home and foreign consumption composites.

Given prices and initial asset holdings B_0, the consumer chooses consumption, labor supply, and asset holdings to maximize Equation 1 subject to Equation 4 and the standard transversality condition.

2.2 Domestic Producers

There is a continuum of firms within each sector in Home, each of which produces a differentiated good (indexed by ω). There also are competitive intermediary firms that aggregate varieties into composite goods, which are consumed, used as inputs, and exported.

2.2.1 Composite Domestic Good

Each competitive intermediary firm purchases output from domestic producers to form a domestic composite, using the production function $Y_t(s) = \left(\int_0^1 Y_t(s,\omega)^{(\varepsilon-1)/\varepsilon} d\omega \right)^{\varepsilon/(\varepsilon-1)}$, where $Y_t(s,\omega)$ is the amount of output purchased from firm ω in sector s, and $\varepsilon > 1$ is the elasticity of substitution. Given prices $P_t(s,\omega)$ for individual domestic varieties, cost minimization yields demands $Y_t(s,\omega) = \left(\frac{P_t(s,\omega)}{P_{Ht}(s)} \right)^{-\varepsilon} Y_t(s)$, where the price of the sector composite good is $P_{Ht}(s) = \left[\int_0^1 P_t(s,\omega)^{1-\varepsilon} d\omega \right]^{1/(1-\varepsilon)}$.

2.2.2 Domestic Firms

Each domestic producer in sector s is able to supply output up to a pre-determined capacity of $\bar{Y}_t(s)$, which we refer to as a firm-level capacity constraint. We assume this capacity level is exogenously determined and equal across firms within each sector.

[15] The price indexes are given by $P_t = \left(\sum_s \zeta_t(s) P_t(s)^{1-\vartheta} \right)^{1/(1-\vartheta)}$ and $P_t(s) = \left(\gamma(s) \left(P_{Ht}(s) \right)^{1-\epsilon(s)} + (1-\gamma(s)) \left(P_{Ft}(s) \right)^{1-\epsilon(s)} \right)^{1/(1-\epsilon(s))}$.

The production function for domestic variety ω in sector s is:

$$Y_t(s,\omega) = Z_t(s,\omega)A(s)\left(L_t(s,\omega)\right)^{1-\alpha(s)}\left(M_t(s,\omega)\right)^{\alpha(s)} \tag{5}$$

$$\text{with}\quad M_t(s,\omega) = \left(\sum_{s'}\left(\alpha(s^{'},s)/\alpha(s)\right)^{1/\kappa} M_t(s^{'},s,\omega)^{(\kappa-1)/\kappa}\right)^{\kappa/(\kappa-1)},$$

$$M_t(s^{'},s,\omega)=\left[\xi(s^{'},s)^{\frac{1}{\eta(s^{'})}}M_{Ht}(s^{'},s,\omega)^{\frac{\eta(s^{'})-1}{\eta(s^{'})}}+(1-\xi(s^{'},s))^{\frac{1}{\eta(s^{'})}}M_{Ft}(s^{'},s,\omega)^{\frac{\eta(s^{'})-1}{\eta(s^{'})}}\right]^{\frac{\eta(s^{'})}{\eta(s^{'})-1}},$$

where $L_t(s,\omega)$ is the quantity of labor used by the firm, $M_t(s,\omega)$ is the firm's use of a composite input, $Z_t(s,\omega)$ is productivity, and $A(s) = \alpha(s)^{-\alpha(s)}(1-\alpha(s))^{-(1-\alpha(s))}$ is a normalization constant. The composite input combines inputs purchased from upstream sectors $M_t(s^{'},s,\omega)$, with elasticity of substitution $\kappa \geq 0$. And those upstream inputs are themselves a CES composite of Home ($M_{Ht}(s^{'},s,\omega)$) and Foreign ($M_{Ft}(s^{'},s,\omega)$) composite inputs. The parameters $\eta(s) \geq 0$ are elasticities of substitution across country sources for inputs (conventionally termed the Armington elasticity), while $\xi(s^{'},s) \in (0,1)$ controls relative demand for home inputs conditional on prices.

Producers set prices in domestic currency under monopolistic competition, and they select the input mix to satisfy the implied demand. As is standard, these two problems can be analyzed separately. The firm chooses $\left\{L_t(s,\omega), M_t(s,\omega), M_t(s^{'},s,\omega), M_{Ht}(s,\omega), M_{Ft}(s',s,\omega)\right\}$ to minimize its cost of production.[16]

Given its marginal costs, the domestic firm chooses a sequence of prices to maximize profits, with knowledge of the demand curve for its output, and subject to quadratic adjustment cost for prices [Rotemberg (1982)]. The pricing problem is:

$$\max_{\{P_t(s,\omega)\}} \mathbf{E}_0 \sum_{t=0}^{\infty} \frac{S_{0,t}}{P_t}\left[P_t(s,\omega)Y_t(s,\omega) - MC_t(s,\omega)Y_t(s,\omega) - \Phi_t(s,\omega)\right]$$

$$\text{s.t.}\quad Y_t(s,\omega) \leq \bar{Y}_t(s) \quad\text{and}\quad Y_t(s,\omega) = \left(\frac{P_t(s,\omega)}{P_{Ht}(s)}\right)^{-\varepsilon} Y_t(s),$$

where $\Phi_t(s,\omega) \equiv \frac{\phi(s)}{2}\left(\frac{P_t(s,\omega)}{P_{t-1}(s,\omega)}-1\right)^2 P_{Ht}(s)Y_t(s)$ captures adjustment costs, $\phi(s)$ governs the degree of price rigidity, and the discount rate for profits reflects the domestic agent's stochastic discounting.[17]

[16] The cost of production is $W_tL_t(s,\omega) + P_{Mt}(s)M_t(s,\omega)$, with $P_{Mt}(s)M_t(s,\omega) = \sum_{s'} P_t(s',s)M_t(s^{'},s,\omega)$ and $P_t(s',s)M_t(s^{'},s,\omega) = P_t(s')M_{Ht}(s',s,\omega) + P_{Ft}(s')M_{Ft}(s',s,\omega)$, where $P_{Ft}(s')$ is the (domestic currency) price of the foreign composite input from sector s'.

[17] In writing this problem in this way, we have implicitly assumed that the firm satisfies its demand (i.e., production equals demand given the price). One could alternatively allow the firm to ration its output, setting a price at which quantity demand (weakly) exceeds the amount it produces. This choice could be rationalized by high costs of adjusting

The firm accounts for the potentially binding constraint in its pricing decisions. Denoting the Lagrange multiplier attached to the capacity constraint $\mu_t(s,\omega)$, optimal prices satisfy:

$$0 = 1 - \epsilon\left(1 - \frac{MC_t(s,\omega)+\mu_t(s,\omega)}{P_t(s,\omega)}\right) - \phi(s)\left(\frac{P_t(s,\omega)}{P_{t-1}(s,\omega)} - 1\right)\frac{P_{Ht}(s)Y_t(s)}{P_{t-1}(s,\omega)Y_t(s,\omega)}$$
$$+\mathbb{E}_t\left[S_{t,t+1}\frac{P_t}{P_{t+1}}\phi(s)\left(\frac{P_{t+1}(s,\omega)}{P_t(s,\omega)} - 1\right)\frac{P_{Ht+1}(s)Y_{t+1}(s)}{P_t(s,\omega)Y_t(s,\omega)}\frac{P_{t+1}(s,\omega)}{P_t(s,\omega)}\right]. \quad (6)$$

The corresponding complementary slackness condition is:

$$\mu_t(s,\omega)\left[Y_t(s,\omega) - \bar{Y}_t(s)\right] = 0. \quad (7)$$

And we require $\mu_t(s,\omega) \geq 0$ and the constraint to hold in equilibrium ($Y_t(s,\omega) \leq \bar{Y}_t(s)$) as usual. When the constraint binds, then $\mu_t(s,\omega) > 0$. In Equation 6, we see this is equivalent to an increase in the marginal cost of the firm, which drives up the optimal price. When the capacity constraint is slack, such that $\mu_t(s,\omega) = 0$, and expected to remain slack, then Equation 6 collapses to a standard intertemporal pricing equation.

2.3 Foreign Producers

Turning to foreign producers, we distinguish between producers of foreign consumption goods versus inputs, which allows us to to analyze data on import prices by end use.

2.3.1 Composite Foreign Goods

For each end use $u \in \{C, M\}$, where C and M denote consumption and intermediate use respectively, there is a unit continuum of foreign firms that produce foreign inputs, indexed by ϖ. A competitive intermediary firm aggregates output produced by each foreign firm, and bundles it into the foreign composite according to the production function: $Y^*_{ut}(s) = \left(\int_0^1 Y^*_{ut}(s,\varpi)^{(\varepsilon-1)/\varepsilon} d\varpi\right)^{\varepsilon/(\varepsilon-1)}$. Demand for each variety then takes the standard CES form: $Y^*_{ut}(s,\varpi) = \left(\frac{P_{uFt}(s,\varpi)}{P_{uFt}(s)}\right)^{-\varepsilon} Y^*_{ut}(s)$, where $P_{uFt}(s,\varpi)$ is the price of variety ϖ and $P_{uFt}(s) = \left(\int_0^1 P_{uFt}(s,\varpi)^{1-\varepsilon} d\varpi\right)^{1/(1-\varepsilon)}$ is the price of the foreign composite, both denominated in Home currency.

2.3.2 Foreign Firms

Each foreign firm (in sector s, producing for end use u) is able to supply output up to a predetermined capacity of $\bar{Y}^*_{ut}(s)$, and this capacity is exogenous and equal across firms. Foreign

prices, for example. In practice, this would dampen the response of prices when constraints bind, so would be counterproductive in terms of accounting for high inflation. For brevity, we omit consideration of this case in the main text.

marginal costs are given by $MC^*(s,\varpi)$, and we assume this cost is exogenous (as in a small open economy), denominated in foreign currency, and equal across end uses.

Each firm chooses a sequence for the price of its variety in Home currency $\{P_{uFt}(s,\varpi)\}$, subject to price adjustment frictions, to solve:

$$\max_{\{P_{Ft}(s,\varpi)\}} \mathbf{E}_0 \sum_{t=0}^{\infty} \frac{S^*_{0,t}}{P^*_t E_t} \left[P_{uFt}(s,\varpi) Y^*_{ut}(s,\varpi) - E_t MC^*_t(s) Y^*_{ut}(s,\varpi) - \Phi_t(s,\varpi)\right]$$

$$\text{s.t.} \quad Y^*_{ut}(s,\varpi) \leq \bar{Y}^*_{ut}(s) \quad \text{and} \quad Y^*_{ut}(s,\varpi) = \left(\frac{P_{uFt}(s,\varpi)}{P_{uFt}(s)}\right)^{-\varepsilon} Y^*_{ut}(s),$$

with $\Phi^*_t(s,\varpi) \equiv \frac{\phi(s)}{2}\left(\frac{P_{uFt}(s,\varpi)}{P_{uFt-1}(s,\varpi)} - 1\right)^2 P_{uFt}(s) Y^*_{ut}(s)$ with knowledge of the demand curve for its output specified above. Here $S^*_{0,t}$ is the foreign stochastic discount factor, P^*_t is the foreign price level (in foreign currency), and E_t is a the nominal exchange rate (units of home currency to buy one unit of foreign currency).

Denoting the Lagrange multiplier attached to the capacity constraint $\mu^*_{ut}(s,\varpi)$, then the first order condition is:

$$\begin{aligned} 1 - \varepsilon\left(1 - \frac{E_t\left(MC^*_t(s,\varpi) + \mu^*_{ut}(s,\varpi)\right)}{P_{uFt}(s,\varpi)}\right) - \phi(s)\left(\frac{P_{uFt}(s,\varpi)}{P_{uFt-1}(s,\varpi)} - 1\right)\frac{P_{uFt}(s)Y^*_{ut}(s)}{P_{uFt-1}(s,\varpi)Y^*_{ut}(s,\varpi)} \\ + \mathbb{E}_t\left[S^*_{t,t+1}\left(\frac{E_t P^*_t}{E_{t+1}P^*_{t+1}}\right)\phi(s)\left(\frac{P_{uFt+1}(s,\varpi)}{P_{uFt}(s,\varpi)} - 1\right)\frac{P_{uFt+1}(s)Y^*_{ut+1}(s)}{P_{uFt}(s,\varpi)Y^*_{ut}(s,\varpi)}\frac{P_{uFt+1}(s,\varpi)}{P_{uFt}(s,\varpi)}\right] = 0. \quad (8) \end{aligned}$$

The complementary slackness condition is:

$$\mu^*_{ut}(s,\varpi)\left[Y^*_{ut}(\varpi) - \bar{Y}^*_{ut}\right] = 0. \quad (9)$$

In equilibrium, $\mu^*_{ut}(\varpi) \geq 0$ and $Y^*_{ut}(\varpi) \leq \bar{Y}^*_{ut}$.

2.4 Closing the Model

We assume that demand for exports of the home composite good takes the CES form:

$$X_t(s) = \left(\frac{P_{Ht}(s)}{P_t Q_t}\right)^{-\sigma(s)} X^*_t(s), \quad (10)$$

where $Q_t \equiv \frac{E_t P^*_t}{P_t}$ is the real exchange rate and $X^*_t(s)$ is an exogenous export demand factor.

The market clearing condition for the home composite good is:

$$Y_t(s) = C_{Ht}(s) + \sum_{s'=1}^{S}\int_0^1 M_{Ht}(s,s',\omega)d\omega + X_t(s) + \int_0^1\left[\frac{\phi(s)}{2}\left(\frac{P_t(s,\omega)}{P_{t-1}(s,\omega)} - 1\right)^2 Y_t(s)\right]d\omega, \quad (11)$$

where the composite good is sold to consumers and domestic producers, exported, and used to cover price adjustment costs. For the foreign composite goods, we impose similar market clearing conditions:

$$Y^*_{Ct}(s) = C_{Ft}(s) + \int_0^1 \left[\frac{\phi(s)}{2} \left(\frac{P_{CFt}(s,\varpi)}{P_{CFt-1}(s,\varpi)} - 1 \right)^2 Y^*_{Ct}(s) \right] d\varpi \tag{12}$$

$$Y^*_{Mt}(s) = \sum_{s'} M_{Ft}(s,s') + \int_0^1 \left[\frac{\phi(s)}{2} \left(\frac{P_{MFt}(s,\varpi)}{P_{MFt-1}(s,\varpi)} - 1 \right)^2 Y^*_{Mt}(s) \right] d\varpi. \tag{13}$$

Labor market clearing is given by:

$$L_t = \sum_{s=1}^{S} L_t(s) \quad \text{with} \quad L_t(s) = \int_0^1 L_t(s,\omega) d\omega. \tag{14}$$

Trade in Arrow-Debreu securities implies that Home and Foreign consumers share risk, such that:

$$\Theta_t \left(\frac{C_t}{C^*_t} \right)^{-\rho} Q_t = \Xi, \tag{15}$$

where Ξ is a constant.

Turning to monetary policy, we specify an extended inflation-targeting rule for interest rates. Since we allow for sector-specific preference shocks, we now distinguish measured price inflation from changes in the welfare-theoretic price index. We define an auxiliary price index under the assumption that preferences are constant over time: $\bar{P}_t = \left(\sum_s \zeta_0(s) \left(P_t(s) \right)^{1-\vartheta} \right)^{1/(1-\vartheta)}$, where $\zeta_0(s)$ are steady-state CES weights. Then $\bar{\Pi}_t = \bar{P}_t / \bar{P}_{t-1}$ is the ratio of measured prices across periods, and the approximate inflation rate is given by $\bar{\pi}_t = \sum_s \left(\frac{P_0(s)C_0(s)}{P_0 C_0} \right) [\ln P_t(s) - \ln P_{t-1}(s)]$.[18] We write the monetary policy rule in terms of measured inflation:

$$1 + i_t = (1 + i_{t-1})^{\varrho_i} \bar{\Pi}_t^{\omega(1-\varrho_i)} \left(Y_t / Y_0 \right)^{(1-\varrho_i)\varrho_y} \mathit{\Psi}_t, \tag{16}$$

where $Y_t = \sum_s P_0(s) Y_t(s)$ is aggregate real gross output and $\mathit{\Psi}_t$ is a monetary policy shock. The parameters ω and ϱ_y determine how aggressively the central bank responds to inflation and the output gap (defined as the deviation of output from steady state), while the parameter ϱ_i controls the degree of interest rate inertia.

[18] The following relationship holds between the ratios of measured and welfare-based price indexes across periods: $\bar{\Pi}_t = \frac{\bar{P}_t / P_t}{\bar{P}_{t-1}/P_{t-1}} \Pi_t$, where $\frac{\bar{P}_t}{P_t} = \left(\sum_s \zeta_0(s) \left(\frac{P_t(s)}{P_t} \right)^{1-\vartheta} \right)^{1/(1-\vartheta)}$ and the ratio of aggregate prices across periods is $\Pi_t \equiv \frac{P_t}{P_{t-1}}$. We include these among auxiliary price definitions in the model equilibrium.

2.5 Solving the Model

We focus on an equilibrium with symmetric producers within each sector and country. Given parameters and exogenous variables, an equilibrium is a sequence of quantities and prices that satisfy the model's equilibrium conditions in Table A1 of the Appendix.

Because the model features occasionally binding constraints, we need to adopt an appropriate solution technique that captures the non-linearities induced by them. Among alternatives, we adopt the piecewise linear solution technique developed by Guerrieri and Iacoviello (2015). The perturbation-based solution algorithm combines first order approximations to the model equilibrium for both the unconstrained and constrained equilibria, where the point of approximation is the unconstrained equilibrium in all cases.[19] The log-linear approximation for the model used in our quantitative analysis, and details regarding the solution procedure, are presented in the Online Appendix.

Collecting log deviations from steady state for endogenous (both control and state) variables in the vector X_t, the general solution for the model can be written as:

$$X_t = \mathbf{J}\left(X_{t-1}, \varepsilon_t; \theta\right) + \mathbf{Q}\left(X_{t-1}, \varepsilon_t; \theta\right) X_{t-1} + \mathbf{G}\left(X_{t-1}, \varepsilon_t; \theta\right) \varepsilon_t, \tag{17}$$

where ε_t is the vector of exogenous shocks in period t, θ is a collection of structural parameters, and $\mathbf{J}(\cdot)$, $\mathbf{Q}(\cdot)$, and $\mathbf{G}(\cdot)$ are time-varying matrices (dependent on the state and current shocks) that describe the optimal policy function.

2.6 Discussion

We now describe the domestic and import price Phillips Curves in the model, where binding constraints appear as markup shocks in reduced form. Then, we discuss how the model compares to a related literature on capacity, capital utilization, and fixed factors in production. Finally, we introduce a stylized fact about profits, which serves to support our emphasis on markups.

2.6.1 Domestic and Import Price Phillips Curves

It is instructive to examine log-linear approximations for the dynamic pricing equations for domestic and imported goods. Noting that $\mu_t(s)/P_t$ and $\mu^*_{ut}(s)/P^*_t$ for $u \in \{C, M\}$ take on zero values in the unconstrained equilibrium, we define auxiliary variables $\tilde{\mu}_t(s) \equiv \mu_t(s)/P_t + 1$ and

[19]The solution procedures requires that the model satisfies two important conditions. First, it is assumed that the model returns to the unconstrained equilibrium in finite time after a once-off shock, if agents expect future shocks to be zero. Second, the unconstrained equilibrium must be stable, in the usual Blanchard-Kahn sense. Both requirements are satisfied for our baseline model and parameter values.

$\tilde{\mu}^*_{ut}(s) \equiv \mu^*_{ut}(s)/P^*_t + 1$, and then we log-linearize the equilibrium with respect to these auxiliary variables. The resulting approximate pricing equations are:

$$\pi_{Ht}(s) = \left(\frac{\varepsilon-1}{\phi(s)}\right)\left(\widehat{rmc}_t(s) - \widehat{rp}_{Ht}(s)\right) + \left(\frac{\varepsilon}{\phi(s)}\frac{P_0}{P_{H0}(s)}\right)\hat{\tilde{\mu}}_t(s) + \beta\mathbb{E}_t\left[\pi_{Ht+1}(s)\right] \tag{18}$$

$$\pi_{uFt}(s) = \left(\frac{\varepsilon-1}{\phi(s)}\right)\left(\widehat{rmc}^*_t(s) + \hat{q}_t - \widehat{rp}_{uFt}(s)\right) + \left(\frac{\varepsilon}{\phi(s)}\frac{P_0}{P_{uF0}(s)}\right)\hat{\tilde{\mu}}^*_{ut}(s) + \beta\mathbb{E}_t\left[\pi_{uFt+1}(s)\right], \tag{19}$$

where hat-notation denotes deviations from steady state, $\pi_t(s) \equiv \ln P_t(s) - \ln P_{t-1}(s)$, $\pi_{Ft}(s) \equiv \ln P_{Ft}(s) - \ln P_{Ft-1}(s)$, $rmc_t(s) = \ln(MC_t(s)/P_t)$, $rmc^*_t(s) = \ln(MC^*_t(s)/P^*_t)$, $rp_{Ht}(s) = \ln(P_{Ht}(s)/P_t)$, $rp_{uFt}(s) = \ln(P_{uFt}(s)/P_t)$, and $q_t = \ln Q_t$. Equations 18-19 are sector-level domestic and import price Phillips curves.

An important conceptual point is that binding constraints – when $\mu_t(s)$ or $\mu^*_{ut}(s)$ are strictly positive – appear as "markup shocks" in reduced form. That is, binding constraints lead inflation to be higher than can be accounted for given parameters, real marginal costs, and expected inflation.

Whereas exogenous markups shocks in New Keynesian models typically are micro-founded via shocks to the elasticity of demand, the apparent "markups shocks" in this model have a different structural interpretation. Markup shocks arise here not because the competitive environment per se has changed – i.e., market structure and demand elasticities are time invariant – rather firm conduct changes when constraints bind. Firms cease to make price changes to target their ideal (flexible price, CES) markups; they instead "price to demand," based on willingness to pay for their constrained output.

The conclusion that capacity constraints influence firm conduct, holding market structure fixed, is not unique to this model of course. For example, Bertrand competition among symmetric firms leads to competitive (marginal cost) pricing when firms are unconstrained, but the Bertrand equilibrium features prices above marginal cost when capacity constraints bind [e.g., Tirole (1988)]. The dependence of firm conduct on whether constraints bind carries over to models with monopolistic competition, when capacity constraints bind at the firm level and firms know whether constraints bind when they set prices.[20]

With occasionally binding constraints, markups may rise and fall sharply as constraints are triggered and relaxed. This implies that the statistical behavior of markups and thus prices will be different during periods when constraints switch on/off than in ordinary times (when constraints are always slack). Building on this observation, it is possible to structurally identify whether variation in reduced-form markups is due to binding constraints or run-of-the-mill exogenous markup shocks.

[20] Fagnart et al. (1999) and Álvarez-Lois (2006) assume that (ex ante symmetric) firms set prices under monopolistic competition prior to realization of firm-specific, idiosyncratic demand shocks. Thus, all firms all set identical prices, based on the expectation of whether constraints will bind, rather than the ex post realization of whether they actually do. In our model, firms set prices with knowledge of whether their own constraint binds, like in Murphy (2017) and Boehm and Pandalai-Nayar (2022).

Looking ahead, we demonstrate this point in the Online Appendix, where we re-estimate the model including both potentially binding constraints and separate exogenous markup shocks. Intuitively, the unusual variation in realized inflation during the post-COVID period is statistically unlikely to be explained by exogenous markup shocks, given historical data that disciplines the stochastic process for those shocks. As such, Bayesian inference favors binding constraints over markup shocks in explaining recent inflation.

2.6.2 Capacity Constraints vs. Capital Utilization and Fixed Factors

We now pause to discuss micro-foundations for capacity constraints and the contrast between capacity constraints and alternative models of capital utilization and (quasi-) fixed factors.

Following Fagnart et al. (1999), output-based capacity constraints may be thought of as manifestation of putty-clay technologies. Specifically, suppose that firms make ex ante flexible decisions about the maximal quantity of variable inputs they can use in production, and that these decisions are locked in before shocks are realized. When the firm cannot freely substitute between variable inputs and other factors ex post, then its precomitted maximal input use defines the firm's capacity to produce output.[21] The capacity constraint that we adopt captures these ideas in a simple, reduced-form way.

Nonetheless, we are deliberately agnostic about the precise micro-foundation for the constraints, and we do not develop an explicit model with capacity investment in this paper. The reason is that we are principally interested in detecting whether constraints bind, and analyzing their impact on macroeconomic outcomes. That is, we seek to analyze the interaction of demand with potentially time-varying constraints. Further, we direct attention to capacity shocks to explain the evolution of capacity over time. Allowing for capacity shocks is important because data – specifically, diminished domestic goods production in the face of surging demand, as in Figure 2b versus Figure 3a – suggest that realized capacity levels may have been markedly lower during 2021-2022 than in normal times. There is ample narrative evidence of COVID-related disruptions to suggest shocks played an important role in this.

Turning to related models, capacity constraints differ from prior literature that emphasizes either variable capital utilization, or fixed factors in production. Starting with models of capital utilization, Greenwood et al. (1988) assume that higher rates of capital utilization lead capital to depreciate

[21] In Fagnart et al. (1999), the particular micro-foundation is that firms choose their capital stock and number of 'workstations', where workstations limit the number of workers the firm can hire. Further, while capital and workstations are substitutes ex ante, they are effectively perfect complements ex post. Broadening out from labor, similar constraints likely apply to input use generally – i.e., firms make ex ante decisions about their input sourcing strategy (which firms to source from, how to customize their production process to particular inputs, the quantity of inputs to order in advance, and so on). As a concrete example drawn from General Motors experience [Bunkley (2011)], an auto firm might design its car use a particular electronic controller, which cannot be easily substituted ex post. When the supply of that component is disrupted, then the quantity of cars produced is effectively constrained.

faster. As a result, higher utilization raises the effective marginal cost for the firm inclusive of increased capital depreciation, so utilization ultimately affects inflation through marginal costs.[22] Gilchrist and Williams (2000) instead assume that higher capital utilization is associated with less productive units of capital being drawn into production, again raising the effective marginal cost of production in the short run.

In a related vein, several recent papers have appealed to quasi-fixed factors to explain the differential behavior of the goods versus services sectors during the post-COVID period.[23] Lorenzoni and Werning (2023) introduce a scarce, non-labor input in fixed supply in their model of wage-price spirals, as a stand-in for factors constraining supply. Baqaee and Fahri (2022) introduce constraints on the allocation of factors (commonly interpreted as labor) to particular sectors, and di Giovanni et al. (2024) build on this formulation. Focusing on labor, Ferrante et al. (2023) introduce upward sloping labor supply curves for individual sectors in model with convex hiring costs.

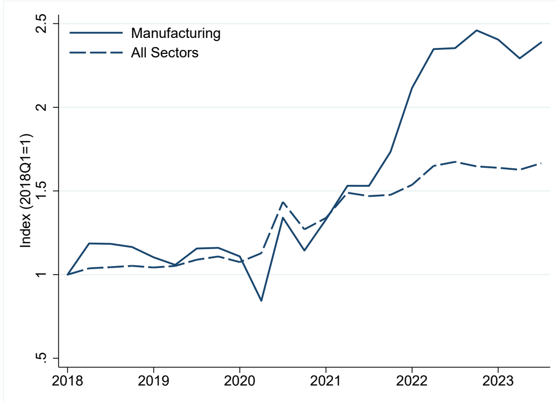
A common underlying narrative emerging from these models is that inflation is principally a cost-push phenomenon, where higher demand for output raises firms' marginal costs as firms move up their short run marginal cost curves, leading to higher output prices. In contrast, capacity constraints in our model lead to changes in pricing conduct, whereby price-cost margins increase (see further discussion in Section 2.6.3).[24] Further, whereas cost-push explanations typically require high realized output to drive firms up their marginal cost curves, the capacity mechanism we emphasize can easily rationalize low realized output together with high prices. Lastly, focusing on sectoral dynamics, cost-push models would explain high inflation for goods relative to services by frictions that limit the reallocation of factors from services to goods sectors. In contrast, our model features free mobility of labor (the only primary factor in the model) across sectors. Consistent with this assumption, relative wages across goods and services sectors were quite stable in the US in recent years, while wages actually grew faster in services than manufacturing in advanced countries overall [International Monetary Fund (2022), Chapter 2 annex]. Motivated by all these observations, we provide space for a separate capacity constraints mechanism to rationalize data.

[22]The vast majority of the literature on capital utilization takes log-linear approximations, using functional form assumptions introduced by Greenwood et al. (1988). Then, the standard log-linear Phillips Curve relationship between marginal costs and inflation (equivalently, utilization and inflation) holds. These log-linear approximations struggle to explain the highly non-linear response of inflation observed in recent data.

[23]Amiti et al. (2023) study the interaction of negative labor supply shocks and shocks to import prices. While there is no explicitly fixed factor in this framework, the negative shock to labor supply effectively restrains labor supply in a way that mimics a fixed factor.

[24]To be clear, our model embeds various cost push forces – via wages, imported input prices, and upstream input prices that all feed into marginal cost – and these are accounted for in the quantitative analysis.

Figure 4: Corporate Profits per Unit of Gross Output



Note: Corporate profits (with inventory valuation adjustments) and gross output are from the US Bureau of Economic Analysis (series identifiers: N400RC and A390RC). The corporate profits per unit of gross output are are reported as an index, measured relative to their value in 2017Q1.

2.6.3 Profits

Our model implies that price-cost margins (realized markups) are high when firms face binding constraints. To examine the plausibility of this channel, we turn to data on profits per unit of output, which serves as an observable proxy for price-cost margins. To formalize this link, note that the absolute markup is equal to profits per unit of output in the steady state: $P_t(s) - MC_t(s) = \frac{\Xi_t(s)}{Y_t(s)}$, where $\Xi_t(s) \equiv P_t(s)Y_t(s) - MC_t(s)Y_t(s)$ is the profit of the representative producer in sector s. Thus, tracking profits per unit over time sheds light on how markups are changing.

In Figure 4, we plot indexes of US corporate profits per unit of gross output for both the manufacturing sector and the aggregate private sector.[25] The takeaway is that profits per unit escalated sharply for manufacturing firms during the pandemic recovery, coinciding with the takeoff in goods price inflation and widespread complaints about binding (supply chain) constraints that limited production. Further, total profits (profits per unit times quantity sold) were at historically high levels in 2021. This pattern of high profitability alongside high inflation is a natural outcome of binding (domestic) constraints in our model. More recently, profit margins appear to be falling as inflation has declined in 2023 [Kerr (2023)]. In Section 3.4.2, we will show that occasionally binding constraints provide a parsimonious explanation for these profit dynamics, by replicating this figure.

3 Accounting for Inflation

We now apply the model to parse recent data. We describe the procedure we use to estimate the model in Section 3.1, with additional details in the Online Appendix. Then, we discuss data, cali-

[25]This corporate profit measure omits profits attributable to non-corporate entities; We focus on corporate profits because data is available for manufacturing on a quarterly frequency in the national accounts.

bration, and estimated parameters in Section 3.2. Section 3.3 reviews model fit. We analyze what the model tells us about recent inflation in Section 3.4.

3.1 Estimation Framework

Referring back to Section 2.5, the impact of a given structural shock in the model depends on whether constraints bind today following the shock, as well as the duration that constraints are expected to continue to bind into the future following that shock. To make this dependence explicit, let us define a set of regimes ($\mathbb{R}_t$), which record which constraints are binding at a given point in time: $\mathbb{R}_t = \left\{\mathbf{1}\left(Y_t(1) = \bar{Y}_t(1)\right), \mathbf{1}\left(Y^*_{Mt}(1) = \bar{Y}^*_{Mt}(1)\right)\right\}$, where the indicator functions switch on when individual constraints bind. Given a sequence $\mathbb{E}_t\left\{\mathbb{R}_{t+j}\right\}$ for $0 \leq j \leq J$, together with the assumption that $\mathbb{E}_t\left\{\mathbb{R}_{t+j}\right\} = \{0, 0\}$ for $j > J$, we can solve for an equilibrium path for $\{X_t\}$, using the method described in Cagliarini and Kulish (2013) and Kulish and Pagan (2017).

Building on this idea, we re-parameterize the model solution in a convenient way. Specifically, let us define the duration that constraints are expected to bind from date t forward as $\mathbf{d}_t = [d_t, d^*_t]$, where each entry is a non-negative integer that records the number of periods that the domestic (d_t) or foreign constraint (d^*_t) binds. By convention, d_t and d^*_t take on zero values when constraints are slack today and expected to remain so in the absence of future shocks, and they are positive when they are binding today. As in Guerrieri and Iacoviello (2015), we construct policy matrices under the assumptions that agents know the state (X_{t-1}) and the current realization of the shocks (ε_t), but that they do not anticipate that future shocks will occur. Under these assumptions, $\mathbf{d}_t$ summarizes all the information about the anticipated sequence of regimes that is needed to solve for equilibrium responses to a one-time shock in our model. Specifically, constraints may switch on immediately in response to shock at date t, then bind for some (non-negative) number of consecutive periods, and switch off thereafter. In the absence of future shocks, constraints do not then switch on again in periods after they switch off (e.g., following a shock ε_t, constraints cannot be slack at date t and then binding at date $t + 1$).[26] With these observations, we re-write the model solution directly in terms of durations:

$$X_t = \mathbf{J}\left(\mathbf{d}_t, \theta\right) + \mathbf{Q}\left(\mathbf{d}_t, \theta\right) X_{t-1} + \mathbf{G}\left(\mathbf{d}_t, \theta\right) \varepsilon_t, \tag{20}$$

where duration $\mathbf{d}_t$ implies a specific anticipated sequence of regimes over time.

Following Kulish et al. (2017), Kulish and Pagan (2017), and Jones et al. (2022a), our estimation

[26]To be careful, this is not a general property of models with potentially binding constraints, but rather one that holds given the structural assumptions in our model about behavior and shock processes. While we lack a general proof of this property, we verify it holds numerically in the model in practice, and we can demonstrate that imposing this criterion in the estimation procedure is reasonable via simulation analysis. One could capture a more complex structure of potential regime changes via introduction of additional parameters (e.g., durations for binding constraints that start one period forward), at the cost of added computational complexity.

framework exploits the fact that durations enter the policy function like parameters. As is standard, let us assume that observables (S_t) are linearly related to the unobserved state, as in $S_t = \mathbf{H}_t X_t + \nu_t$, where ν_t is an i.i.d. vector of normally distributed measurement errors. Given $\mathbf{d} \equiv \{\mathbf{d}_t\}_{t=1}^{T}$ and θ, we can construct the piecewise linear solution with time-varying coefficients, and then apply the Kalman filter to construct the Likelihood function $\mathcal{L}(\theta, \mathbf{d} | \{S_t\}_{t=1}^{T})$. We put priors over structural parameters and independent priors over durations to construct the posterior, and then estimate the model via Bayesian Maximum Likelihood.

In implementing this approach to estimation, we are careful to account for the fact that the duration of binding constraints is an equilibrium object in the model – i.e., $\mathbf{d}_t$ depends on both the state X_{t-1} and current shock ε_t in our model. Thus, we impose a rational expectations equilibrium restriction on admissible durations, which requires that agents' forecasts about how long constraints bind following a given shock are consistent with equilibrium model responses. To impose this restriction, we proceed as follows. For each proposed duration and parameter draw, we filter the data for smoothed shocks. We then evaluate whether the equilibrium model response to those smoothed shocks is consistent with the proposed duration draw. We retain the proposed draw if this requirement is satisfied; otherwise, we reject it and draw again.

In the Online Appendix, we study the performance of this procedure using simulated data, for which we know the true data generating process and the exact incidence of endogenously binding constraints. First, we confirm that our estimation procedure is able to recover unobserved durations from the observables that we use, by directly examining likelihood functions. Then, we also show that the reduced-form multipliers implied by the duration and parameter estimates align with true latent multipliers, which summarize the impacts of binding constraint on inflation, our key outcome.

Lastly, as a practical matter to restrict the size of the parameter space, we impose priors that allow capacity constraints to bind only periods from 2020:Q2 forward. Put differently, we impose dogmatic priors that assign zero probably to binding constraints prior to 2020:Q2, thus focusing on the role of capacity in explaining the unusual post-pandemic inflation dynamics.[27]

3.2 Data and Parameters

To populate Y_t, we collect standard macro variables together with particular series that serve to identify whether constraints are binding and shocks to them. Among standard macro variables, we include consumption price inflation and the growth rates of consumption expenditure for goods and services. We also use data on aggregate nominal GDP growth, the growth rate of (real) industrial production (which we treat as a proxy for output of the goods sector), and labor productivity growth

[27] As a robustness check, we have estimated the model allowing constraints to potentially bind starting in 2018:Q1, prior to the pandemic. We find that the mode of estimated durations before 2020:Q2 is zero, and that the mode of estimated durations after 2020:Q2 is not affected by the initial date when capacity constraints can bind.

by sector (measured as real value added per worker).[28] On the international side, we use data on import price inflation for consumption goods, and we proxy input price inflation in the model using data on inflation for imported industrial materials (excluding fuels). We then also use data on the growth of total expenditure on imported consumption goods and imported materials inputs (again excluding fuels), which we associate with imported inputs of goods.[29]

These data are all obtained from quarterly US national accounts produced by the Bureau of Economic Analysis, with the exception of labor productivity data from the Bureau of Labor Statistics and industrial production from the Federal Reserve Board (G.17 program). Having constructed growth rates for individual variables from the first quarter of 1990 through the fourth quarter of 2023, we detrend the data by removing the mean growth rate from each series. Finally, because our estimation sample includes a significant period during which interest rates are at the zero lower bound, we use data on the "shadow Fed Funds rate" to estimate parameters in the monetary policy rule.[30]

We present the full set of parameters for the model in the Online Appendix, which we obtain through a mix of estimation and calibration. We calibrate key value shares in the model – e.g., consumer expenditure, input use, export and import shares, etc. – to match US national accounts and input-output data. We set a subset of the structural parameters to standard values from the literature, including preference parameters and some elasticities of substitution.

We also calibrate the level of excess capacity for domestic and foreign firms, setting $\bar{Y}_0(1) = 1.05Y_0(1)$ and $\bar{Y}^*_{M0}(1) = 1.10Y^*_{M0}(1)$. These levels are chosen to be sufficiently high that constraints are slack prior to 2020:Q2, given our maintained assumption that there are no capacity shocks prior to that period.[31] Further, note that the model and data allows us to estimate the level

[28] We use data on labor productivity growth in manufacturing and total (private sector) labor productivity growth from the Bureau of Labor Statistics. We assume that labor productivity growth in manufacturing coincides with goods labor productivity (growth in real value added per worker) in the model, while also matching aggregate (economy-wide) labor productivity growth in the model. While the definition of industrial production and goods output do not align exactly, the dynamics of gross output for the goods sector and industrial production are similar.

[29] We use data for consumer goods (except food and automotive) to proxy for consumption imports, and we construct proxies for imported inputs (excluding fuels) by removing the subcategory of petroleum and products from industrial materials and supplies using standard chain index formulas and auxiliary NIPA data on the sub-categories of imports.

[30] During periods where the nominal Fed Funds rate is at zero, we replace it with the shadow rate from Wu and Xia (2016): `https://www.atlantafed.org/cqer/research/wu-xia-shadow-federal-funds-rate`. Changes in the shadow rate capture the consequences of unconventional policy actions taken by the Federal Reserve, such as forward guidance or quantitative easing policies. We have checked the results using an alternative shadow rate series from Jones et al. (2022b) as well, which yields similar results. We have also checked the results by re-estimating the model with an occasionally binding, zero lower bound constraint on the policy rate, as in Kulish et al. (2017) and Jones et al. (2022a).

[31] This amount of domestic excess capacity is consistent with historical fluctuations in capacity utilization for the US, as measured by the Federal Reserve's G.17 data series, for which the maximal value for capital utilization about five percent higher than the minimum. Further, cyclical fluctuations in this capacity utilization measure are almost entirely driven by changes in industrial production itself, rather than the Fed's estimate of capacity (based on firm survey data). Thus, our calibration accommodates historically normal fluctuations in industrial production, absent shocks to capacity.

of capacity that actually prevailed during the pandemic. Alternative values for steady state capacity then re-scale the size of the capacity shocks needed to achieve this realized capacity level.[32]

Turning to the final set of parameters, we estimate (a) the elasticities of substitution between home and foreign goods, in consumption and production separately; (b) the parameters in the extended Taylor rule governing the response of interest rates to inflation and output, as well as interest rate inertia, (c) parameters governing the stochastic processes for exogenous variables, and (d) the variance of measurement errors. Regarding (c), we assume that exogenous variables evolve according to AR1 stochastic processes.

We obtain an estimated mean value for the elasticity of substitution between home and foreign goods of about 1.5 in consumption and 0.5 for inputs, so consumer goods are substitutes while inputs are complements. These values are not far from standard values estimated using aggregate time series variation in the macroeconomic literature, though there is limited prior work that distinguishes consumption and input elasticities. We find that the policy rule displays inertia, and it responds to both inflation and output gaps with reasonable magnitudes. There is significant persistence in most exogenous variables, and measurement error variances are plausible. See the Online Appendix for the estimated parameters.

3.3 Model Fit

Applying the quantitative model framework to the data, we construct Kalman-smoothed values for endogenous variables and observables. In Figure 5, we plot data and smoothed values for several key observables – goods, services, and aggregate price inflation for consumers, and imported input price inflation – over the 2017-2023 period, where each data point is the annualized value of quarterly inflation. To compute the smoothed inflation series, we take 1000 draws from the posterior distribution for model parameters, compute Kalman-smoothed inflation for each draw, and then plot statistics (the median, 5th, and 95% percentiles) for the distribution of smoothed values.

The model fits the dynamics of aggregate consumer price inflation well, accounting for essentially all of the four percentage point increase in headline inflation after 2020 (Figure 5a).[33] It also

[32]Consistent with this observation, the level of calibrated steady state capacity is not an important parameter in understanding the key quantitative results. To demonstrate this robustness, we estimate steady-state capacity levels directly in the Online Appendix, using data from the pandemic period, and show that our main counterfactual results go through with this alternative parameterization.

[33]Recall that aggregate consumer price inflation is treated as an unobserved variable. In the model, it is constructed by aggregating sector-level consumer price growth using fixed (steady-state) expenditure weights. In the data, however, the PCE deflator is a chain-weighted index, which features time-varying weights. Thus, part of the discrepancy between aggregate inflation in the model and data is likely due to differing index number concepts. Specifically, the dramatic increase in the goods expenditure share, combined with high goods price inflation, likely pushed measured inflation up relative to our fixed-weight index. Going forward, we focus entirely on decomposing model-based measures of inflation, so we do not belabor this point.

accounts well for the two percentage point rise in inflation for the services sector (Figure 5b). Because goods price inflation is substantially more volatile than that for services, the model attributes more of its variation to measurement error. Nonetheless, smoothed values for goods price inflation also track the data well (Figure 5c). The model replicates the initial surge in goods price inflation in 2021, and goods price inflation then remains elevated into 2022. The model undershoots the level of goods price inflation in 2022, attributing the gap to measurement error, but captures the general hump shape in goods price inflation during the post-pandemic period. The model also matches inflation for imported goods inputs well (Figure 5d), matching both levels and dynamics closely.

We present similar figures illustrating model fit for the remaining observables in the Online Appendix. Together with the inflation figures here, we assess that the model captures the behavior of economic variables well during the pandemic, so it is a useful laboratory for exploring the driving forces underlying the inflation surge.

3.4 Explaining the Inflation Surge

We provide three sets of results. The first two illustrate the role of constraints in explaining inflation. First, we examine the dynamics of the multipliers on the constraints. Second, we present counterfactuals in which we switch off the constraints, comparing model responses to the same set of shocks with and without constraints. The third set of results focuses on how individual shocks and constraints shape inflation outcomes, both individually and via interactions between them.

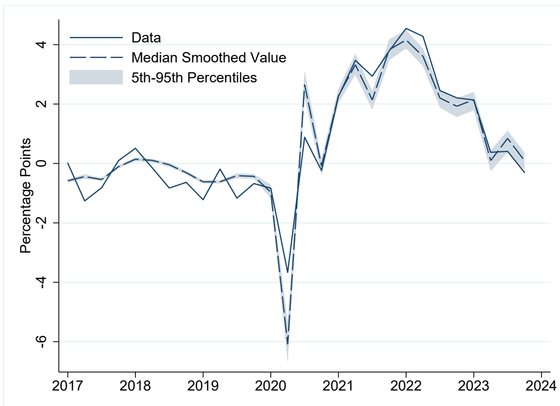
3.4.1 Multipliers on Constraints

To start, we can directly illustrate the impact of constraints by examining the smoothed value of multipliers on the domestic and foreign constraints. Because the multipliers themselves do not have intuitive economic units, we plot the reduced-form markup shocks implied by the value of the multipliers – given by $\left(\frac{\varepsilon}{\phi(s)}\frac{P_0}{P_{H0}(s)}\right)\hat{\tilde{\mu}}_t(s)$ in Equation 18 and $\left(\frac{\varepsilon}{\phi(s)}\frac{P_0}{P_{uF0}(s)}\right)\hat{\tilde{\mu}}^*_{ut}(s)$ in Equation 19 – which summarize the impulse of binding constraints for domestic and import price inflation. As is evident, the values of the multipliers rise in 2021, coincident with the rise in headline inflation.[34] On the import side, constraints appear to be slack in 2020, then bind sharply at the start of 2021, relax somewhat, then bind sharply again into 2022, and ease in the latter half of 2022. Domestic

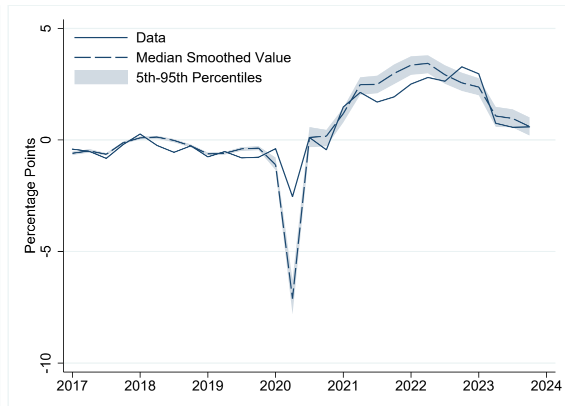
[34] We place positive mass in our priors on positive values for d_t only starting in 2020:Q2, so multipliers are identically zero before that date. Further, while multipliers are typically positive, they sometimes take on negative values in the simulations. This is due to model approximation error, due to the piecewise linear solution technique that we employ. When constraints bind, the multipliers are computed as residuals in the log-linearized Phillips Curves. As such, the computed multipliers are approximations to the exact equilibrium multipliers; further, we do not impose a zero lower bound on them, as would be required in the full non-linear solution to the model. Despite this, the estimated multipliers are typically positive, consistent with the underlying theory.

Figure 5: Consumer Price Inflation in Model and Data

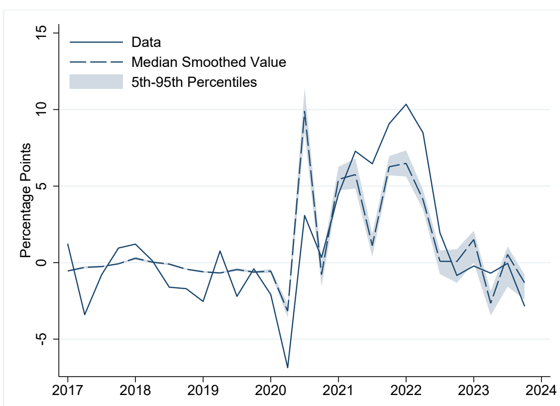
(a) Aggregate Consumer Inflation



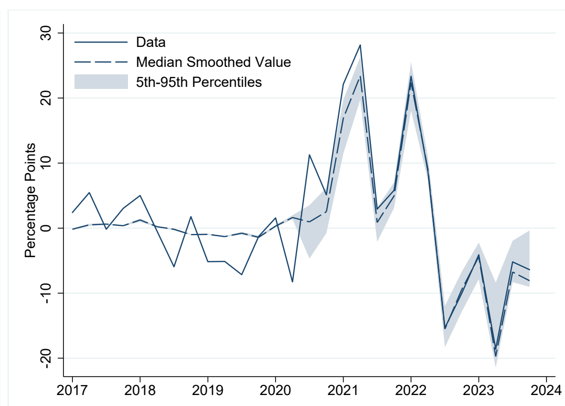
(b) Consumer Services Inflation



(c) Consumer Goods Inflation



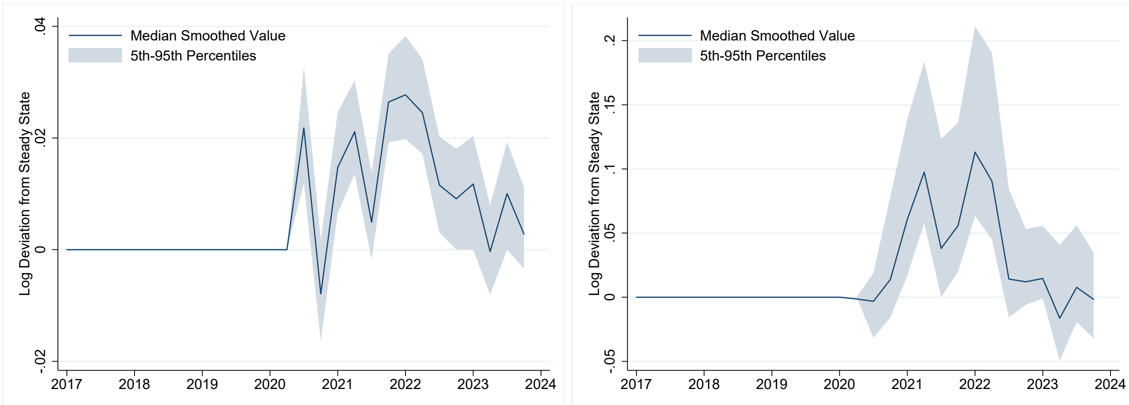
(d) Inflation for Imported Goods Inputs



Note: Inflation at each date is the annualized value for demeaned quarterly inflation, in percentage points. If demeaned quarterly inflation is $\pi_t(s) = \ln P_t(s) - \ln P_{t-1}(s)$ where t indexes quarters, then the annualized inflation rate is $4\pi_t(s)$. Data is raw data. We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, and then plot the median smoothed value as the dashed line. We shade the area covering the 5% to 95% percentile for smoothed values (the interval is imperceptibly small prior to 2020).

Figure 6: Smoothed Values for the Reduced-Form Markup Shock Implied by the Multipliers on Constraints

(a) Domestic Constraint (b) Foreign Constraint



Note: Figure 6a plots composite variable $\left(\frac{\varepsilon}{\phi(s)} \frac{P_0}{P_{H0}(s)}\right) \hat{\hat{\mu}}_t(s)$ and Figure 6b plots composite variable $\left(\frac{\varepsilon}{\phi(s)} \frac{P_0}{P_{uF0}(s)}\right) \hat{\hat{\mu}}^*_{ut}(s)$, which are the reduced-form markup shocks in domestic and import price Phillips Curves induced by binding constraints. We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, and then plot the median smoothed value as the solid line. We shade the area covering the 5% to 95% percentile for smoothed values.

multipliers fluctuate in 2020 with gyrations in the US economy, but are near zero heading into 2021. They rise steadily through 2021 into 2022, and then slacken into 2023.

While there is limited external data to which we can benchmark the estimated multipliers, we show that their joint dynamics align well with fluctuations in the New York Federal Reserve's Global Supply Chain Pressure Index over the post-2020 period in the Online Appendix. For both multipliers, the high frequency dynamics also correspond to fluctuations in goods price inflation and imported input price inflation in Figure 5, which foreshadows the quantitative role of the constraints in explaining inflation.

3.4.2 Relaxing Constraints

We now provide counterfactual analysis as to how inflation would have evolved in the absence of capacity constraints, given the path of realized shocks that we infer hit the US economy after 2020.

To describe this exercise more precisely, the mechanics of each iteration are as follows. We first draw model parameters from the estimated posterior distributions, including the durations for binding constraints. Given these parameters, we apply the Kalman-filter to the data and construct smoothed model outcomes and shocks. Note that we construct smoothed shocks here assuming that constraints are potentially binding, in line with posterior duration estimates. Using these smoothed shocks, we then simulate the path of the economy under the counterfactual assumption that constraints are slack throughout, such that the solution conforms to the unconstrained equilibrium dynamics of the model. We repeat this procedure for one thousand posterior draws, and we plot statistics (means and percentiles) across these simulations in Figures 7 and 8.

Figure 7 presents results for consumer price inflation. The figures present raw data on annualized values of (de-meaned) quarterly inflation, along with data from counterfactual simulations in which we allow for measurement error in these observables.[35] In Figure 7a, we see that realized inflation for consumer goods is substantially higher than counterfactual inflation with slack capacity constraints during 2021 and into 2022, with the absolute gap peaking near six percentage points in early 2021. Put differently, given the shocks we infer from data, binding constraints account for about half of the acceleration in goods price inflation from 2020:Q2 through 2021:Q2. Likewise, they appear to explain about half of the decline in goods price inflation in the latter half of 2022.

Under the hood, these inflation outcomes are tied to the impact of binding constraints in holding back production of domestic goods and foreign goods inputs. In Figure 8a, we plot the path for smoothed domestic goods output along with counterfactual output. As is evident, in the absence of constraints, goods output would have risen significantly in 2021 relative to its pre-pandemic level, as a result of the other shocks (principally, demand shocks) that hit the economy. The fact that output did not rise in reality speaks directly to the role of constraints. Output of foreign goods inputs is similarly constrained in Figure 8b. Correspondingly, smoothed inflation for both domestically-produced goods and foreign-produced inputs is substantially higher than counterfactual inflation in Figures 8c and 8d.

Interestingly, binding constraints also play an important role in driving price inflation for services in Figure 7b. While services price inflation initially accelerates due to the underlying shocks, it is between one and two percentage points higher in 2021 as a result of binding constraints. This reflects the fact services use goods as inputs, so there is a direct inflation spillover from binding constraints in the goods sector via input-output linkages. Further, binding constraints serve to tighten the labor market as well, as the price increases they trigger substitution from goods inputs toward labor in production.

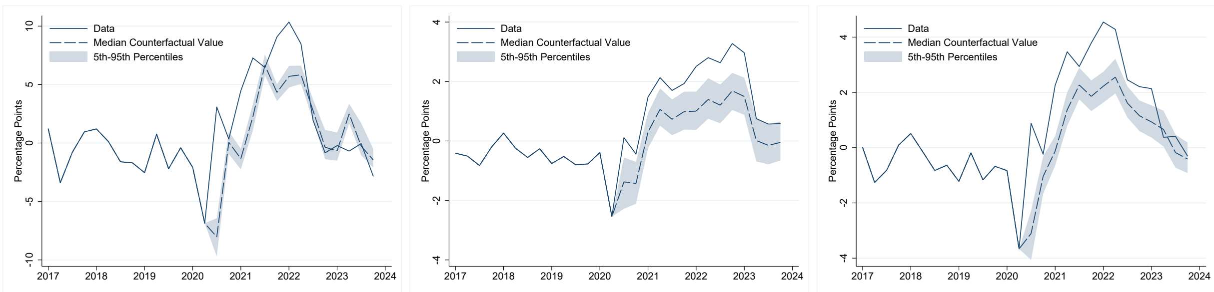
Adding up these results in Figure 7c, headline consumer price inflation is between one and two percentage points higher than counterfactual inflation during 2021-2022. And binding constraints account for about one half of the acceleration in headline goods price inflation. The relaxation of constraints, starting in the latter half of 2022 and extending into 2023, then leads actual and counterfactual inflation converge again.

Finally, we revisit the discussion about profits per unit. In Figure 4, we presented an index of

[35] For each iteration, we draw the variance of the measurement error from the posterior and filter the data for smoothed shocks. We then add a draw from the measurement error to the smoothed counterfactual endogenous variables to get counterfactual values for the observables that are comparable to data.

Figure 7: Counterfactual Consumer Price Inflation without Capacity Constraints

(a) Goods Inflation (b) Services Inflation (c) Aggregate Inflation



Note: We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, add measurement error to the observables, and then plot the median smoothed value as the solid line. We shade the area covering the 5% to 95% percentile for smoothed values.

nominal profits per unit of gross output for manufacturing and the aggregate economy. In Figures 8e and 8f, we present analogous results from the model for goods and services.[36] Similar to the data, our model yields a sharp increase in profits for the goods sector during the 2021-2022 period, even though this is not a targeted data moment. In contrast, the counterfactual economy with slack constraints yields no such goods profit surge. Moreover, profits per unit are essentially flat through the pandemic period (outside the 2020 spike), for both the economies with and without capacity constraints. We conclude that the model provides a plausible explanation for the run-up in profits for goods producers that occurred alongside the inflation takeoff, where both are explained in large measure by binding capacity constraints.

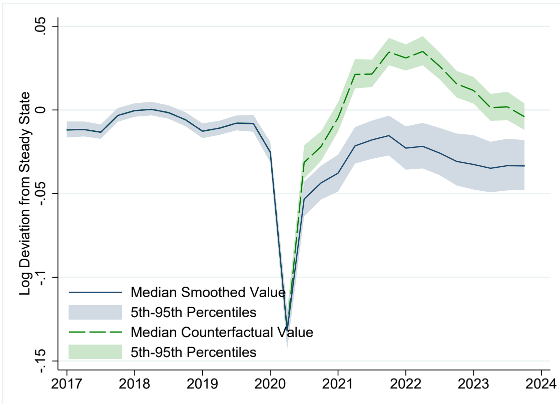
3.4.3 Decomposing the Role of Individual Shocks

We now examine the role of individual shocks in explaining inflation outcomes. We start by filtering smoothed shocks from the data. Specifically, we take a draw from the posterior distributions for structural parameters and durations, use this draw to parameterize the state equation (Equation 20), and then apply the Kalman filter to the data to construct smoothed shocks. We then feed the smoothed shocks into the structural model (summarized by Equation 17) to compute counterfactual model outcomes. In each counterfactual scenario, we solve for the simulated equilibrium path using Dynare's OccBin procedure, which ensures that whether constraints bind at particular points in time in response to shocks is endogenous. We repeat this procedure 1000 times, compute the

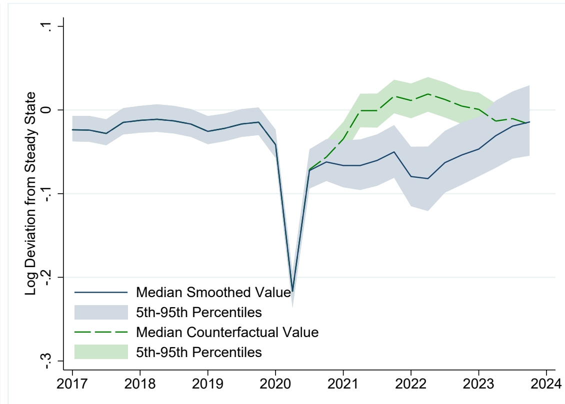
[36] In the model, the log change in nominal profits per unit of output from a given base period ($t = 0$) is given by: $\left[\hat{\Xi}_t(s) - \hat{y}_t(s)\right] - \left[\hat{\Xi}_0(s) - \hat{y}_0(s)\right] = [\hat{p}_{Ct} - \hat{p}_{C0}] + \epsilon\,[\widehat{rp}_t(s) - \widehat{rp}_0(s)] - (\epsilon - 1)\,[\widehat{rmc}_t(s) - \widehat{rmc}_0(s)]$, where $\hat{p}_{Ct} - \hat{p}_{C0} = \sum_{s=0}^{t} \pi_{Cs}$. We add trend inflation to these log changes to make it comparable to the data in Figure 4, and then we convert the log change to levels to plot the index.

Figure 8: Counterfactual Quantities, Inflation, and Profits without Capacity Constraints

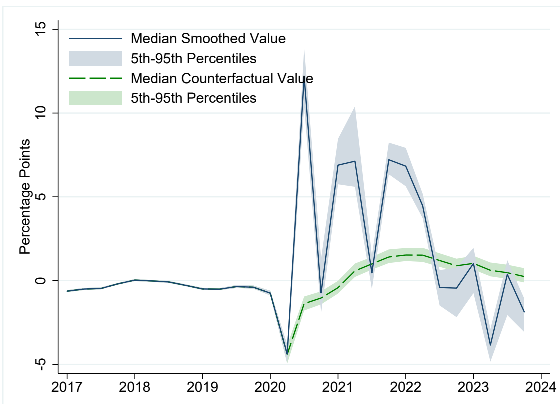
(a) Domestic Goods Output ($Y_t(1)$)



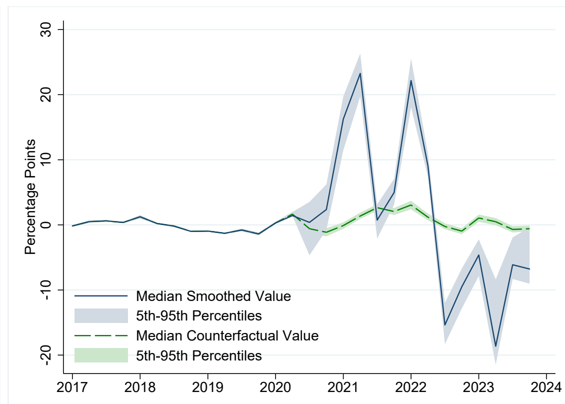
(b) Imported Goods Inputs $Y^*_{Mt}(1)$



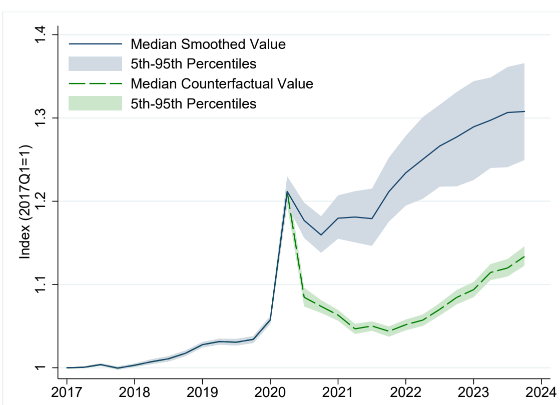
(c) Domestic Goods Price Inflation ($\pi_t(1)$)



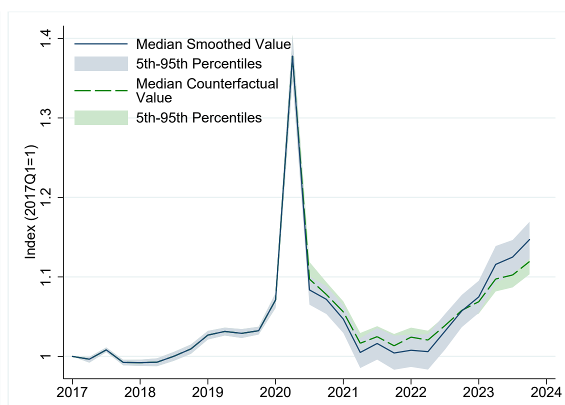
(d) Imported Goods Input Inflation ($\pi_{Mt}(1)$)



(e) Profits per Unit: Goods



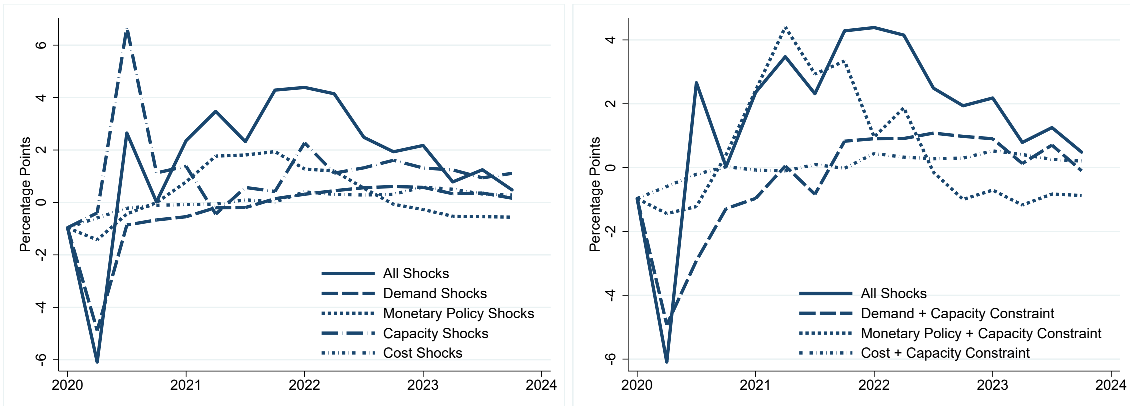
(f) Profits per Unit: Services



Note: We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, and then plot the median smoothed value as the solid line. We shade the area covering the 5% to 95% percentile for smoothed values. Counterfactual assumes that constraints are slack in all periods.

Figure 9: Counterfactual Consumer Price Inflation for Individual Shocks

(a) Individual Shocks (b) Individual Shocks with Capacity Constraints



Note: In Panel (a), each series represents the simulated path of consumer price inflation (quarterly value, annualized) for the indicated subset of smoothed shocks during 2020-2023. In Panel (b), each series represents the simulated path of inflation for the indicated subset of shocks with capacity constraints set to their smoothed value.

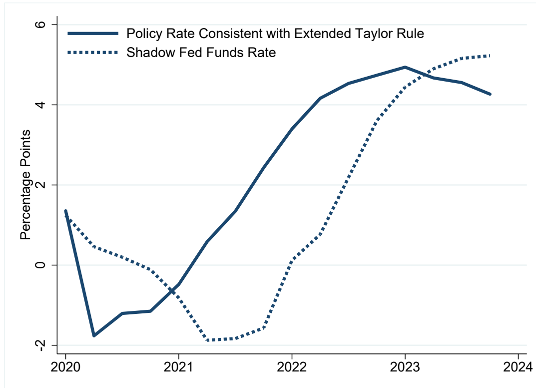
median across the simulated counterfactual series, and report the results in Figure 9.

In Figure 9a, we plot the path of aggregate consumer price inflation following four types of shocks, each fed individually into the model: demand shocks (including both the discount rate and goods-biased preference shocks), monetary policy shocks, capacity shocks, and cost shocks (including domestic productivity and foreign cost shocks).[37] In 2020, temporary negative demand shocks yield a decline then rebound of inflation in 2020, while negative capacity shocks in the third quarter of 2020 (as the economy re-opens) raise inflation. Into 2021, however, no single shock appears to play a dominant role in explaining the path of inflation on its own.

In Figure 9b, we plot a second set of counterfactuals, which illustrate how particular shocks interact with the capacity constraints. Because the model is non-linear, interactions between the shocks and constraints are important for parsing the underlying sources of inflation. To illustrate the impact of demand, monetary policy, and cost shocks, we feed these shocks into the model together with shocks to capacity, which reproduce the path of capacity that we have filtered from data. Call the simulated outcome $\tilde{X}_t^j$ for shock j. We then separately simulate the impact of those capacity shocks alone, as above, and call this simulated series X_t^{cap}. We then plot the difference across these simulations: $\tilde{X}_t^j - X_t^{cap}$. In doing so, we are capturing the non-linear interaction of each shock with the capacity constraints, while differencing away the direct impact of capacity

[37] In the demand, monetary policy, and cost shock counterfactuals, domestic and foreign capacity are set to their steady state levels in all periods. Constraints therefore bind only if the filtered shocks push the economy to trigger the constraints. In the capacity shock scenario, the shocks themselves move the realized level of capacity around, such that capacity shocks themselves trigger the constraints. The final series is the value for inflation when all shocks are fed simultaneously into the model.

Figure 10: Comparing the Policy Interest Rate to the Extended Taylor Rule



constraint shocks, which is common across all these simulations.

In contrast to Figure 9a, monetary policy stands out in Figure 9b. While monetary policy shocks play essentially no role in 2020, expansionary monetary policy shocks in 2021 – combined with tight capacity, as inferred from the data – lead inflation to increase by about 4 percentage points in 2021. Further, this accounts for essentially all of the surge in inflation, with both demand and cost shocks playing small roles. Into 2022, the monetary policy shocks dissipate – as the Federal Reserve raises interest rates – and inflation falls rapidly. Thus, the dynamics of monetary policy, interacted with shocks that tightened capacity, appear to be the most important driving force behind the rapid rise and subsequent fall in inflation.

To examine the conduct of monetary policy directly, we compare the realized (shadow) Fed Funds rate to the (counterfactual) policy rate required by the extended Taylor rule in our model.[38] In Figure 10, the realized policy rate was substantially lower than that called for by the extended Taylor rule during 2021-2022. Plainly, the Federal Reserve left rates low in 2021, in the face of rapidly rising inflation. In 2022, it raised rates rapidly, bringing them back in line with the extended Taylor rule by early 2023. Coincident with this policy tightening, inflation falls rapidly in both our simulations and the data.

4 Extensions

In this section, we present three extensions. First, we examine whether our results change when we account more carefully for energy shocks. Second, we examine the role of fiscal policy shocks, in addition to monetary policy shocks, in a two-agent New Keynesian (TANK) model. Third, we

[38] We use smoothed inflation ($\hat{\tilde{\pi}}_t$) and output ($\hat{y}_t$) to compute the policy rate ($\tilde{i}_t$) using the extended Taylor rule: $\tilde{i}_t = \varrho_i \tilde{i}_{t-1} + \omega(1-\varrho_i)\hat{\tilde{\pi}}_t + (1-\varrho_i)\varrho_y \hat{y}_t$, with parameters set to estimated values. We plot the median value of $\tilde{i}_t$ across 1000 model simulations, each with a different draw from the posterior distribution of model parameters.

enrich the labor market, which allows us to study the impact of labor market disruptions during the pandemic period.

4.1 Accounting for Energy Shocks

During 2021-2022, global energy prices escalated, as strong demand for energy combined with supply disruptions (e.g., following from the Ukraine war) to drive energy prices up. Further, since the middle of 2022, energy prices have receded rapidly as inflation has cooled. A natural question arises then about whether the dynamics of inflation that we attribute to occasionally binding constraints might instead be driven by these energy price fluctuations.

To frame this discussion, note that our model abstracts from the peculiar features of energy markets – i.e., we do not attempt to model energy prices, production, and demand explicitly. Therefore, we think it reasonable to estimate our model using data that also excludes energy prices. In part, we have already done this in prior sections, in that we have stripped out petroleum and fuels when we constructed the price index for imported materials. Here we also remove energy prices from the domestic price indexes used in estimation – constructing PCE inflation for goods and services, excluding energy. Specifically, we remove prices for "gasoline and other energy goods" (which includes motor vehicle fuels and lubricants, fuel oil, and other fuels) from the goods PCE price index, and then we remove prices for electricity and gas utilities from the services PCE price index. We then re-estimate the model using the modified domestic price indexes.

In Figure 11a, we plot the adjusted PCE inflation series for goods prices and overall consumption.[39] Goods price inflation is virtually indistinguishable with/without energy through 2021:Q3, during the initial inflation takeoff. Thereafter, energy prices push inflation up during early 2022, and then rapidly bring goods price inflation down thereafter. Nonetheless, the basic inverted U-shape for goods price inflation appears in both series, with non-energy goods price inflation falling from 8 percent to near zero during the course of 2022. Overall PCE price inflation then reflects these deviations in goods price inflation.

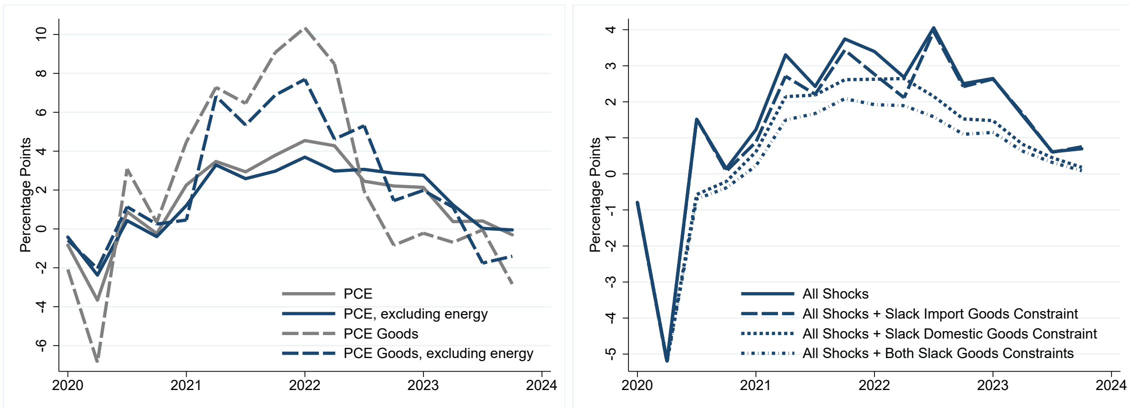
In Figure 11b, we investigate the role of these differences for our conclusions about the role of constraints in explaining inflation dynamics. The simulations here follow the same scheme as in Section 3.4.3: we compare simulated inflation when all shocks are fed through the model to counterfactual inflation when one or both constraints are relaxed. As in the prior counterfactuals, binding constraints continue to play a large quantitative role in driving inflation. Further, note that here we decompose the role of binding constraints for domestic goods production versus imports. Both constraints appear to be important, though the domestic constraint has a larger impact on inflation than the import constraint in most periods.

[39] Services inflation looks very similar with and without energy prices, so we omit it for clarity in the figure.

Figure 11: Accounting for Energy Shocks

(a) PCE Inflation with and without Energy

(b) Capacity Shocks plus Individual Shocks



Note: In panel (b), each series represents the simulated path of consumer price inflation (quarterly value, annualized) for all shocks and the indicated set of constraints during 2020-2022.

4.2 Fiscal Shocks in a TANK Model

In addition to monetary policy, the US government used fiscal policy to blunt the economic impact of the pandemic. First, there was a large increase in transfer payments, including unemployment benefits, tax rebate checks, and social spending [Romer (2021)]. Second, this spending was financed by debt: there was a large, temporary increase in the federal budget deficit. Both elements of this fiscal response likely stimulated the economy, so omitting them could potentially lead us to overstate the role of constraints and monetary policy in driving inflation.

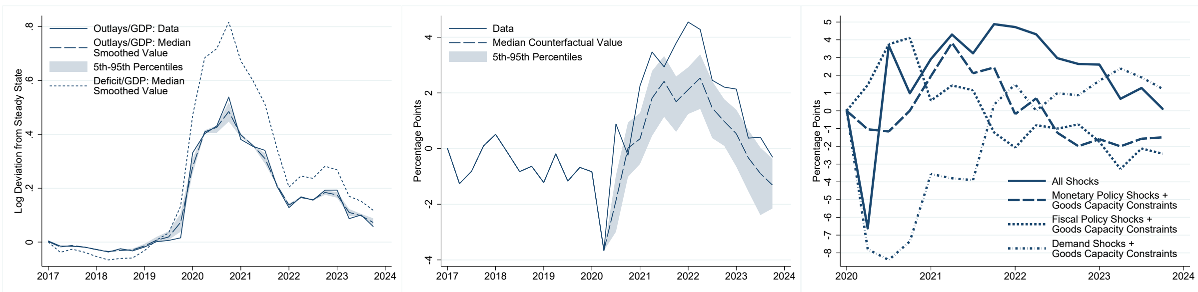
To address these concerns, we extend the model to incorporate fiscal policy (details are provided in the Online Appendix). First, we add a fiscal authority to the model, which collects income taxes and makes transfer payments to households, and we assume it is able to borrow to cover budget deficits.[40] To ensure that the fiscal authority remains solvent (i.e., to stabilize the stock of real debt) in the long run, we adopt the following fiscal rule: $\hat{rt}_t = \varphi_1 \hat{rt}_{t-1} - \varphi_2 \hat{rb}_t + \varepsilon_t$, where $\hat{rt}_t$ is the log deviation in the real value of transfer payments from steady state, and $\hat{rb}_t$ is the log deviation in real debt (nominal debt divided by the consumer price level) from steady state. Fiscal policy shocks (ε_t) combine with endogenous changes in income tax revenue and government financing costs to account for fluctuations in the fiscal deficit.

Second, to capture the re-distributive aspects of transfer payments, we assume there are two types of households: some households are hand-to-mouth consumers (consuming all their income each period, with no borrowing/saving), while the remainder have access to complete financial markets (as in the baseline model). We then assume that fiscal transfers are made exclusively to

[40] We assume the government issues one period debt, and it is able to borrow at the risk-free interest rate.

Figure 12: Model and Counterfactuals with Fiscal Policy

(a) Gov. Spending and Deficits (b) Inflation without Constraints (c) Inflation: Individual Shocks



Note: In panel (c), we simulate the impact of individual shocks given the smoothed levels for domestic and foreign capacity, as in Figure 9b. Consumer price inflation is the quarterly value, expressed at an annualized rate.

the hand-to-mouth households, capturing the idea that fiscal transfers were directed to consumers with high marginal propensities to consume. Obviously, these assumptions serve to strengthen the stimulative impact of deficit-financed transfers.

To estimate parameters in the government's fiscal rule and recover fiscal shocks, we use data on federal outlays as a share of GDP (in log deviations from steady state) as an additional observable data series.[41] We set the steady-state ratio of transfers to GDP equal to its time-series average (0.2106), and the steady-state ratio of debt to GDP is set to 0.6. We assume 30% of households are hand-to-mouth consumers, following Kaplan et al. (2018). The income tax rate (τ) is set to 0.3. We estimate the parameters in the fiscal rule that govern the size of the transfer shocks (the variance of ε_t), the persistence of transfers (φ_1), and the responsiveness of transfers to debt (φ_2).

Having re-estimated the model with fiscal shocks, we now present a few key results. In Figure 12a, we plot the data series we use to estimate the fiscal rule, along with smoothed values for this observable series and the government budget deficit as a share of GDP. The smoothed deficit peaks in 2020:Q4 and then contracts sharply during 2021, mimicking the data on government outlays. Thus, the model now includes the large fiscal impulse implied by the data. In Figure 12b, we show that constraints continue to play an important role in this extended model, by exogenously relaxing the capacity constraints (as in Figure 7). Similar to the baseline model, inflation is about 2 percentage points lower over the 2021-2022 period when constraints are relaxed.

In Figure 12c, we evaluate the relative roles for fiscal, monetary, and preference-based demand shocks in accounting for inflation.[42] The fiscal shock drives inflation up in 2020, coincident with

[41] We use total federal outlays from the Monthly Treasury Statement (FRED code MTSO133FMS) and nominal GDP (FRED code NA000334Q). Neither series is seasonally adjusted, so we use a four-quarter moving average of each. We append measurement error to the observed outlays-to-GDP series and estimate its variance.

[42] As in Figure 9b, we simulate the effect of each shock together with the capacity shocks that we filter from the data, and then we difference out the direct impact of the capacity shocks on inflation. The resulting simulation captures how individual shocks influence inflation when they are allowed to interact with the capacity constraints.

the large increase in debt-financed transfers during the pandemic. Because both transfers and the deficit declined rapidly in 2021, the role for fiscal policy in driving inflation abates and turns slightly negative in 2022-2023. In contrast, monetary policy has little impact on inflation in 2020, but then plays an important role in explaining its rise in 2021 and subsequent decline after 2022.

As a final point in Figure 12c, preference-based demand shocks have a large negative impact on inflation in 2020-2021, and then these demand shocks propel inflation higher during 2022-2023. Though this may seem inconsistent with small role for the demand shocks in the baseline model, it is not. In the baseline model, the discount rate shock largely subsumes the impact of both fiscal shocks and discount rate shocks on consumption.[43] Now having separated these shocks, we see that they offset each other. All together, we conclude that incorporating fiscal policy into the model does not substantially modify the conclusions drawn from the baseline model.

4.3 Enriching the Labor Market

Motivated by pervasive disruptions in labor markets during the pandemic period and recovery, we enrich the labor market of the model in three ways. First, we allow for adjustment frictions for nominal wages, in addition to price adjustment frictions. Second, we introduce shocks to the disutility of labor supply, which stand in for various pandemic-related supply shocks (e.g., responses to disease risk, the great resignation, etc.). Third, we incorporate an occasionally binding constraint on labor supply, in addition to the goods market capacity constraints considered previously. Unlike normal times, labor supply constraints plausibly loomed large during the COVID period, where stay-at-home orders, school closures, and other abnormal policies constrained households' ability to supply labor to the market.

For brevity, we consign the details about this extended model to the Online Appendix, and we instead focus on one key result here. The model yields a wage Phillips Curve:

$$\pi_{Wt} = \left(\frac{\epsilon_L - 1}{\phi_W}\right)\left[\widehat{mrs}_t - \widehat{rw}_t\right] + \left(\frac{\epsilon_L}{\phi_W}\frac{P_0}{W_0}\right)\hat{\tilde{\mu}}_{Lt} + \beta\mathbb{E}_t\left(\pi_{Wt+1}\right), \tag{21}$$

where π_{Wt} is nominal wage inflation, mrs_t is the log of the marginal rate of substitution between labor supply and consumption in preferences, rw_t is the log real wage, and $\tilde{\mu}_{Lt} \equiv 1 + (\mu_{Lt}/C_t{}^{-\rho})$ is a function of the multiplier on the labor constraint (μ_{Lt}).[44]

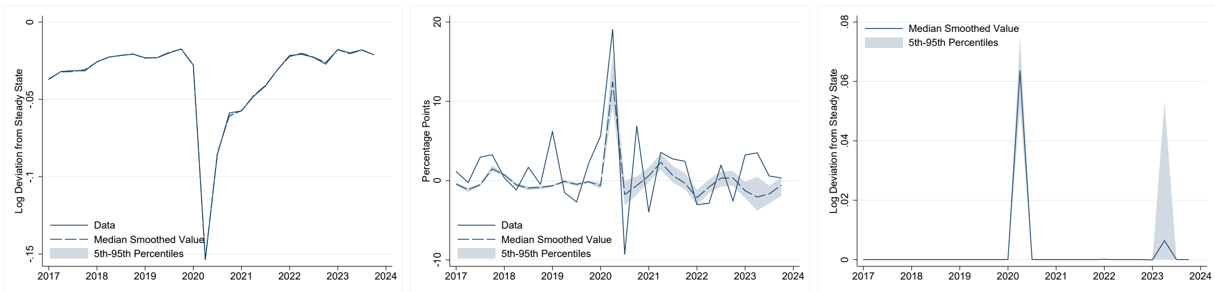
Two important results follow from inspection of Equation 21. The first (standard) result is that labor (disutility) supply shocks enter the wage Phillips curve via the marginal rate of substi-

[43] This is consistent with arguments in Gabaix (2020) and Angeletos et al. (2023), where fiscal shocks appear like discount rate shocks in reduced form when Ricardian equivalence fails in New Keynesian models.

[44] For completeness, the parameter ϵ_L controls steady-state wage markups (the degree of market power exercised by workers) and the parameter ϕ_W controls the flexibility of wages. See the Online Appendix for the details underlying derivation of Equation 21, and how it fits into the remainder of the model.

Figure 13: Model Fit with Labor Market Extensions

(a) Hours Worked (b) Real Wage Growth (c) Reduced-Form Wage Markup



Note: We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, and then plot the median smoothed value as the solid line. We shade the area covering the 5% to 95% percentile for smoothed values. In Figure 13c, we plot the reduced form labor markup shock term $\left(\frac{\epsilon_L}{\phi_W}\frac{P_0}{W_0}\right)\hat{\tilde{\mu}}_{Lt}$.

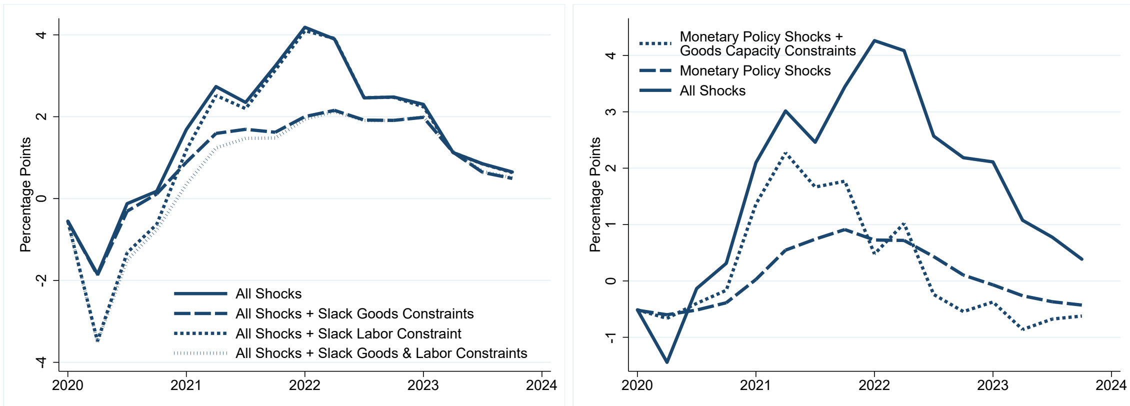
tution ($\widehat{mrs}_t$), where increased disutility of supplying labor raises $\widehat{mrs}_t$ and thus wage inflation. Elsewhere in the model, increases in the disutility of labor supply also naturally lower the equilibrium quantity of labor employed as well. The second (non-standard) result is that binding labor constraints appear as reduced-form "markup shocks" in the wage Phillips Curve. As a result, binding labor constraints drive up wage inflation, conditional on the other labor market fundamentals. With these results in hand, we turn to quantitative analysis. We calibrate several new parameters (e.g., ϵ_L and ϕ_W) based on external references. We then re-estimate the extended model along with stochastic processes for labor disutility and labor constraint shocks using two new observable data series: aggregate hours worked and real wage growth, which are constructed using data from the US Bureau of Labor Statistics. Details on these steps are provided in the Online Appendix.

Turning to results, we illustrate model fit and smoothed multipliers on the labor constraint in Figure 13. In Figure 13a, there is an obvious dramatic collapse in hours in early 2020:Q2, a rapid partial rebound in Q3, and then a slow recovery thereafter through 2021. The model matches these dynamics well, in large part through shocks to labor supply. In addition, Figure 13b illustrates that there were sharp gyrations in real wage growth during the early pandemic. However, real wage growth from 2020:Q4 forward was similar to the pre-pandemic period. Turning to Figure 13c, the model clearly favors a binding labor constraint in 2020:Q2, in order to explain the spike and subsequent collapse in real wage growth. Labor constraints then play a less important role in 2021-2022. The median simulation has a slack or nearly slack labor constraint in most periods, though labor constraints do appear to bind in 2022 for a non-trivial share of the simulations.

To evaluate how incorporating labor supply shocks and constraints affect our prior results, we

Figure 14: Counterfactuals with Labor Market Extensions

(a) Relax Goods Capacity or Labor Constraints (b) Monetary Policy Shocks and Capacity Shocks



Note: Each series represents the simulated path of consumer price inflation (quarterly value, annualized) for the indicated subset of smoothed shocks and constraints during 2020-2022.

present two sets of counterfactuals.[45] First, in Figure 14a, we illustrate how relaxing the goods and labor constraints separately and in combination affects inflation. When the labor constraint is assumed to be slack, inflation falls substantially at the outset of the pandemic, which is counterfactual; thus, binding labor constraints help explain the absence of disinflation in 2020. However, their impact dissipates rapidly, such that inflation is essentially similar across versions of the model with and without labor constraints in 2021-2022. In contrast, assuming goods constraints are slack has little impact on inflation in 2020, but then inflation would have been significantly lower in 2021-2022 with slack goods constraints (this echoes Figure 7c). Further, we point out that the quantitative impact of removing the goods market constraints is essentially the same in this model with labor supply (disutility) shocks as in the baseline without them.

Second, we investigate again how monetary policy interacts with constraint shocks in Figure 14b. The first simulation shuts off all shocks except for the monetary policy shocks, and the second considers the joint impact of monetary policy shocks and capacity shocks for both domestic and imported goods. As in the prior simulations, monetary policy alone has a moderate effect on inflation, while monetary policy combined with capacity shocks lead to a rapid increase in inflation in 2021, sustained high inflation through 2021 into 2022, and then a collapse in inflation from

[45] Like prior counterfactuals, we draw form the posterior to parameterize the model and filter smoothed shocks from data, and we then simulate responses to subsets of the smoothed shocks under various assumptions about whether constraints bind. Repeating this procedure 1000 times, we report median outcomes in the figures. As a technical matter, we allow goods constraints to bind endogenously in all these simulations. The labor constraint is a third constraint, which complicates simulation, as the Dynare implementation of OccBin only admits two potentially binding constraints. Therefore, we impose the labor constraint by assuming that there are reduced-form wage markup shocks, which are tied to the smoothed values of the multiplier on the labor constraint. We then solve for whether the two goods constraints are binding endogenously.

2022:Q3 forward.

5 Concluding Remarks

We have developed a quantitative framework that places potentially-binding capacity constraints at center stage. We show that binding constraints alter the Phillips Curve relationship between inflation and real marginal costs, introducing a term that looks like a markup shock in reduced form. Applying the framework, we find that binding constraints are important drivers of US inflation during 2021-2022, explaining half of the rise in inflation. We also find that negative capacity shocks, which tightened constraints during the post-pandemic period, set the stage for demand shocks to have outsized impacts on inflation. In particular, monetary policy shocks loom in accounting for the rise and fall of US inflation.

Going forward, there are various extensions of this framework that would be useful to consider. While we have modelled capacity as an exogenous, stochastic variable, we see high returns to extending the model to allow for investment in capacity. Further, while we have focused on recent US inflation outcomes in this paper, we intend to apply the model to parse data for other countries (e.g., the UK and euro area) that experienced similar high inflation episodes. Because energy prices likely played a larger role in these related contexts, we also see value in extending the model to treat energy supply and use more carefully.

More generally, the framework we have developed can be deployed to study optimal policy, as well as potential policy mistakes during the pandemic recovery. Given the importance of monetary policy shocks in our quantitative analysis, a critical analysis of policy is warranted. In our framework, demand shocks shift the Phillips Curves when constraints bind. This complicates policy design, relative to canonical frameworks in which only supply shocks shift the Phillips Curve. Further, since reduced-form markups may reflect either the influence of exogenous markup shocks, or the impact of binding constraints, how policy ought to respond to apparent markup shocks will depend on the central bank's ability to discriminate between the underlying causes of them. Both these issues raise novel questions for policy.

References

Alessandria, George A., Shafaat Y. Khan, Armen Khederlarian, Carter B. Mix, and Kim J. Ruhl, "The Aggregate Effects of Global and Local Supply Chain Bottlenecks: 2020-2022," *Journal of International Economics*, 2023, p. 103788.

Álvarez-Lois, Pedro P., "Endogeneous Capacity Utilization and Macroeconomic Persistence," *Journal of Monetary Economics*, 2006, *53*, 2213–2237.

Amiti, Mary, Sebastian Heise, and Aidan Wang, "High Import Prices along the Global Supply Chain Feed Through to U.S. Domestic Prices," Liberty Street Economics 20211108, Federal Reserve Bank of New York 2021.

_ , _ , Fatih Karahan, and Ayşegül Şahin, "Inflation Strikes Back: The Role of Import Competition and the Labor Market," 2023. NBER Working Paper No. 31211.

Angeletos, George-Marios, Chen Lian, and Christian K. Wolf, "Can Deficits Finance Themselves?," 2023. NBER Working Paper No. 31185.

Bai, Xiwen, Jesús Fernández-Villaverde, Yiliang Li, and Francesco Zanetti, "The Causal Effects of Global Supply Chain Disruptions on Macroeconomic Outcomes: Evidence and Theory," 2023. Unpublished Manuscript.

Baqaee, David and Emmaneul Fahri, "Supply and Demand in Disaggregated Keynesian Economies with an Application to the Covid-19 Crisis," *American Economic Review*, 2022, *112* (5), 1397–1436.

Benigno, Gianluca, Julian di Giovanni, Jan J.J. Groen, and Adam I. Noble, "The GSCPI: A New Barometer of Global Supply Chain Pressures," May 2022. Federal Reserve Bank of New York Staff Reports, no. 1017.

Bernanke, Ben and Olivier Blanchard, "What Caused the U.S. Pandemic-Era Inflation?," 2023. NBER Working Paper No. 31417.

Bianchi, Francesco, Renato Faccini, and Leonardo Melosi, "A Fiscal Theory of Persistent Inflation," *The Quarterly Journal of Economics*, 2023, *138*, 2127–2179.

Boehm, Christoph E. and Nitya Pandalai-Nayar, "Convex Supply Curves," *The American Economic Review*, 2022, *112* (12), 3941–3969.

Bonadio, Barthélémy, Zhen Huo, Andrei A. Levchenko, and Nitya Pandalai-Nayar, "Global Supply Chains in the Pandemic," *Journal of International Economics*, 2021, *133*, 103534.

Born, Benjamin and Johannes Pfeifer, "The New Keynesian Wage Phillips Curve: Calvo vs. Rotemberg," *Macroeconomic Dynamics*, 2020, *24* (55), 1017–1041.

Bunkley, Nick, "Piecing Together a Supply Chain," The New York Times May 12 2011. https://www.nytimes.com/2011/05/13/business/global/13auto.html.

Cagliarini, Adam and Mariano Kulish, "Solving Linear Rational Expectations Models with Predictable Structural Changes," *The Review of Economic Studies*, 2013, *95* (1), 328–336.

Celasun, Oya, Niels-Jakob Hansen, Aiko Mineshima, MarianoSpector, and Jing Zhou, "Supply Bottlenecks: Where, Why, How Much,and What Next?," 2022. IMF Working Paper WP/22/31.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 2005, *113* (1), 1–45.

Comin, Diego A. and Robert C. Johnson, "Offshoring and Inflation," 2020. NBER Working Paper 27957.

Comin, Diego, Robert C. Johnson, and Callum Jones, "Import Constraints," *AEA Papers and Proceedings*, May 2024, *114*, 118–23.

de Soyres, Francois, Ana Maria Santacreu, and Henry Young, "Demand-Supply Imbalance during the Covid-19 Pandemic: The Role of Fiscal Policy," *Federal Reserve Bank of St. Louis Review*, 2023, *105* (1), 21–50.

Del Negro, Marco, Aidan Gleich, Shlok Goyal, Alissa Johnson, and Andrea Tambalotti, "Drivers of Inflation: The New York Fed DSGE Model's Perspective," Liberty Street Economics 20220301, The Federal Reserve Bank of New York 2022.

di Giovanni, Julian, Şebnem Kalemi-Özcan, Alvaro Silva, and Muhammed A. Yildirim, "Quantifying the Inflationary Impact of Fiscal Stimulus Under Supply Constraints," *AEA Papers and Proceedings*, 2023, *113.*

_ , _ , _ , and _ , "Pandemic-Era Inflation Drivers and Global Spillovers," 2024. Unpublished Manuscript, Brown University.

_ , Şebnem Kalemli-Özcan, Alvaro Silva, and Muhammed A. Yildirim, "Global Supply Chain Pressures, International Trade, and Inflation," in "Challenges for Monetary Policy in a Rapidly Changing World" ECB Forum on Central Banking 2022.

Fagnart, Jean-François, Omar Licandro, and Franck Portier, "Firm Heterogeneity, Capacity Utilization, and the Business Cycle," *Review of Economic Dynamics*, 1999, *2*, 433–455.

Ferrante, Francesco, Sebastian Graves, and Matteo Iacoviello, "The Inflationary Effects of Sectoral Reallocation," *Journal of Monetary Economics*, 2023, *140*, 64–81.

Finck, David and Peter Tillmann, "The Macroeconomic Effects of Global Supply Chain Disruptions," 2023. Unpublished Manuscript.

Fitzgerald, Terry, Callum Jones, Mariano Kulish, and Juan Pablo Nicolini, "Is There a Stable Relationship Between Unemployment and Future Inflation?," *American Economic Journal: Macroeconomics*, forthcoming.

Gabaix, Xavier, "A Behavioral New Keynesian Model," *American Economic Review*, 2020, *110* (8), 2271–2327.

Gagliardone, Luca and Mark Gertler, "Oil Prices, Monetary Policy and Inflation Surges," 2024. Unpublished Manuscript, New York University.

Gilchrist, Simon and John C. Williams, "Putty-Clay and Investment: A Business Cycle Analysis," *Journal of Political Economy*, 2000, *108* (5), 928–960.

Goodman, Peter, "How the Supply Chain Broke, and Why It Won't Be Fixed Anytime Soon," The New York Times October 22 2021. https://www.nytimes.com/2021/10/22/business/shortages-supply-chain.html.

Gopinath, Gita, Emine Boz, Camila Casas, Federico Díez, Pierre-Olivier Gourinchas, and Mikkel Plaborg-Møller, "Dominant Currency Paradigm," *American Economic Review*, 2020, *110* (3), 677–719.

Gourinchas, Pierre-Olivier, Şebnem Kalemli-Özcan, Veronika Penciakova, and Nick Sander, "Fiscal Policy in the Age of COVID: Does it "Get in all of the Cracks"?," 2021. NBER Working Papers 29293.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory Huffman, "Investment, Capacity Utilization, and the Real Business Cycle," *American Economic Review*, 1988, *78* (3), 402–17.

Guerrieri, Luca and Matteo Iacoviello, "OccBin: A Toolkit for Solving Dynamic Models with Occasionally Binding Constraints Easily," *Journal of Monetary Economics*, 2015, *70*, 22–38.

_ and _ , "Collateral Constraints and Macroeconomic Asymmetries," *Journal of Monetary Economics*, 2017, *90*, 28–49.

International Monetary Fund, "Recovery During a Pandemic: Health Concerns, Supply Disruptions, and Price Pressures," World Economic Outlook October 2021.

_ , "Countering the Cost-of-Living Crisis," World Economic Outlook October 2022.

Jones, Callum, Mariano Kulish, and Daniel M. Rees, "International Spillovers of Forward Guidance Shocks," *Journal of Applied Econometrics*, 2022, *37* (1), 131–160.

_ , _ , and James Morley, "A Structural Measure of the Shadow Federal Funds Rate," 2022. Finance and Economics Discussion Series 2021-064. Washington: Board of Governors of the Federal Reserve System.

Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante, "Monetary Policy According to HANK," *American Economic Review*, March 2018, *108* (3), 697–743.

Kerr, Jaren, "US Companies Face Biggest Decline in Profits since Covid Shutdowns," The Financial Times April 2023. https://on.ft.com/41dIw6n.

Kulish, Mariano and Adrian Pagan, "Estimation and Solution of Models with Expectations and Structural Changes," *Journal of Applied Econometrics*, 2017, *32*, 255–274.

_ , James Morley, and Tim Robinson, "Estimating DSGE Models with Zero Interest Rate Policy," *Journal of Monetary Economics*, 2017, *88*, 35–49.

Lafrogne-Joussier, Raphaël, Julien Martin, and Isabelle Mejean, "Supply Shocks in Supply Chains: Evidence from the Early Lockdown in China," *IMF Economic Review*, 2023, *71*, 170–215.

Lane, Philip, "Bottlenecks and Monetary Policy," The ECB Blog, 10 February., The European Central Bank February 2022.

La'o, Jennifer and Alireza Alireza Tahbaz-Salehi, "Optimal Monetary Policy in Production Networks," *Econometrica*, 2022, *90* (3), 1295–1336.

Lorenzoni, Guido and Ivan Werning, "Wage-Price Spirals," *Brookings Papers on Economic Activity*, 2023.

Murphy, Daniel, "Excess Capacity in a Fixed-Cost Economy," *European Economic Review*, January 2017, *91*, 245–260.

Romer, Christina, "The Fiscal Policy Response to the Pandemic," *Brookings Papers on Economic Activity*, 2021, *Spring*, 89–110.

Rotemberg, Julio J., "Monopolistic Price Adjustment and Aggregate Output," *The Review of Economic Studies*, 1982, *49* (4), 517–531.

Rubbo, Elisa, "Networks, Phillips Curves, and Monetary Policy," *Econometrica*, 2023, *91* (4), 1417–1455.

Santacreu, Ana Maria and Jesse LaBelle, "Global Supply Chain Disruptions andInflation During the COVID-19 Pandemic," *Federal Reserve Bank of St. Louis Review*, 2022, *104* (2), 78–91.

Smialek, Jeanna and Eshe Nelson, "The world's top central bankers see supply chain problems prolonging inflation," The New York Times September 29 2021. https://www.nytimes.com/2021/09/29/business/central-bankers-supply-chains-inflation.html.

The Economist, "The World Economy's Shortage Problem," Print Edition October 9 2021. https://www.economist.com/leaders/2021/10/09/the-world-economys-shortage-problem.

Tirole, Jean, *The Theory of Industrial Organization*, Vol. 1 of *MIT Press Books*, The MIT Press, December 1988.

Wu, Jing Cynthia and Fan Dora Xia, "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound," *Journal of Money, Credit and Banking*, 2016, *48* (2-3), 253–291.

APPENDIX [For Online Publication]

A Quantitative Model

We start by presenting equilibrium conditions for the model, and then we present the calibration and estimation procedure. We proceed to discuss parameter estimates, model fit, and robustness checks.

A.1 Model Equilibrium Conditions

As noted in the text, we study an equilibrium with symmetric producers within each sector and country. Further, we write all prices relative to the domestic price level, and we define $\Pi_t \equiv \frac{P_t}{P_{t-1}}$. Given parameters and exogenous variables, an equilibrium is a sequence of aggregate quantities $\{C_t \,, L_t\}$, sector-level quantities $\{C_t(s)\,, C_{Ht}(s)\,, C_{Ft}(s)\,, L_t(s)\,, Y_t(s)\,, M_t(s)\,, X_t(s)\,, Y^*_{Ct}(s)\,, Y^*_{Mt}(s)\}_s$, input use $\{\{M_t(s',s)\,, M_{Ht}(s',s)\,, M_{Ft}(s',s)\}_{s'}\}_s$, aggregate prices $\{W_t/P_t \,, i_t \,, Q_t \,, \Pi_t \,, \bar{\Pi}_t \,, \bar{P}_t/P_t\}$, sector-level relative prices $\{P_t(s)/P_t, MC_t(s)/P_t, P_{Mt}(s)/P_t, P_{Ht}(s)/P_t, P_{CFt}(s)/P_t, P_{MFt}(s)/P_t\}_s$, sector-level inflation $\{\Pi_t(s)\,, \Pi_{CFt}(s)\,, \Pi_{MFt}(s)\}_s$, input prices $\{\{P_t(s',s)/P_t\}_{s'}\}_s$, and (normalized) multipliers $\{\mu_t(s)/P_t \,, \mu^*_{Ct}(s)/P^*_t \,, \mu^*_{Mt}(s)/P^*_t\}_s$ that satisfy the model's equilibrium conditions in Table A1. This system is $8 + 21S + 4S^2$ equations in the same number of unknowns.

Table A1: Equilibrium Conditions

Labor Supply	$C_t^{-\rho}\frac{W_t}{P_t} = \chi L_t^{\psi}$
Consumption Allocation	$C_t(s) = \zeta_t(s)\left(\frac{P_t(s)}{P_t}\right)^{-\vartheta} C_t$
	$C_{Ht}(s) = \gamma(s)\left(\frac{P_{Ht}(s)/P_t}{P_t(s)/P_t}\right)^{-\epsilon(s)} C_t(s)$
	$C_{Ft}(s) = (1-\gamma(s))\left(\frac{P_{CFt}(s)/P_t}{P_t(s)/P_t}\right)^{-\epsilon(s)} C_t(s)$
Euler Equation	$1 = \mathbb{E}_t\left[\beta\frac{\Theta_{t+1}}{\Theta_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\left(\frac{1+i_t}{\Pi_{t+1}}\right)\right]$
Consumer Prices	$1 = \left(\sum_s \zeta_t(s)\left(\frac{P_t(s)}{P_t}\right)^{1-\vartheta}\right)^{1/(1-\vartheta)}$
	$\frac{P_t(s)}{P_t} = \left(\gamma(s)\left(\frac{P_{Ht}(s)}{P_t}\right)^{1-\epsilon(s)} + (1-\gamma(s))\left(\frac{P_{CFt}(s)}{P_t}\right)^{1-\epsilon(s)}\right)^{1/(1-\epsilon(s))}$
Labor Demand	$\frac{W_t}{P_t}L_t(s) = (1-\alpha(s))\frac{MC_t(s)}{P_t}Y_t(s)$
	$\frac{P_{Mt}(s)}{P_t}M_t(s) = \alpha(s)\frac{MC_t(s)}{P_t}Y_t(s)$
Input Demand	$M_t(s',s) = \frac{\alpha(s',s)}{\alpha(s)}\left(\frac{P_t(s',s)/P_t}{P_{Mt}(s)/P_t}\right)^{-\kappa} M_t(s)$
	$M_{Ht}(s',s) = \xi(s',s)\left(\frac{P_{Ht}(s')/P_t}{P_t(s',s)/P_t}\right)^{-\eta(s')} M_t(s',s)$
	$M_{Ft}(s',s) = (1-\xi(s',s))\left(\frac{P_{MFt}(s')/P_t}{P_t(s',s)/P_t}\right)^{-\eta(s')} M_t(s',s)$
Marginal Cost	$\frac{MC_t(s)}{P_t} = \frac{1}{Z_t(s)}\left(\frac{W_t}{P_t}\right)^{1-\alpha(s)}\left(\frac{P_{Mt}(s)}{P_t}\right)^{\alpha(s)}$

Input Prices	$\frac{P_{Mt}(s)}{P_t} = \left(\sum_{s'} \left(\frac{\alpha(s',s)}{\alpha(s)}\right) \left(\frac{P_t(s',s)}{P_t}\right)^{1-\kappa}\right)^{1/(1-\kappa)}$ $\frac{P_t(s',s)}{P_t} = \left[\xi(s',s) \left(\frac{P_{Ht}(s')}{P_t}\right)^{1-\eta(s')} + (1-\xi(s',s)) \left(\frac{P_{MFt}(s')}{P_t}\right)^{1-\eta(s')}\right]^{1/(1-\eta(s'))}$
Domestic Pricing	$0 = 1 - \varepsilon \left(1 - \frac{MC_t(s)/P_t + \mu_t(s)/P_t}{P_{Ht}(s)/P_t}\right) - \phi(s) \left(\Pi_{Ht}(s) - 1\right) \Pi_{Ht}(s)$ $+\mathbb{E}_t \left[\beta \frac{\Theta_{t+1}}{\Theta_t} \left(\frac{C_{t+1}}{C_t}\right)^{-\rho} \frac{\phi(s)}{\Pi_{t+1}} \left(\Pi_{Ht+1}(s) - 1\right) \Pi_{Ht+1}(s)^2 \frac{Y_{t+1}(s)}{Y_t(s)}\right]$
Consumption Import Pricing	$0 = 1 - \varepsilon \left(1 - \frac{Q_t}{P_{CFt}(s)/P_t} \frac{MC_t^*(s) + \mu_{Ct}^*(s)}{P_t^*}\right) - \phi(s) \left(\Pi_{CFt}(s) - 1\right) \Pi_{CFt}(s)$ $+\mathbb{E}_t \left[\beta \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\rho} \frac{Q_t}{Q_{t+1}} \frac{\phi(s)}{\Pi_{t+1}} \left(\Pi_{CFt+1}(s) - 1\right) \Pi_{CFt+1}(s)^2 \frac{Y_{Ct+1}^*(s)}{Y_{Ct}^*(s)}\right]$
Input Import Pricing	$0 = 1 - \varepsilon \left(1 - \frac{Q_t}{P_{MFt}(s)/P_t} \frac{MC_t^*(s) + \mu_{Mt}^*(s)}{P_t^*}\right) - \phi(s) \left(\Pi_{MFt}(s) - 1\right) \Pi_{MFt}(s)$ $+\mathbb{E}_t \left[\beta \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\rho} \frac{Q_t}{Q_{t+1}} \frac{\phi(s)}{\Pi_{t+1}} \left(\Pi_{MFt+1}(s) - 1\right) \Pi_{MFt+1}(s)^2 \frac{Y_{Mt+1}^*(s)}{Y_{Mt}^*(s)}\right]$
Complementary Slackness	$\min \left\{\mu_t(s), \bar{Y}_t(s) - Y_t(s)\right\} = 0$ $\min \left\{\mu_{Ct}^*(s), \bar{Y}_{Ct}^*(s) - Y_{Ct}^*(s)\right\} = 0$ $\min \left\{\mu_{Mt}^*(s), \bar{Y}_{Mt}^*(s) - Y_{Mt}^*(s)\right\} = 0$
Market Clearing	$Y_t(s) = C_{Ht}(s) + \sum_{s'} M_{Ht}(s,s') + X_t(s) + \frac{\phi(s)}{2} \left(\frac{P_t(s)}{P_{t-1}(s)} - 1\right)^2 Y_t(s)$ $X_t(s) = \left(\frac{P_{Ht}(s)}{P_t Q_t}\right)^{-\sigma(s)} X_t^*(s)$ $Y_{Ct}^*(s) = C_{Ft}(s) + \frac{\phi(s)}{2} \left(\Pi_{CFt}(s) - 1\right)^2 Y_{Ct}^*(s)$ $Y_{Mt}^*(s) = \sum_{s'} M_{Ft}(s,s') + \frac{\phi(s)}{2} \left(\Pi_{MFt}(s) - 1\right)^2 Y_{Mt}^*(s)$ $\Theta_t \left(\frac{C_t}{C_t^*}\right)^{-\rho} Q_t = \Xi$ $\sum_s L_t(s) = L_t$
Monetary Policy	$1 + i_t = (1 + i_{t-1})^{\varrho_i} \bar{\Pi}_t^{\omega(1-\varrho_i)} \left(Y_t/Y_0\right)^{(1-\varrho_i)\varrho_y} \Psi_t$ with $Y_t = \sum_s P_0(s) Y_t(s)$
Auxiliary Definitions	$\Pi_{Ht}(s) = \left(\frac{P_{Ht}(s)/P_t}{P_{Ht-1}(s)/P_{t-1}}\right) \Pi_t$ $\Pi_{CFt}(s) = \left(\frac{P_{CFt}(s)/P_t}{P_{CFt-1}(s)/P_{t-1}}\right) \Pi_t$ $\Pi_{MFt}(s) = \left(\frac{P_{MFt}(s)/P_t}{P_{MFt-1}(s)/P_{t-1}}\right) \Pi_t$ $\bar{\Pi}_t = \frac{\bar{P}_t/P_t}{\bar{P}_{t-1}/P_{t-1}} \Pi_t$ $\frac{\bar{P}_t}{P_t} = \left(\sum_s \zeta_0(s) \left(\frac{P_t(s)}{P_t}\right)^{1-\vartheta}\right)^{1/(1-\vartheta)}$

To construct the piece-wise linear solution to the model, we log-linearize the equilibrium conditions for both the unconstrained and constrained equilibria around the steady state. We normalize Home prices relative to the domestic price level, and we denote "real" prices with the letter r attached to the price. Further, lower case variables with hats denote log deviations from steady state. For example, the log deviation in the real wage from steady state is given by $\widehat{rw}_t = \hat{w}_t - \hat{p}_t$, while the real price of home output in sector s is $\widehat{rp}_{Ht}(s) = \hat{p}_{Ht}(s) - \hat{p}_t$, and so on.[46] Foreign currency

[46]For completeness, $\widehat{rp}_t(s) = \hat{p}_t(s) - \hat{p}_t$, $\widehat{rp}_{Ft}(s) = \hat{p}_{Ft}(s) - \hat{p}_t$, $\widehat{rmc}_t(s) = \widehat{mc}_t(s) - \hat{p}_t$, $\widehat{rp}_{Mt}(s) = \hat{p}_{Mt}(s) - \hat{p}_t$,

Table A2: Common Equilibrium Conditions for Unconstrained and Constrained Equilibria

Labor Supply	$-\rho\hat{c}_t + \widehat{rw}_t = \psi\hat{l}_t$
Consumption Allocation	$\hat{c}_t(s) = \hat{\zeta}_t(s) - \vartheta\widehat{rp}_t(s) + \hat{c}_t$ with $\sum_s \zeta_0(s)\hat{\zeta}_t(s) = 0$
	$\hat{c}_{Ht}(s) = -\epsilon(s)\left(\widehat{rp}_{Ht}(s) - \widehat{rp}_t(s)\right) + \hat{c}_t(s)$
	$\hat{c}_{Ft}(s) = -\epsilon(s)\left(\widehat{rp}_{Ft}(s) - \widehat{rp}_t(s)\right) + \hat{c}_t(s)$
Euler Equation	$0 = \mathbb{E}_t\hat{\Theta}_{t+1} - \hat{\Theta}_t - \rho\left(\mathbb{E}_t\hat{c}_{t+1} - \hat{c}_t\right) + i_t - \mathbb{E}_t\pi_{t+1}$
Consumer Prices	$0 = \sum_s \left[\zeta_0(s)\left(\frac{P_0(s)}{P_0}\right)^{1-\vartheta}\right]\left[\widehat{rp}_t(s) + \frac{1}{1-\vartheta}\hat{\zeta}_t(s)\right]$
	$\widehat{rp}_t(s) = \gamma(s)\left(\frac{P_{H0}(s)}{P_0(s)}\right)^{1-\epsilon(s)}\widehat{rp}_{Ht}(s) + (1-\gamma(s))\left(\frac{P_{CF0}(s)}{P_0(s)}\right)^{1-\epsilon(s)}\widehat{rp}_{Ft}(s)$
Labor Demand	$\widehat{rw}_t + \hat{l}_t(s) = \widehat{rmc}_t(s) + \hat{y}_t(s)$
	$\widehat{rp}_{Mt}(s) + \hat{m}_t(s) = \widehat{rmc}_t(s) + \hat{y}_t(s)$
Input Demand	$\hat{m}_t(s', s) = -\kappa\left(\widehat{rp}_{Mt}(s', s) - \widehat{rp}_{Mt}(s)\right) + \hat{m}_t(s)$
	$\hat{m}_{Ht}(s', s) = -\eta(s')\left(\widehat{rp}_{Ht}(s') - \widehat{rp}_{Mt}(s', s)\right) + \hat{m}_t(s', s)$
	$\hat{m}_{Ft}(s', s) = -\eta(s')\left(\widehat{rp}_{FMt}(s') - \widehat{rp}_{Mt}(s', s)\right) + \hat{m}_t(s', s)$
Marginal Cost	$\widehat{rmc}_t(s) = -\hat{z}_t(s) + (1-\alpha(s))\widehat{rw}_t(s) + \alpha(s)\widehat{rp}_{Mt}(s)$
Input Prices	$\widehat{rp}_{Mt}(s) = \sum_{s'}\left(\frac{\alpha(s',s)}{\alpha(s)}\right)\left(\frac{P_0(s',s)}{P_{M0}(s)}\right)^{1-\kappa}\widehat{rp}_{Mt}(s', s)$
	$\widehat{rp}_{Mt}(s', s) = \xi(s', s)\left(\frac{P_{H0}(s')}{P_0(s',s)}\right)^{1-\eta(s')}\widehat{rp}_{Ht}(s') + (1 -$
	$\xi(s', s))\left(\frac{P_{MFt}(s')}{P_0(s',s)}\right)^{1-\eta(s')}\widehat{rp}_{FMt}(s')$
Consumption Import Pricing	$\pi_{Ft}(s) = \frac{\epsilon-1}{\phi(s)}\left(\widehat{rmc}^*_t(s) + \hat{q}_t - \widehat{rp}_{Ft}(s)\right) + \beta\mathbb{E}_t\pi_{Ft+1}(s)$
Domestic Pricing for Services	$\pi_{Ht}(2) = \frac{\epsilon-1}{\phi(2)}\left(\widehat{rmc}_t(2) - \widehat{rp}_{Ht}(2)\right) + \beta\mathbb{E}_t\pi_{Ht+1}(2)$
Input Import Pricing for Services	$\pi_{MFt}(2) = \frac{\epsilon-1}{\phi(2)}\left(\widehat{rmc}^*_t(2) + \hat{q}_t - \widehat{rp}_{FMt}(2)\right) + \beta\mathbb{E}_t\pi_{FMt+1}(2)$
Market Clearing	$\hat{y}_t(s) = \left(\frac{C_{H0}(s)}{Y_0(s)}\right)\hat{c}_{Ht}(s) + \sum_{s'}\left(\frac{M_{H0}(s,s')}{Y_0(s)}\right)\hat{m}_{Ht}(s, s') + \left(\frac{X_0(s)}{Y_0(s)}\right)\hat{x}_t(s)$
	$\hat{x}_t(s) = -\sigma(s)\left(\widehat{rp}_{Ht}(s) - \hat{q}_t\right) + \hat{c}^*_t$
	$\hat{y}^*_{Ct}(s) = \hat{c}_{Ft}(s)$
	$\hat{y}^*_{Mt}(s) = \sum_{s'}\left(\frac{M_{F0}(s,s')}{Y^*_{M0}(s)}\right)\hat{m}_{Ft}(s, s')$
	$\hat{\Theta}_t - \rho\left(\hat{c}_t - \hat{c}^*_t\right) + \hat{q}_t = 0$
	$\sum_s\left(\frac{L_0(s)}{L_0}\right)\hat{l}_t(s) = \hat{l}_t$
Monetary Policy Rule	$i_t = \varrho_i i_{t-1} + \omega(1-\varrho_i)\hat{\bar{\pi}}_t + (1-\varrho_i)\varrho_y\hat{y}_t + \hat{\Psi}_t$
	with $\hat{y}_t = \sum_s\left(\frac{P_0(s)Y_0(s)}{Y_0}\right)\hat{y}_t(s)$
Auxiliary Inflation Definitions	$\pi_{Ht}(s) = \widehat{rp}_{Ht}(s) - \widehat{rp}_{Ht-1}(s) + \pi_t$
	$\pi_{Ft}(s) = \widehat{rp}_{Ft}(s) - \widehat{rp}_{Ft-1}(s) + \pi_t$
	$\pi_{FMt}(s) = \widehat{rp}_{FMt}(s) - \widehat{rp}_{FMt-1}(s) + \pi_t$
	$\bar{\pi}_t = \pi_t + \sum_s \zeta_0(s)\left(\frac{P_0(s)}{P_0}\right)^{1-\vartheta}\left(\widehat{rp}_t(s) - \widehat{rp}_{t-1}(s)\right)$

Table A3: Equilibrium Conditions with Binding Constraints for Goods

Panel A: Only Domestic Constraint Binds	
Domestic Pricing	$\pi_{Ht}(1) = \left(\frac{\epsilon-1}{\phi(1)}\right)(\widehat{rmc}_t(1) - \widehat{rp}_{Ht}(1)) + \left(\frac{\varepsilon}{\phi(1)}\frac{P_0}{P_{H0}(1)}\right)\hat{\hat{\mu}}_t(1) + \beta\mathbb{E}_t\pi_{Ht+1}(1)$
Input Import Pricing	$\pi_{MFt}(1) = \left(\frac{\epsilon-1}{\phi(1)}\right)(\widehat{rmc}^*_t(1) + \hat{q}_t - \widehat{rp}_{MFt}(1)) + \beta\mathbb{E}_t\pi_{FMt+1}(1)$
Domestic Constraint	$\hat{y}_t(1) = \hat{\bar{y}}_t(1) + \ln(\bar{Y}_0(1)/Y_0(1))$
Panel B: Only Foreign Constraint Binds	
Domestic Pricing	$\pi_{Ht}(1) = \left(\frac{\epsilon-1}{\phi(1)}\right)(\widehat{rmc}_t(1) - \widehat{rp}_{Ht}(1)) + \beta\mathbb{E}_t\pi_{Ht+1}(1)$
Input Import Pricing	$\pi_{MFt}(1) = \left(\frac{\epsilon-1}{\phi(1)}\right)(\widehat{rmc}^*_t(1) + \hat{q}_t - \widehat{rp}_{MFt}(1)) + \left(\frac{\varepsilon}{\phi(1)}\frac{P_0}{P_{MF0}(1)}\right)\hat{\hat{\mu}}^*_t(1) + \beta\mathbb{E}_t\pi_{MFt+1}(1)$
Import Constraint	$\hat{y}^*_t(1) = \hat{\bar{y}}^*_t(1) + \ln(\bar{Y}^*_0(1)/Y^*_0(1))$
Panel C: Both Constraints Bind	
Domestic Pricing	$\pi_{Ht}(1) = \left(\frac{\epsilon-1}{\phi(1)}\right)(\widehat{rmc}_t(1) - \widehat{rp}_{Ht}(1)) + \left(\frac{\varepsilon}{\phi(1)}\frac{P_0}{P_{H0}(1)}\right)\hat{\hat{\mu}}_t(1) + \beta\mathbb{E}_t\pi_{Ht+1}(1)$
Input Import Pricing	$\pi_{MFt}(1) = \left(\frac{\epsilon-1}{\phi(1)}\right)(\widehat{rmc}^*_t(1) + \hat{q}_t - \widehat{rp}_{MFt}(1)) + \left(\frac{\varepsilon}{\phi(1)}\frac{P_0}{P_{MF0}(1)}\right)\hat{\hat{\mu}}^*_t(1) + \beta\mathbb{E}_t\pi_{MFt+1}(1)$
Domestic Constraint	$\hat{y}_t(1) = \hat{\bar{y}}_t(1) + \ln(\bar{Y}_0(1)/Y_0(1))$
Import Constraint	$\hat{y}^*_t(1) = \hat{\bar{y}}^*_t(1) + \ln(\bar{Y}^*_0(1)/Y^*_0(1))$

prices (denoted by stars) are normalized relative to the foreign price level; for example, foreign real marginal costs are $\widehat{rmc}^*_t(s) = \widehat{mc}^*_t - \hat{p}^*_t$. We also define deviations in the value of constraints from steady state: $\hat{\bar{y}}_t(1) = \ln \bar{Y}_t(1) - \ln \bar{Y}_0(1)$ and $\hat{\bar{y}}^*_t(1) = \ln \bar{Y}^*_t(1) - \ln \bar{Y}^*_0(1)$. Finally, to reduce the number of potential foreign shocks, we assume that foreign export demand is given by $X^*_t(s) = \varpi(s)\left(\frac{P^*_t}{P^*_t(s)}\right)^{-\sigma(s)} C^*_t$, where we treat $\frac{P^*_t}{P^*_t(s)}$ and $\varpi(s)$ as constants, so $\hat{x}^*_t(s) = \hat{c}^*_t$.

We present the log-linear equilibrium conditions in Tables A2 and A3. Table A2 contains equilibrium conditions that hold in both unconstrained and constrained equilibria. Table A3 collects equilibrium conditions that differ across equilibria, depending on which constraints are slack or binding.

A.2 Stochastic Processes

We collect log deviations in exogenous domestic and foreign variables – $\hat{\Theta}_t$, $\hat{\zeta}_t(1)$, $\hat{c}^*_t$, and $\{\hat{z}_t(s), \widehat{rmc}^*_t\}_s$ – into vector $\hat{F}_t$, and we assume that $\hat{F}_t$ is a first-order vector autoregressive process, as in $\hat{F}_t = \Lambda\hat{F}_{t-1} + \varepsilon_t$, where Λ is a diagonal matrix of autoregressive coefficients (denoted λ_x for variable x) and ε_t is a vector of shocks.[47] We assume the vector of shocks has a multivariate normal distri-

$\widehat{rp}_{Mt}(s', s) = \hat{p}_{Mt}(s', s) - \hat{p}_t$.

[47] We assume that foreign real marginal costs are equal for goods and services: $\widehat{rmc}^*_t(s) = \widehat{rmc}^*_t$. Because the services sector is relatively closed, this restriction is unimportant.

Table A4: Calibration

Parameter	Value	Reference/Target
ψ	2	Labor supply elasticity of 0.5
ρ	2	Intertemporal elasticity of substitution of 0.5
β	.995	Annual risk-free real rate of 2%
ϑ	0.5	Elasticity of substitution across sectors in consumption
ε	4	Elasticity of substitution between varieties
κ	0.3	Elasticity of substitution for inputs across sectors
$\sigma(s)$	1.5	Export demand elasticity
ϕ	35.468	To yield first order equivalence to Calvo pricing, with average price duration of 4 quarters.

bution, with $var\left(\varepsilon_t\right)=\Sigma$ having diagonal elements σ_x^2 for each variable x and zeros off diagonal, and $cov\left(\varepsilon_t,\varepsilon_{t+s}\right)=0$ at all leads and lags ($s\neq 0$).

We assume that the constraint for imports of consumption goods is not binding in all periods. Similarly, we assume that constraints are not binding for services. This leaves $\bar{Y}_t(1)$ and $\bar{Y}^*_{Mt}(1)$ as the remaining constraints.[48] We assume they follow autoregressive processes: $\hat{\bar{y}}_t(1)=\rho_{\bar{y}}\hat{\bar{y}}_{t-1}(1)+\varepsilon_{\bar{y}t}(1)$ and $\hat{\bar{y}}^*_t(1)=\rho_{\bar{y}^*}\hat{\bar{y}}^*_{t-1}(1)+\varepsilon_{\bar{y}^*t}(1)$, where $\gamma\in(0,1)$ and $\varepsilon_{\bar{y}t}(1)$ and $\varepsilon_{\bar{y}^*t}(1)$ denote capacity shocks. We assume the capacity shocks are independent, mean zero normal random variables, with variances $var\left(\varepsilon_{\bar{y}t}(1)\right)=\sigma^2_{\bar{y}}$ and $var\left(\varepsilon_{\bar{y}^*t}(1)\right)=\sigma^2_{\bar{y}^*}$, and $cov\left(\varepsilon_{\bar{y}t}(1),\varepsilon_{\bar{y},t+s}(1)\right)=cov\left(\varepsilon_{\bar{y}^*t}(1),\varepsilon_{\bar{y}^*,t+s}(1)\right)=0$ at all leads and lags ($s\neq 0$).

A.3 Calibration

We set values for a subset of the structural parameters based on standard values in the literature, which we collect in Table A4. We use input-output data compiled by the US Bureau of Economic Analysis to pin down values for steady-state expenditure shares. We report these shares, which reflect mean values over the 1997-2018 period, in Table A5, along with their corresponding definitions in the model.

[48] Reliable data on capacity at high frequencies is generally not available, so we cannot include capacity among the observable variables. Existing data on capacity, such as the series used by the Federal Reserve Board to produce its G.17 series, are not well suited to our exercise. One reason is that the Federal Reserve's survey data is collected at an annual frequency, while we are interested in capacity dynamics at higher frequencies. Further, the capacity estimates are nearly time invariant at medium term (multi-year) frequencies, which means that capacity utilization mainly reflects the dynamics of industrial production. A second problem concerns how capacity survey questions are posed to firms. Specifically, the survey instrument asks firms to report how much they could produce if they had access to all the labor and materials they need to produce. This question fails to capture key aspects of production that effectively limit true capacity. For example, firms make predetermined choices about essential labor, material inputs, and other aspects of the production process that limit their ability to produce today, but this would be not be picked up by the survey.

Table A5: Steady State Shares

Model and Data	Description
$\begin{bmatrix} \zeta_0(1)\left(\frac{P_0(1)}{P_0}\right)^{1-\vartheta} \\ \zeta_0(2)\left(\frac{P_0(2)}{P_0}\right)^{1-\vartheta} \end{bmatrix} = \begin{bmatrix} 0.26 \\ 0.74 \end{bmatrix}$	Sector shares in consumption expenditure
$\begin{bmatrix} \gamma(1)\left(\frac{P_{H0}(1)}{P_0(1)}\right)^{1-\epsilon} \\ \gamma(2)\left(\frac{P_{H0}(2)}{P_0(2)}\right)^{1-\epsilon} \end{bmatrix} = \begin{bmatrix} 0.80 \\ 0.995 \end{bmatrix}$	Home shares in consumption expenditure by sector
$\begin{bmatrix} \alpha(1) \\ \alpha(2) \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$	Input expenditure share of gross output
$\begin{bmatrix} \left(\frac{\alpha(1,1)}{\alpha(1)}\right)\left(\frac{P_0(1,1)}{P_{M0}(1)}\right)^{1-\kappa} & \left(\frac{\alpha(1,2)}{\alpha(2)}\right)\left(\frac{P_0(1,2)}{P_{M0}(2)}\right)^{1-\kappa} \\ \left(\frac{\alpha(2,1)}{\alpha(1)}\right)\left(\frac{P_0(2,1)}{P_{M0}(1)}\right)^{1-\kappa} & \left(\frac{\alpha(2,2)}{\alpha(2)}\right)\left(\frac{P_0(2,2)}{P_{M0}(2)}\right)^{1-\kappa} \end{bmatrix} = \begin{bmatrix} 0.70 & 0.20 \\ 0.30 & 0.80 \end{bmatrix}$	Sector shares in input expenditure
$\begin{bmatrix} \xi(1,1)\left(\frac{P_{H0}(1)}{P_0(1,1)}\right)^{1-\eta} & \xi(1,2)\left(\frac{P_{H0}(1)}{P_0(1,2)}\right)^{1-\eta} \\ \xi(2,1)\left(\frac{P_{H0}(2)}{P_0(2,1)}\right)^{1-\eta} & \xi(2,2)\left(\frac{P_{H0}(2)}{P_0(2,2)}\right)^{1-\eta} \end{bmatrix} = \begin{bmatrix} 0.77 & 0.84 \\ 0.99 & 0.98 \end{bmatrix}$	Home shares in input expenditure
$\begin{bmatrix} \frac{C_{H0}(1)}{Y_0(1)} & \frac{M_{H0}(1,1)}{Y_0(1)} & \frac{M_{H0}(1,2)}{Y_0(1)} & \frac{X_0(1)}{Y_0(1)} \\ \frac{C_{H0}(2)}{Y_0(2)} & \frac{M_{H0}(2,1)}{Y_0(2)} & \frac{M_{H0}(2,2)}{Y_0(2)} & \frac{X_0(2)}{Y_0(2)} \end{bmatrix} = \begin{bmatrix} 0.41 & 0.32 & 0.16 & 0.11 \\ 0.61 & 0.07 & 0.29 & 0.03 \end{bmatrix}$	Domestic output allocation
$\begin{bmatrix} \frac{M_{F0}(1,1)}{Y^*_{M0}(1)} & \frac{M_{F0}(1,2)}{Y^*_{M0}(1)} \\ \frac{M_{F0}(2,1)}{Y^*_{M0}(2)} & \frac{M_{F0}(2,2)}{Y^*_{M0}(2)} \end{bmatrix} = \begin{bmatrix} 0.76 & 0.24 \\ 0.08 & 0.92 \end{bmatrix}$	Foreign output allocation for inputs
$\begin{bmatrix} \frac{P_{H0}(1)Y_0(1)}{P_{H0}(1)Y_0(1)+P_{H0}(2)Y_0(2)} \\ \frac{P_{H0}(2)Y_0(2)}{P_{H0}(1)Y_0(1)+P_{H0}(2)Y_0(2)} \end{bmatrix} = \begin{bmatrix} 0.29 \\ 0.71 \end{bmatrix}$	Sector shares in gross output

A.4 Estimation Procedure

Building on Kulish et al. (2017) and Kulish and Pagan (2017), we treat the duration of the potentially binding constraints as an estimable parameter. To explain the method, we first discuss how to solve the model for given constraint durations, and then we describe the estimation procedure.

A.4.1 Solving the Model for Given Durations

To construct a piecewise linear solution to the model, we take linear approximations of the model equilibrium for four regimes: the unconstrained regime, a second regime in which only domestic constraints bind, a third regime in which foreign constraints bind, and a fourth regime in which both constraints bind. Further, the linear approximations are all taken around the non-stochastic (unconstrained) steady state of the model. The solution procedure combines these local approximations to solve for the policy function.

The linear approximation to the unconstrained system can be written as:

$$\mathbf{A}X_t = \mathbf{C} + \mathbf{B}X_{t-1} + \mathbf{D}\mathbb{E}_t X_{t+1} + \mathbf{F}\varepsilon_t,$$

where x_t is an $n \times 1$ vector of model variables, ε_t is an $l \times 1$ vector of structural shocks, and $\mathbf{A}$, $\mathbf{C}$, $\mathbf{B}$, $\mathbf{D}$, and $\mathbf{F}$ are conformable matrices determined by the structural equations. If agents expect the economy to remain unconstrained from date t forward, then standard rational expectations solution procedures imply that the reduced-form solution is: $X_t = \mathbf{J} + \mathbf{Q}X_{t-1} + \mathbf{G}\varepsilon_t$, where $\mathbf{J}$, $\mathbf{Q}$, and $\mathbf{G}$ describe the policy function and model dynamics.

There are three regimes in which one or both constraints bind, and let us index these by $r \in \{1, 2, 3\}$. Then we can express the linear approximation to the model equilibrium in each case as:

$$\bar{\mathbf{A}}_r X_t = \bar{\mathbf{C}}_r + \bar{\mathbf{B}}_r X_{t-1} + \bar{\mathbf{D}}_r \mathbb{E}_t X_{t+1} + \bar{\mathbf{F}}_r \varepsilon_t,$$

where $\bar{\mathbf{A}}_r$, $\bar{\mathbf{C}}_r$, $\bar{\mathbf{B}}_r$, $\bar{\mathbf{D}}_r$, and $\bar{\mathbf{F}}_r$ are conformable matrices that correspond to the structural equations for each.

We summarize the expected evolution of regimes from a given date t forward by the durations that the individual constraints are expected to bind, as in $\mathbf{d}_t = [d_t, d_t^*]$. To fix ideas, suppose that $d_t = 1$, which means that the domestic constraint binds today, and then is expected to be slack in the future. Further, suppose that $d_t^* = 0$, so the foreign constraint is slack today and in the future. This implies that the constrained system governs model responses in period t and then the unconstrained system applies thereafter. Working backwards from the unconstrained solution, then $\mathbb{E}_t X_{t+1} = \mathbf{J} + \mathbf{Q}X_t$, so then $\bar{\mathbf{A}}_1 X_t = \bar{\mathbf{C}}_1 + \bar{\mathbf{B}}_1 X_{t-1} + \bar{\mathbf{D}}_1 (\mathbf{J} + \mathbf{Q}X_t) + \bar{\mathbf{F}}_1 \varepsilon_t$, where $r = 1$ is the system that applies when the domestic constraint binds and the foreign constraint is slack. Solving this linear equation yields the reduced form solution for X_t.

Generalizing this idea, the system will evolve according to:

$$\mathbf{A}_t X_t = \mathbf{C}_t + \mathbf{B}_t X_{t-1} + \mathbf{D}_t \mathbb{E}_t X_{t+1} + \mathbf{F}_t \varepsilon_t, \tag{22}$$

where $\mathbf{A}_t$, $\mathbf{C}_t$, $\mathbf{B}_t$, $\mathbf{D}_t$, and $\mathbf{F}_t$ are the structural matrices that apply at date t. Then the piecewise linear solution is given by:

$$X_t = \mathbf{J}_t + \mathbf{Q}_t X_{t-1} + \mathbf{G}_t \varepsilon_t, \tag{23}$$

where $\mathbf{J}_t$, $\mathbf{Q}_t$, and $\mathbf{G}_t$ are determined via the following backward recursion, which is initialized as starting from the unconstrained solution:

$$\begin{aligned} \mathbf{Q}_t &= \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}\right]^{-1} \mathbf{B}_t \\ \mathbf{J}_t &= \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}\right]^{-1} \left(\mathbf{C}_t + \mathbf{D}_t \mathbf{J}_{t+1}\right) \\ \mathbf{G}_t &= \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}\right]^{-1} \mathbf{F}_t . \end{aligned} \tag{24}$$

At this point, it is useful to note that this recursive solution coincides with the recursion employed by the Dynare OccBin toolkit to obtain policy functions for a given guess about the sequence of regimes. The Occbin toolkit then proceeds to verify whether the guess about the sequence of regimes is consistent with model equilibrium, given the current value of the shocks. Put differently, it solves for endogenous constraint durations. While we do not discuss this second step here, we do solve for endogenous durations (using Occbin) when we analyze counterfactuals in the model. We also take the dependence of $\mathbf{d}_t$ on ε_t into account in the estimation procedure, with details below.

While Equations 23 and 24 present the model solution for a given anticipated sequence of regimes, note that the anticipated sequence changes as durations evolve over time. The duration $\mathbf{d}_t$ implies a particular sequence of regimes anticipated at dates $t+1$, $t+2$, etc. Given this sequence and the maintained assumption that agents do not anticipate future shocks, one then uses the recursion above to solve for the associated policy matrices: $\mathbf{J}\left(\mathbf{d}_t, \theta\right), \mathbf{Q}\left(\mathbf{d}_t, \theta\right)$, and $\mathbf{G}\left(\mathbf{d}_t, \theta\right)$, where the notation captures the dependence of these matrices on $\mathbf{d}_t$. At date $t+1$, a new value for durations ($\mathbf{d}_{t+1}$) will be realized, and one then solves the recursion anew to obtain $\mathbf{J}\left(\mathbf{d}_{t+1}, \theta\right), \mathbf{Q}\left(\mathbf{d}_{t+1}, \theta\right)$, and $\mathbf{G}\left(\mathbf{d}_{t+1}, \theta\right)$. And so on. The state (transition) equation of the model then features time-varying coefficients:

$$X_t = \mathbf{J}\left(\mathbf{d}_t, \theta\right) + \mathbf{Q}\left(\mathbf{d}_t, \theta\right) X_{t-1} + \mathbf{G}\left(\mathbf{d}_t, \theta\right) \varepsilon_t . \tag{25}$$

When $\mathbf{d}_t = 0$, the unconstrained solution applies, so $\mathbf{J}\left(\mathbf{d}_t, \theta\right) = \mathbf{J}\left(\theta\right)$, $\mathbf{Q}\left(\mathbf{d}_t, \theta\right) = \mathbf{Q}\left(\theta\right)$, and $\mathbf{G}\left(\mathbf{d}_t, \theta\right) = \mathbf{G}\left(\theta\right)$ are time invariant.

A.4.2 Joint Estimation of Durations and Structural Parameters

We assume that a vector of observables (S_t) are linked to underlying model states via the measurement equation: $S_t = \mathbf{H}_t X_t + \nu_t$, where ν_t is an i.i.d. vector of normally distributed measurement errors and $\mathbf{H}_t$ is a conformable (potentially time-varying) matrix linking states to observables. Using this state space representation of the model, we can apply the Kalman filter to construct the Likelihood function $\mathcal{L}(\theta, \mathbf{d}| \{S_t\}_{t=1}^T)$, where $\mathbf{d} = \{\mathbf{d}\}_{t=1}^T$ is the sequence of durations.

We put priors over structural parameters and independent priors over durations to construct the posterior, and then estimate the model via Bayesian Maximum Likelihood. We construct draws from the joint posterior distribution $p\left(\theta, \mathbf{d}| \{S_t\}_{t=1}^T\right)$ using a Metropolis-Hastings algorithm with two blocks – one for the structural parameters, which are continuous, and a second for the discrete duration parameters – as in Kulish et al. (2017). We use a uniform proposal density for the durations, between 0 (unconstrained) and a sufficiently large maximum duration. We discuss the priors in Section A.4.4 below.

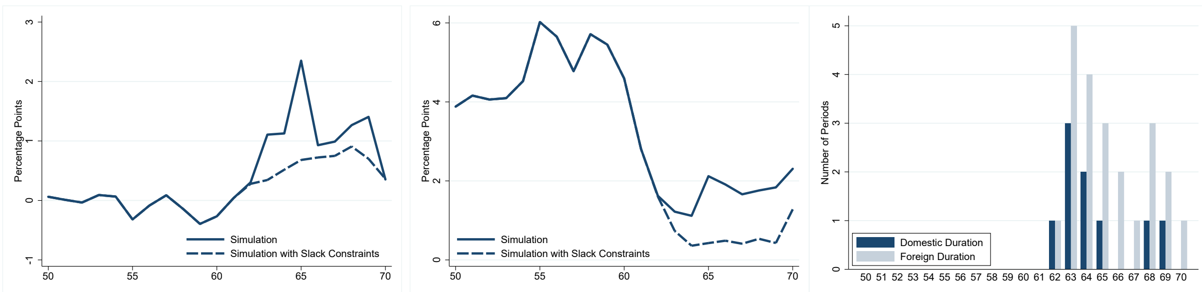
In evaluating proposed parameter and durations draws, we recognize that it is desirable for posterior estimates of constraint durations to be consistent with agents' forecasts about how long constraints will endogenously bind given shocks. To this end, we constrain admissible draws to enforce this constraint, in an approximate sense. For a given proposed joint parameter (θ^i) and duration draw ($\mathbf{d}^i$), we construct the piecewise linear solution for the model and use the Kalman filter to obtain smoothed structural shocks $\{\tilde{\varepsilon}_t^i\}_{t=1}^T$ and equilibrium variables $\{\tilde{X}_t^i\}_{t=1}^T$ given the data. At each sample period $\tau \in [1, \ldots, T]$, we then use the piecewise linear solution to project model outcomes forward given the state and current shock – $\left(\tilde{X}_{\tau-1}^i, \tilde{\varepsilon}_\tau^i\right)$, assuming that there are no anticipated future shocks.[49] We then check for violations of the output capacity constraints. If projected home or foreign output violates the constraints, then we reject the proposed parameter draw as inconsistent with model equilibrium. Otherwise, we accept the parameter draw, evaluate the likelihood, and proceed through the estimation algorithm. Under this procedure, we accept about 25% of the proposed parameter/duration draws, so the estimation proceeds at reasonable computational pace.

In this procedure, we reject the proposed draw when it implies that constraints will be violated in expectation. In turn, we accept draws for which constraints are satisfied. Strictly speaking, we do not explicitly check whether the duration $\mathbf{d}_\tau$ is equal to the endogenous equilibrium duration consistent with $\left(\tilde{X}_{\tau-1}^i, \tilde{\varepsilon}_\tau^i\right)$ in the model. Nonetheless, our approach provides a good approximation to model outcomes with endogenously binding constraints. To demonstrate this, we turn to simulation evidence.

[49] Recall that in the absence of future shocks, agents anticipate that the model will return to the unconstrained state over time, where the duration of binding constraints ticks down toward zero in each passing period. We project model outcomes forward using this expected path for durations.

Figure A1: Simulation

(a) Inflation (b) Interest Rate (c) Domestic / Foreign Durations



Note: Inflation is reported at a quarterly rate in percentage points. The interest rate is in percentage points.

A.4.3 Validating the Estimation Procedure

We provide results for two exercises to evaluate the accuracy of the estimation procedure. First, using simulated data, we demonstrate that the procedure is capable of identifying latent durations. Moreover, we show that it accurately recovers the corresponding multipliers on the constraints. Second, using results from the full estimation of the model with real world data, we compare smoothed inflation to simulated model results.

Estimation using Simulated Data The first step is to generate simulated data from the model, for given parameters.[50] Specifically, we draw a set of i.i.d. shocks for all variables over 70 quarters, and then impose a sequence of large, expansionary monetary policy shocks for quarters 61 to 69 (the size of the monetary policy shocks is set to three standard deviations). These shocks are large enough to trigger the capacity constraints. Since we can identify when the constraints are anticipated to bind in the simulation, we know the true sequence of durations.

We plot several simulated data series in Figure A1 to illustrate the set up, under both the maintained assumption that constraints are potentially binding and the counterfactual assumption that constraints are slack in all periods. The top two panels contain simulated inflation and the policy interest rate, while the implied durations for domestic and foreign constraints are recorded in the bottom two panels. The expansionary policy shocks evidently cause the policy rate to be low in periods 62 through 70, where inflation more than doubles at its peak relative to a simulation without capacity constraints.

Treating the simulated series as observable data, we illustrate that our empirical model is capa-

[50] In contrast to the main quantitative model, we assume there is zero measurement error, so observable variables are equal to corresponding objects in the simulated data. Further, we set steady-state capacity levels so that there is 4% excess capacity for both home and foreign goods firms, so that we can trigger binding constraints with demand shocks alone (i.e., without negative capacity shocks). Remaining parameters are set to the mode of our baseline estimates.

ble of identifying the true durations by directly examining model likelihood functions. Setting all parameters in the state and observation equations (other than durations) to their true values used to generate the simulated data, we compute the likelihood of the model for different values of the domestic and foreign durations, at given points in time. For example, setting the duration of the foreign constraint to its true value in a given period, we then trace out the likelihood over alternative values of the duration of the domestic constraint. And vice versa. We present the results from period 60, before the constraints become binding, through period 70, when the domestic capacity constraint stops binding and the foreign capacity constraint binds for one more quarter.

Figure A2 plots the inverse of the likelihood value across durations of the domestic constraint, where each panel corresponds to a period and the vertical line identifies the true duration.[51] The inverse likelihood is minimized at the true values in every quarter, which confirms that the likelihood procedure we implement is able to discriminate between durations of different length. Importantly, for periods when the constraint does not bind, the likelihood is maximized at a duration value of zero.

Turning to estimation of the multipliers, we conduct a full estimation of the model using the simulated data, in which we estimate both the structural parameters and durations, as in the main analysis.[52] Here we focus on the estimated (smoothed) multipliers on the capacity constraints, as these play a key role in the framework. In Figure A3, we plot the true paths for the multipliers in the simulation, along with smoothed multipliers recovered via estimation. As is evident, the smoothed values of the multipliers match the exact simulation values closely, meaning the procedure does a good job at pinning down the reduced-form impact of constraints on inflation.

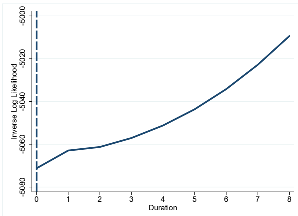
Smoothed vs. Simulated Inflation Drawing on results presented below and in the main text, we briefly compare smoothed inflation outcomes obtained via our estimation procedure with outcomes from the full structural model with endogenously binding constraints. This comparison serves to check that the empirical model with estimated durations replicates the outcomes of the structural model with endogenously binding constraints. Specifically, suppose we feed the structural shocks $\{\tilde{\varepsilon}_t^i\}_{t=1}^T$ obtained from our estimation procedure through the model, where structural parameters are set to their modal values and we use the OccBin procedure to solve for the endogenous duration of binding constraints in each period following the realization of shocks. We then plot this simulated inflation series to the smoothed inflation series from our estimation in Figure A4. As is evident, the two series track each other closely, so we conclude that our approach to capturing endogenously binding constraints in the estimation routine performs well.

[51] Results for the duration of the foreign constraint look similar; while we omit them here, they are included in prior working paper drafts.

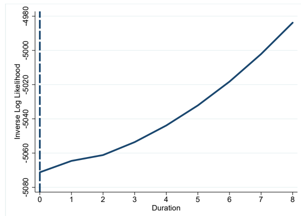
[52] In this estimation, we allow constraints to potentially bind for two quarters before the first period in which they actually bind in the simulated data. Further, we use the same priors here as in the baseline estimation.

Figure A2: Likelihood Over Domestic Durations

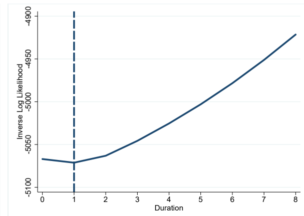
(a) Period 60



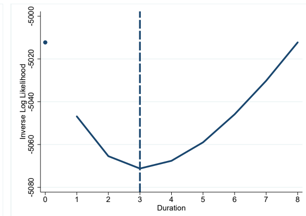
(b) Period 61



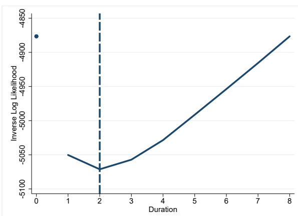
(c) Period 62



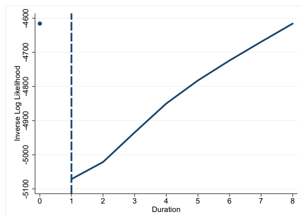
(d) Period 63



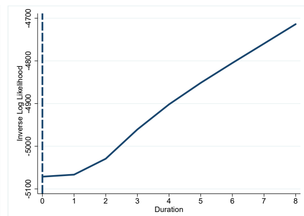
(e) Period 64



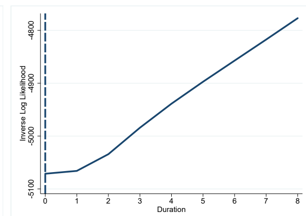
(f) Period 65



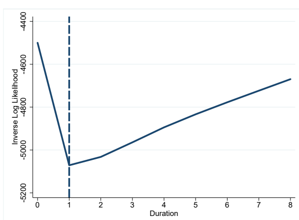
(g) Period 66



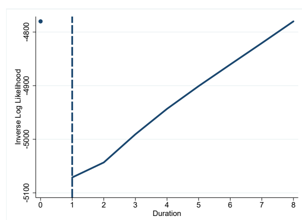
(h) Period 67



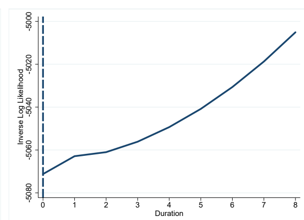
(i) Period 68



(j) Period 69



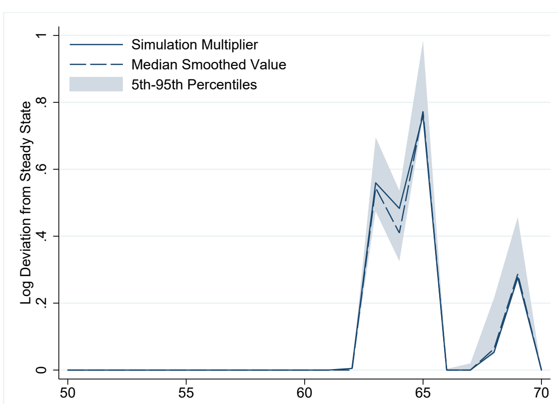
(k) Period 70



Note: The vertical dashed line marks the true duration of the constraint in the simulation for each period. In some figures, the dot denotes a value of the inverse likelihood that is substantially higher than the other values plotted in the figure; the dot is located at the maximal value depicted in the figure for visual reference.

Figure A3: Multipliers Capacity Constraints: Simulation vs. Estimation

(a) Multiplier on the Domestic Constraint



(b) Multiplier on the Foreign Constraint

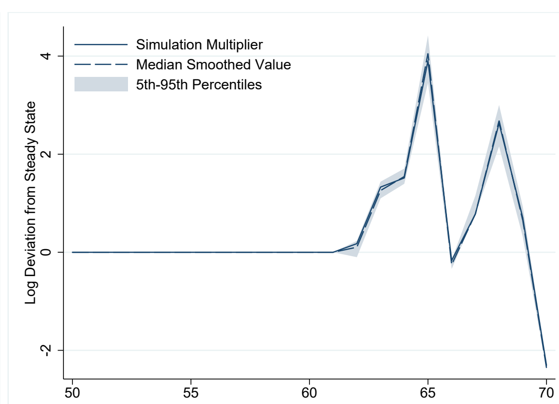
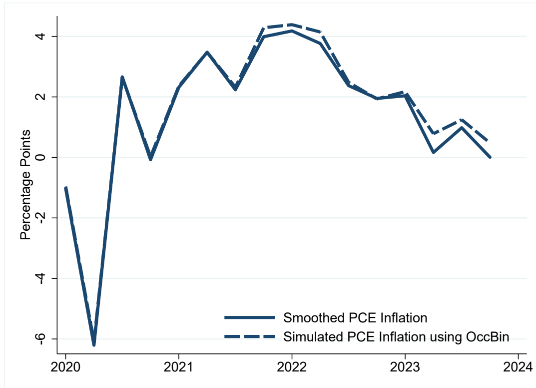


Figure A4: Comparison Between Smoothed Inflation and OccBin Simulated Inflation



Note: Smoothed PCE Inflation is Kalman-smoothed consumer price inflation, where the filter is parameterized using the modal values of structural parameters and durations from the empirical estimation. Simulated PCE Inflation using OccBin is obtained by simulating model responses to smoothed shocks.

A.4.4 Priors

The full set of priors for structural parameters is included in Table A6. We use standard priors on autoregressive persistence of exogenous variables, parameters in the monetary policy rule, elasticities, and the standard deviations of most structural shocks. We set priors on the persistences of the exogenous capacity shocks that are wider than the priors on the other exogenous variables, as well as wide (uniform) priors on the standard deviations of the capacity shocks, since these are nonstandard parameters.

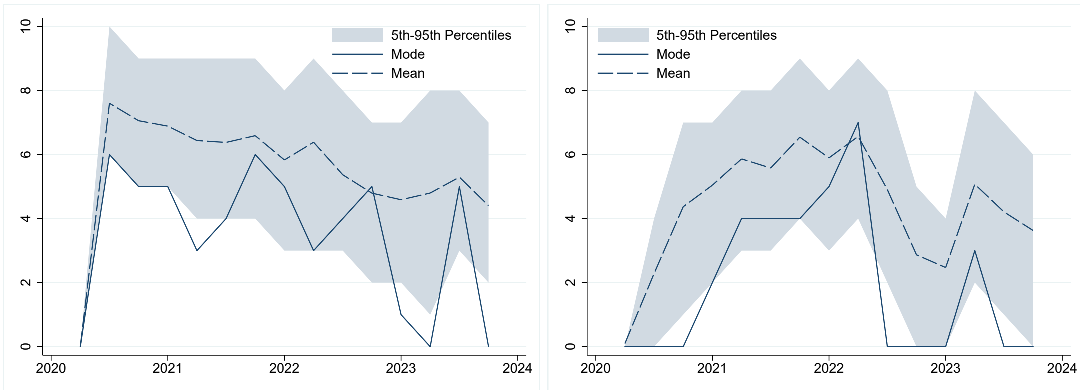
As noted in the main text, we allow constraints to potentially bind only starting in the second quarter of 2020. That is, we put zero mass on positive durations at all dates at/before 2020:Q1, which can be thought of as a dogmatic prior that constraints were not substantively important prior to the pandemic. Thereafter in each period, we place equal mass on durations of 0 to 4 quarters, summing to 60% total (12% on each discrete duration). We place 30% mass on durations of 5, 6, 7, and 8 quarters, again equally spread (7.5% each). The remaining 10% mass is spread equally over durations 9 through 12, and we place zero mass on durations longer than 12 quarters.

A.5 Estimation Results

In Table A6, we provide the mode, mean, and 5th-95th percentiles for the posterior distributions of the structural parameters. As noted in the text, we find that domestic and foreign goods inputs are complements on the production side, while domestic and foreign goods are substitutes in consumption. The Taylor rule coefficient on inflation is near 1.5, which is standard. Interest rates also depend positively on deviations of output from steady state, and the policy rule features a significant degree of inertia. The stochastic processes for shocks generally feature persistence, with

Figure A5: Posterior Distributions for Constraint Durations

(a) Domestic Constraint Durations (b) Foreign Constraint Durations



Note: At each date, there is a posterior distribution for constraint durations. Each figure presents the mean, mode, and interquartile range for this posterior distribution.

auto-regressive coefficients generally between 0.7 and 0.9. Building on the discussion of measurement error above, we note that posterior estimates for measurement errors on consumer goods price inflation and import price inflation are pushed toward the boundary of their prior distributions, reflecting tension in the model between fitting data before and during the COVID period. For all the other parameters, posterior distributions are generally well behaved, with single peaks well inside the allowable parameter space and reasonably tight distributions.

Turning to duration estimates, we plot statistics for the posterior distributions of domestic and foreign constraint durations in Figure A5. Due to skewness in the distributions, modal values for the duration (our preferred approach to summarizing the posterior distribution) are below the mean value in most periods. The time path for the duration estimates mimics the path of estimated multipliers on the constraints, as reported in the main text.

A.6 Model Fit

In the main text, we presented results on model fit for core inflation series. To evaluate model fit more broadly, we present data and smoothed values for the remaining observable variables in Figure A6.[53] For legibility in the figures, we focus on the 2017-2022 period – the key period leading up to and through our analysis. The model fits most series well, even capturing the whiplash dynamics of the data in 2020. The model struggles to replicate data on US labor productivity, particularly in 2020 for services. Through the lens of the model, this implies that the data contains substantial measurement error during the pandemic period, which seems plausible to us. More broadly, a more

[53] We assume the interest rate is measured without error, so it is omitted here.

Table A6: Prior and Posterior Distributions for Structural Parameters

	Prior			Posterior			
Panel A: Elasticity and Taylor Rule	Dist	Mean	SD	Mode	Mean	5%	95%
Consumption Armington Elasticity: ι	G	1.50	0.25	1.508	1.540	1.168	1.951
Input Armington Elasticity: η	G	0.50	0.25	0.636	0.680	0.412	1.003
Taylor Rule Inflation: ω	N	1.50	0.12	1.469	1.491	1.312	1.677
Taylor Rule Inertia:α_i	B	0.75	0.10	0.866	0.865	0.844	0.885
Taylor Rule Output: α_y	G	0.12	0.05	0.228	0.241	0.176	0.311
Panel B: Stochastic Processes							
Preference for Goods: σ_ζ	IG	1	2	0.290	0.334	0.180	0.528
Discount Rate: σ_Θ	IG	1	2	3.270	3.343	2.965	3.754
Foreign Costs: σ_{rmc^*}	IG	1	2	1.960	1.972	1.588	2.399
Goods Productivity: $\sigma_{z(1)}$	IG	1	2	0.187	0.188	0.123	0.266
Services Productivity: $\sigma_{z(2)}$	IG	1	2	0.190	0.197	0.126	0.280
Foreign Constraint: $\sigma_{\bar{y}^*}$	U	1	0.58	0.028	0.047	0.019	0.106
Domestic Constraint: $\sigma_{\bar{y}}$	U	1	0.58	0.012	0.015	0.008	0.024
Monetary Policy Shock: σ_i	IG	1	2	0.153	0.155	0.137	0.176
Preference for Goods: ρ_ζ	B	0.50	0.10	0.602	0.583	0.406	0.755
Discount Rate: ρ_Θ	B	0.50	0.15	0.751	0.750	0.708	0.787
Foreign Costs: ρ_{rmc^*}	B	0.50	0.15	0.938	0.933	0.895	0.965
Goods Productivity: $\rho_{z(1)}$	B	0.50	0.10	0.520	0.529	0.370	0.682
Services Productivity: $\rho_{z(2)}$	B	0.50	0.10	0.526	0.550	0.370	0.745
Foreign Constraint: $\rho_{\bar{y}^*}$	B	0.50	0.15	0.897	0.816	0.553	0.952
Domestic Constraint: $\rho_{\bar{y}}$	B	0.50	0.15	0.953	0.933	0.865	0.977
Panel C: Measurement Error							
Goods PCE: $\sigma^{\text{me}}_{\text{pceg}}$	IG	1	2	0.988	0.995	0.855	1.130
Services PCE: $\sigma^{\text{me}}_{\text{pces}}$	IG	1	2	0.618	0.630	0.540	0.726
Goods PCE Inflation: $\sigma^{\text{me}}_{\pi(1)}$	IG	1	2	0.765	0.774	0.697	0.860
Services PCE Inflation: $\sigma^{\text{me}}_{\pi(2)}$	IG	1	2	0.191	0.191	0.160	0.223
Imp. Input Goods Expenditure: $\sigma^{\text{me}}_{\text{inp}}$	IG	1	2	3.265	3.248	2.925	3.589
Imp. Consumption Goods Expenditure: $\sigma^{\text{me}}_{\text{finp}}$	IG	1	2	2.903	2.901	2.614	3.214
Imp. Input Goods Inflation: $\sigma^{\text{me}}_{\text{inpp}}$	IG	1	2	2.013	2.053	1.847	2.278
Imp. Consumption Goods Inflation: $\sigma^{\text{me}}_{\text{fimp}}$	IG	1	2	0.233	0.233	0.158	0.311
Goods Productivity: $\sigma^{\text{me}}_{\text{prod1}}$	IG	1	2	1.114	1.135	0.989	1.291
Services Productivity: $\sigma^{\text{me}}_{\text{prod2}}$	IG	1	2	1.054	1.047	0.935	1.172
Industrial Production: $\sigma^{\text{me}}_{\text{ip}}$	IG	1	2	0.967	0.992	0.871	1.122
Aggregate Nominal GDP: $\sigma^{\text{me}}_{\text{nva}}$	IG	1	2	0.434	0.447	0.378	0.525

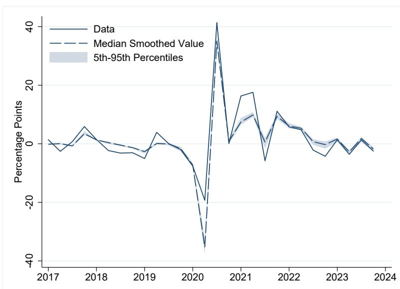
Note: G denotes the gamma distribution, IG denotes the inverse gamma distribution, U denotes the uniform distribution, B denotes the beta distribution, and N denotes the normal distribution.

sensitive treatment of the impact of lockdowns on the services sector would likely be needed to match data in the middle quarters of 2020. Nonetheless, the model is able to capture the dynamics of services inflation well overall, particularly in 2021-2022 when inflation rises.

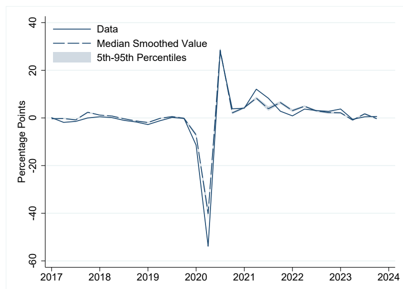
Turning to non-targeted data, we now compare smoothed values for multipliers attached to the constraints to an external measure of supply chain disruptions. Specifically, we use the Global Supply Chain Pressure Index (GSCPI), developed by the New York Federal Reserve [Benigno et al. (2022)], which combines data on transportation costs (sea and air freight rates) with elements of Purchasing Managers' Index surveys pertaining to supply chain management from major industrial countries (China, the Eurozone, Japan, United States, etc.). To be clear, this data is not directly

Figure A6: Data and Smoothed Model Observables

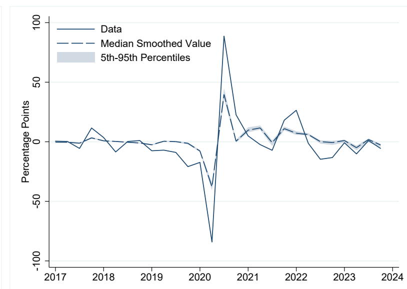
(a) Goods Cons. Exp.



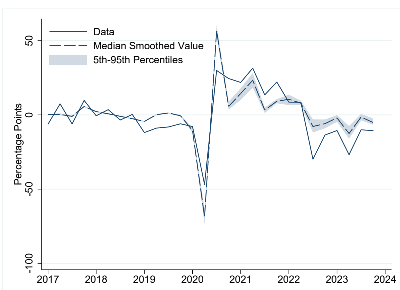
(b) Services Cons. Exp.



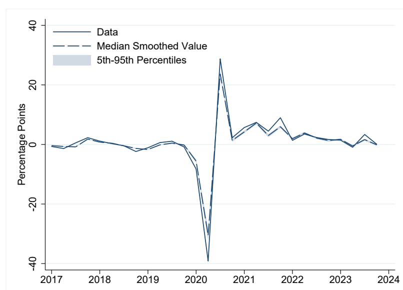
(c) Import Cons. Goods Exp.



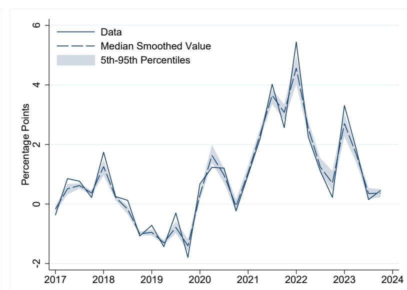
(d) Import Goods Input Exp.



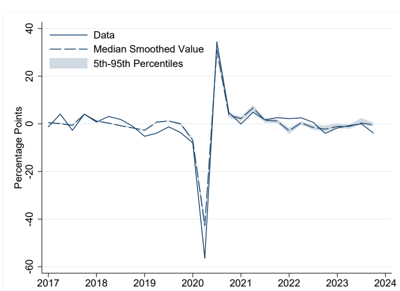
(e) Nominal GDP



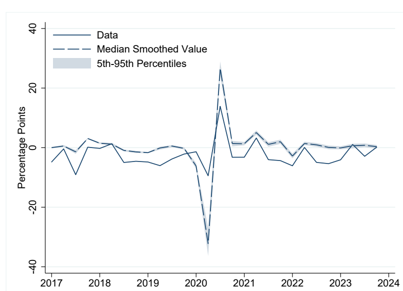
(f) Import Cons. Goods Inflation



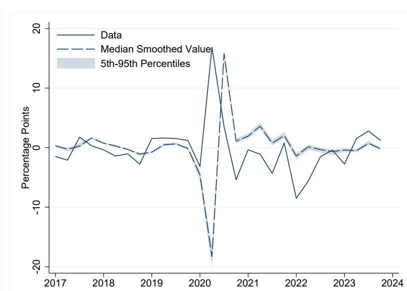
(g) Industrial Production



(h) Goods Productivity

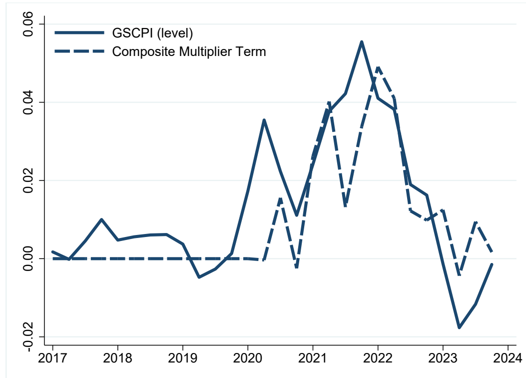


(i) Aggregate Productivity



Note: All data and simulated series are annualized values for de-meaned quarterly growth rates in percentage points. Data is raw data. We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, and then plot the median smoothed value as the dashed line. We shade the area covering the the 5% to 95% percentile for smoothed values.

Figure A7: Comparing the NY Fed GSCPI to the Weighted Mean of Constraint Multipliers



Note: To make the scale of the GSCPI index comparable to the multiplier, we plot the raw level of the GSCPI index divided by 75. The Composite Multiplier is computed as $0.75\left(\frac{\varepsilon}{\phi(s)}\frac{P_0}{P_{H0}(s)}\right)\hat{\tilde{\mu}}_t(s)+0.25\left(\frac{\varepsilon}{\phi(s)}\frac{P_0}{P_{uF0}(s)}\right)\hat{\tilde{\mu}}^*_{ut}(s)$. The weight on the domestic term is 0.75 and the weight on the foreign term is 0.25, which roughly correspond to shares of total spending allocated to domestic and foreign goods.

related to the theoretical construct that we recover from the data.[54] Further, it is a proxy for global conditions, which doesn't distinguish between US-based and foreign supply chain pressures, so we compare it to a weighted mean of the median multipliers on the domestic and foreign constraints. With all these caveats, we plot the GSCPI and the weighted mean multiplier in Figure A7. As is evident, both the composite multiplier and the GSCPI index rise and fall around similar dates. Thus, our approach to structurally estimating when constraints bind produces results that line up relatively well with purely data-based approaches to diagnosing constraints.

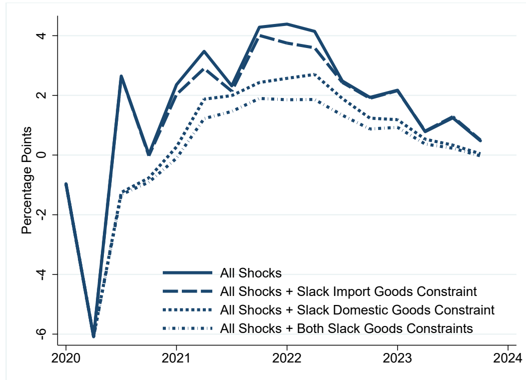
A.7 Additional Counterfactual: Domestic vs. Import Constraints

In the main text, we provide counterfactual results for scenarios in which we relax both the domestic and foreign constraints in the goods market. Here, we assess the relative contribution of each constraint by relaxing one constraint at a time, and then in tandem. To generate the counterfactuals, we draw parameters from their posterior distributions to parameterize the model, then we filter the data to recover smoothed shocks, then we feed the shocks through the model allowing a particular set of constraints to be potentially binding (solving for whether constraints bind endogenously, using OccBin). We repeat this procedure 1000 times and report median outcomes in Figure A8.

Similar to Figure 7, relaxing both constraints together evidently lowers realized inflation. The

[54]It also is not scaled in way that is directly comparable to our estimates. The raw GSCPI index is reported as deviations from its mean value, in units of the standard deviation of the series. The NY Fed does not report either the mean or standard deviation, so we cannot compute log changes in the underlying index. Further, there is no obvious relationship between units attached to the multipliers – which summarize impacts of constraints on inflation – and units on the GSCPI. Because the GSCPI is reported at the monthly frequency, we take simple means across three month intervals to form quarterly values.

Figure A8: Counterfactual Inflation With Relaxed Domestic versus Foreign Constraints



Note: Each series represents the median value across simulated paths for consumer price inflation (quarterly value, annualized). Each simulation solves for inflation given smoothed shocks, filtered from the data for a given draw from the posterior distribution of model parameters.

domestic constraint plays a more important role in explaining the joint gap, accounting for roughly three-quarters of the overall impact of constraints. This is to be expected, in that imports account for a minority of overall spending on inputs, which limits the quantitative role of import constraints relative to domestic input constraints. Nonetheless, both constraints play independent roles.

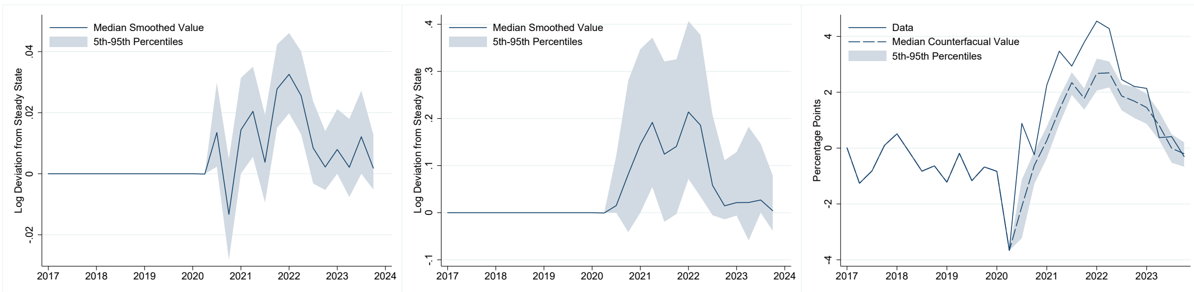
A.8 Estimated Capacity Levels

For results presented in the main text (and above), we calibrate the levels of domestic and foreign goods capacity in steady state. However, we could instead estimate those levels, with an important caveat. The caveat is that we allow constraints to bind only after 2020 in the estimation. The steady-state capacity level is the level to which capacity reverts in the long run, in the absence of shocks. We are able to estimate this level conditional on the data in periods in which constraints are potentially binding. Thus, if we estimate capacity levels, we are attempting to infer the capacity level only using post-2020 data. Naturally, since constraints were binding for much of this period, plausibly due to negative shocks that pushed realized capacity down, using only this data will tend to lead us to estimate a relatively low level for steady-state capacity. And in fact, this is that we find when we treat capacity levels as parameters to be estimated: steady state goods capacity is roughly 1% above the steady state level of goods output, which is lower than the calibrated value we have used previously. Nonetheless, this difference in the level of steady-state capacity has little import for our quantitative assessment.

To demonstrate this, we provide supplemental figures illustrating results from a version of the model in which capacity is estimated in Figure A9. In Figures A9a and A9b, presented estimated multipliers on the constraints. In Figure A9c, we then replicate the counterfactual in which we relax

Figure A9: Model Counterfactuals with Estimated Steady-State Capacity Levels

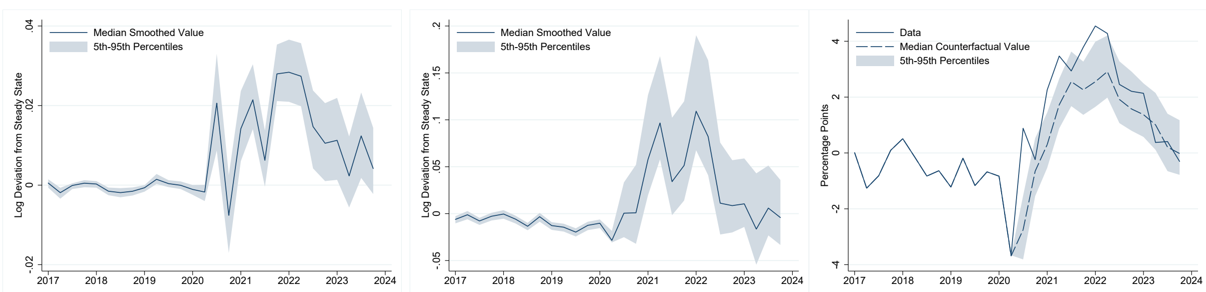
(a) Domestic Multiplier (b) Foreign Multiplier (c) Counterfactual Inflation



Note: The multipliers on the domestic and foreign constraints are reported in markup shock equivalent units, as in Figure 6. Panel (c) presents counterfactual consumer price inflation when both constraints are relaxed, as in Figure 7.

Figure A10: Model Counterfactuals with Exogenous Markup Shocks

(a) Domestic Multiplier (b) Foreign Multiplier (c) Counterfactual Inflation



Note: The multipliers on the domestic and foreign constraints are reported in markup shock equivalent units, as in Figure 6. Panel (c) presents counterfactual consumer price inflation when both constraints are relaxed, as in Figure 7.

both the domestic and import goods constraints. The results are both qualitatively and quantitatively similar to prior results.

A.9 Exogenous Markup Shocks

As discussed in Section 2.6.1, binding constraints manifest as reduced-form markup shocks in the domestic and import price Phillips Curves. So, it is natural to ask whether binding constraints may be separately identified from exogenous markup shocks, as would typically be included in DSGE estimation of New Keynesian models. To investigate this, we introduce exogenous markup shocks into both the domestic and foreign price Phillips Curves of the baseline model, and we assume the markup shocks follow an AR1 stochastic process. We then re-estimate this extended model over the full sample; for intuition, note that pre-COVID data serves to discipline the parameters in the stochastic processes for markup shocks. We report selected results from this re-estimated model in

Figure A10.

In Figures A10a and A10b, we plot smoothed values for the multipliers on the domestic and foreign constraints, in markup shock equivalent units (as in the main text). The takeaway is that the impact of constraints is identified by the combination of model and data, even when we allow for exogenous markup shocks. Further, the dynamics of the multipliers is little changed from the baseline estimation. In Figure A10c, we report counterfactual inflation when the constraints are exogenously relaxed, allowing all other shocks (including exogenous markup shocks) to be active. Relaxing constraints lowers inflation during 2021-2022, by magnitudes similar to the baseline model. Thus, we conclude that our main results are robust to allowing for exogenous markup shocks.

B Fiscal Policy Extension

As described in the main text, we extend the model on the household side to introduce two types of households: hand-to-mouth households (denoted by superscript m) and "Euler consumers" who have access to complete financial markets (denoted by superscript e). L_0^m and L_0^e are the (time-invariant) measures of consumers of each type, with $L_0 = L_0^m + L_0^e$ equal to the total population (normalized so that $L_0 = 1$). We summarize the modifications to the model by presenting the log-linearized conditions added to the equilibrium system.

Hand-to-mouth consumers choose their labor supply and then consume their entire income. Their income includes both labor income ($W_t L_t^m$), which is subject to a proportional income tax at rate $\tau \in (0,1)$, and time-varying transfer payments T_t received from the government. The log-linear labor supply equation and budget constraint for the hand-to-mouth household are:

$$-\rho \hat{c}^m + \hat{rw}_t = \psi \hat{l}_t^m \tag{26}$$

$$C_0^m \hat{c}_t^m = (1-\tau)\frac{W_0}{P_0} L_0^m (\hat{rw}_t + \hat{l}_t^m) + \frac{T_0}{P_0}\hat{rt}_t. \tag{27}$$

In the budget constraint, $\hat{rt}_t = \ln(T_t/P_t) - \ln(T_0/P_0)$ is log deviation of the real transfer received from steady state.

The Euler households choose their consumption, asset positions, and labor supply like the representative household in the baseline model. Adding up across households, we obtain aggregate labor supply and consumption:

$$\hat{l}_t = \left(\frac{L_0^m}{L_0}\right)\hat{l}_t^m + \left(\frac{L_0^e}{L_0}\right)\hat{l}_t^e \tag{28}$$

$$\hat{c}_t = \left(\frac{C_0^m}{C_0}\right)\hat{c}_t^m + \left(\frac{C_0^e}{C_0}\right)\hat{c}_t^e, \tag{29}$$

where $\hat{l}_t^e$ and $\hat{c}_t^e$ reflect the labor and consumption choices for Euler households. C_0^m and C_0^e are the levels of consumption for consumers for each type in steady state, with $C_0 = C_0^m + C_0^e$.

Turning to the government, we assume it issues one period bonds and is able to borrow as the risk-free interest rate (i_t). The nominal value of government bonds at the end of period t, given by B_t, evolves according to:

$$B_t - B_{t-1} = [i_{t-1}B_{t-1} + T_t] - \tau W_t L_t, \tag{30}$$

where the right-hand side of the equation is the nominal budget deficit. Defining the real stock of debt as $RB_t \equiv B_t/P_t$, then the log-linearized government budget constraint is:

$$RB_0\hat{rb}_t = (1+i_0)\, RB_0\left(i_{t-1} - \pi_t + \hat{rb}_{t-1}\right) + RT_0\hat{rt}_t - \tau RW_0L_0\left(\widehat{rw}_t + \hat{l}_t\right), \tag{31}$$

where $\hat{rb}_t = \ln(RB_t/RB_0)$ and $\hat{rt}_t = \ln(T_t/P_t)$.[55] As discussed in the main text, the government implements a fiscal rule that serves to stabilize the real stock of debt. Together with the budget constraint, the fiscal rule ensures that the debt-to-GDP ratio is stationary, so the usual no-Ponzi scheme condition for government debt is satisfied.

With these modifications, the equilibrium conditions for this model differ from those for the baseline model, presented in Tables A2 and A3, as follows. First, we modify the consumer module, dropping the labor supply and Euler equation for the representative consumer, and replacing them with equilibrium conditions for Euler and hand-to-mouth households.[56] Second, we add the government's fiscal policy rule and budget constraint to the system. The new equilibrium conditions are collected in Table A7.

C Labor Market Extension

This section extends the baseline model to incorporate sticky wages, potentially binding labor market constraints, and shocks to the disutility of labor. We now assume there is a unit continuum of

[55] As in the baseline model, we assume zero inflation in steady state.

[56] Since we assume that both types of consumers have identical preferences across sectors and Home/Foreign goods, we do not need to modify equations that pin down consumer prices.

Table A7: Equilibrium Conditions with Fiscal Policy

Hand-to-Mouth Households	$-\rho\hat{c}^m + \hat{rw}_t = \psi\hat{l}^m_t$ $\hat{c}^m_t = (1-\tau)\left(\frac{W_0 L^m_0}{P_0 C^m_0}\right)(\hat{rw}_t + \hat{l}^m_t) + \left(\frac{T_0}{P_0 C^m_0}\right)\hat{rt}_t$
Euler Households	$-\rho\hat{c}^e + \hat{rw}_t = \psi\hat{l}^e_t$ $0 = E_t\hat{\Theta}_{t+1} - \hat{\Theta}_t - \rho(\hat{c}^e_{t+1} - \hat{c}^e_t) + i_t - E_t\hat{\pi}_{t+1}$ $\hat{c}^e_t = \hat{c}^*_t + \frac{1}{\rho}\left(\hat{q}_t + \hat{\Theta}_t\right)$
Household Aggregation	$\hat{l}_t = \left(\frac{L^m_0}{L_0}\right)\hat{l}^m_t + \left(\frac{L^e_0}{L_0}\right)\hat{l}^e_t$ $\hat{c}_t = \left(\frac{C^m_0}{C_0}\right)\hat{c}^m_t + \left(\frac{C^e_0}{C_0}\right)\hat{c}^e_t$
Government Debt	$B_t - B_{t-1} = i_{t-1}B_{t-1} + T_t - \tau W_t L_t$
Government Budget Constraint	$\hat{rb}_t = (1+i_0)\left(i_{t-1} - \pi_t + \hat{rb}_{t-1}\right) + \left(\frac{RT_0}{RB_0}\right)\hat{rt}_t - \tau\left(\frac{RW_0 L_0}{RB_0}\right)\left(\widehat{rw}_t + \hat{l}_t\right)$
Fiscal Rule for Transfers	$\hat{rt}_t = \varphi_1\hat{rt}_{t-1} - \varphi_2\hat{rb}_t + \varepsilon_t$

consumers, indexed by $j \in (0,1)$. Consumers are identical, with one exception: each is the monopolistic supplier of its differentiated labor services to the market. Further, the amount of labor that each consumer is able to supply is bound above by $\overline{L}_t$, which is exogenous and time varying. Differentiated labor services supplied by consumers are costlessly aggregated into a composite bundle by competitive intermediaries and sold to firms. The labor aggregation technology is given by $L_t = \left(\int_0^1 L_t(j)^{(\varepsilon_L-1)/\varepsilon_L} dj\right)^{\varepsilon_L/(\varepsilon_L-1)}$, where $\varepsilon_L > 1$ is the elasticity of substitution between differentiated labor services and the price index for the labor composite is $W_t = \left(\int_0^1 W_t(j)^{1-\varepsilon_L} dj\right)^{1/(1-\varepsilon_L)}$. Finally, each consumer pays Rotemberg-type adjustment costs to modify the nominal wage at which it supplies labor, as in Born and Pfeifer (2020).

Consumer j chooses its consumption, wage, and asset holdings to maximize utility, subject to its budget constraint, the demand curve for its labor, and the labor supply constraint:

$$\max_{\{C_t(j), W_t(j), B_{t+1}(j)\}_{t=0}^{\infty}} \quad E_0 \sum_{t=0}^{\infty} \beta^t \Theta_t \left[\frac{(C_t(j))^{1-\rho}}{1-\rho} - \Lambda_t \frac{L_t(j)^{1+\psi}}{1+\psi}\right] \tag{32}$$

$$\text{s.t.} \quad P_t C_t(j) + \mathbb{E}_t\left[S_{t,t+1}B_{t+1}(j)\right] \leq B_t(j) + W_t(j)L_t(j) - \frac{\phi_W}{2}\left(\frac{W_t(j)}{W_{t-1}(j)} - 1\right)^2 W_t L_t,$$

$$L_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_L} L_t, \quad \text{and} \quad L_t(j) \leq \overline{L}_t,$$

where ϕ_W is a parameter governing wage adjustment costs and Λ_t governs the disutility of labor

Table A8: Equilibrium Conditions with Binding Constraints for Labor

Panel A: Labor Constraint is Slack	
Wage Setting	$\pi_{Wt} = \left(\frac{\epsilon_L - 1}{\phi_W}\right)[\widehat{mrs}_t - \widehat{rw}_t] + \beta \mathbb{E}_t(\pi_{Wt+1})$
Marginal Rate of Substitution	$\widehat{mrs}_t = \hat{\lambda}_t + \psi \hat{l}_t - \rho \hat{c}_t$
Auxiliary Inflation Definition	$\pi_{Wt} = \widehat{rw}_t - \widehat{rw}_{t-1} + \pi_t$
Panel B: Labor Constraint Binds	
Wage Setting	$\pi_{Wt} = \left(\frac{\epsilon_L - 1}{\phi_W}\right)[\widehat{mrs}_t - \widehat{rw}_t] + \left(\frac{\epsilon_L}{\phi_W}\frac{P_0}{W_0}\right)\hat{\tilde{\mu}}_{Lt} + \beta \mathbf{E}_t(\pi_{Wt+1})$
Marginal Rate of Substitution	$\widehat{mrs}_t = \hat{\lambda}_t + \psi \hat{l}_t - \rho \hat{c}_t$
Auxiliary Inflation Definition	$\pi_{Wt} = \widehat{rw}_t - \widehat{rw}_{t-1} + \pi_t$
Labor Market Constraint	$\hat{l}_t = \hat{\bar{l}}_t + \ln(\bar{L}_0/L_0)$

supply. In a symmetric equilibrium, the first order condition for the wage is:

$$1 - \varepsilon_L \left(1 - \frac{MRS_t + \left(\mu_{Lt}/C_t{}^{-\rho}\right)}{W_t/P_t}\right) - \phi_W (\Pi_{Wt} - 1)\Pi_{Wt} + \mathbb{E}_t\left[\beta \frac{\Theta_{t+1}}{\Theta_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho} \frac{1}{\Pi_{t+1}} \phi_W (\Pi_{Wt+1} - 1)\Pi_{Wt+1}^2 \frac{L_{t+1}}{L_t}\right] = 0, \quad (33)$$

where μ_{Lt} is the multiplier on the labor constraint, $\Pi_{Wt} \equiv \frac{W_t}{W_{t-1}}$, and $MRS_t = \frac{\Lambda_t L_t^{\psi}}{C_t^{-\rho}}$ is the marginal rate of substitution between consumption and labor supply in preferences. Further, the complementary slackness condition applies: $\left(L_t - \overline{L}_t\right)\mu_{Lt} = 0$, with $\mu_{Lt} \geq 0$.

Taking a log linear approximation for this equation, we arrive at the wage Phillips Curve presented in the main text:

$$\pi_{Wt} = \left(\frac{\epsilon_L - 1}{\phi_W}\right)[\widehat{mrs}_t - \widehat{rw}_t] + \left(\frac{\epsilon_L}{\phi_W}\frac{P_0}{W_0}\right)\hat{\tilde{\mu}}_{Lt} + \beta \mathbb{E}_t(\pi_{Wt+1}), \quad (34)$$

where $\pi_{Wt} \equiv \hat{w}_t - \hat{w}_{t-1} = \widehat{rw}_t - \widehat{rw}_{t-1} + \pi_t$ is nominal wage inflation, $\widehat{rw}_t \equiv \hat{w} - \hat{p}_t$, $\widehat{mrs}_t = \hat{\lambda}_t + \psi \hat{l}_t - \rho \hat{c}_t$ with $\hat{\lambda}_t \equiv \ln \Lambda_t - \ln \Lambda_0$, and $\hat{\tilde{\mu}}_{Lt} \equiv \ln \tilde{\mu}_{Lt} - \ln \tilde{\mu}_{L0}$ where $\tilde{\mu}_{Lt} \equiv 1 + (\mu_{Lt}/C_t{}^{-\rho})$ is a function of the multiplier on the labor constraint.

To define equilibrium in this model, we modify the equilibrium conditions from Tables A2 and A3 as follows. First, we drop the "labor supply" condition from the baseline model, as labor supply is no longer determined by equating the marginal rate of substitution to the real wage. Second, we add the equilibrium conditions in Table A8, where Panel A corresponds to an equilibrium when labor constraints are slack, and Panel B corresponds to the case when they are binding. The new endogenous variables in the equilibrium system are: $\{\pi_{Wt}, \widehat{mrs}_t\}$ when the labor constraint is slack, and $\left\{\pi_{Wt}, \widehat{mrs}_t, \hat{\tilde{\mu}}_{Lt}\right\}$ when the labor constraint binds. Combined with the goods constraints, this defines eight model regimes with different combinations of binding and slack constraints.

Turning to quantitative implementation of this model, we start by describing new calibrated parameters. We set $\epsilon_L = 21$, following Christiano et al. (2005). We then choose ϕ_W so that the slope of the wage Phillips Curve is equivalent to a Calvo model with wage adjustment parameter 0.4, when $\epsilon_L = 21$. This Calvo wage adjustment target is taken from Fitzgerald et al. (forthcoming), who estimate it based on state-level data. The implied slope of the wage Phillips Curve is then about 0.02, which is relatively flat. We calibrate the level of the labor constraint $(\bar{L}_0)$ to be 1% higher than steady state labor supply. Because the actual level of the constraint at a given point in time is a realization of a stochastic process, results are not sensitive to this value.

We assume the disutility of labor evolves according to $\hat{\lambda}_t = \rho_\lambda \hat{\lambda}_{t-1} + \varepsilon_{\lambda t}$, where $var\left(\varepsilon_{\lambda t}\right) = \sigma_\lambda^2$ and $cov\left(\varepsilon_{\lambda t}, \varepsilon_{\lambda t+s}\right) = 0$ for $s \neq 0$, and we estimate ρ_λ and σ_λ. Further, we assume that the labor constraint is subject to shocks, such that $\ln \bar{L}_t - \ln \bar{L}_0 \equiv \hat{\bar{l}}_t = \varepsilon_{\bar{l}t}$ with $var\left(\varepsilon_{\bar{l}t}\right) = \sigma_{\bar{l}}^2$ and $cov\left(\varepsilon_{\bar{l}t}, \varepsilon_{\bar{l},t+s}\right) = 0$ for $s \neq 0$, and we estimate $\sigma_{\bar{l}}$.

We assume observables (aggregate hours worked and real wage growth) are measured with error and estimate the variance of the measurement errors. We also re-estimate all the same structural parameters and stochastic processes using this version of the model. To do so, we assemble data on aggregate hours worked and real wage growth from raw data provided by the US Bureau of Labor Statistics. To construct real wage growth, we use hourly compensation data for the non-farm business sector to proxy for nominal wage growth (FRED series id: COMPNFB), taking log growth rates of that quarterly index. We then deflate this nominal wage growth using the aggregate PCE price index, used in prior sections. To build an aggregate hours series, we combine several series. We use average weekly hours of production and nonsupervisory works in the private sector (FRED series id: AWHNONAG) to proxy hours per worker. We then compute the ratio of employment (FRED series id: CE16OV) to population (FRED series id: CNP16OV), were we smooth population estimates by taking means within two-year moving windows in order to eliminate jumps due to data revisions. We then multiply average weekly hours by the employment to population ratio, take logs of that index, and compute deviations from the sample mean of the index over the 1992:Q2 to 2019:Q4 (the pre-COVID sample).