# Detection of anticipated structural changes in a rational expectations environment

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When agents have rational expectations, anticipated changes in the structure of the economy have an immediate affect on their behaviour. In this article, we investigate the interplay between a linear rational expectation model with predictable structural changes and reduced-form evidence of structural breaks. In our study, we vary the length of time between the announcement and the implementation of an inflation target change. Using a model similar to Ireland (2007) and the method presented in Bai and Perron (1998) and Bai and Perron (2003) to estimate unknown structural breaks, Monte Carlo simulation results suggest that reduced-form evidence of structural breaks are broadly in line with what is predicted by forward-looking rational expectation models; that is, as the transition period increases, break estimates gradually move farther from the policy announcement date.

**Keywords:** DSGE model; inflation target; Monte Carlo simulation; structural breaks

JEL Classification: C13; C15; E22; E32; E37

## I. Introduction

When agents have rational expectations, anticipated changes in the structure of the economy have an immediate effect on their behaviour. For example, if the central bank credibly announces a future change in the parameters of its policy rule, the economy moves to its new steady state prior to the implementation.

We are interested in when reduced-form structural break tests identify structural change in an economy when agents are forward looking. If the variables move before the change in the structure of the economy, we should expect break tests to register before the implementation of the new structure. We investigate this across the dimension of the length of time between the announcement and the implementation of a monetary policy change. We use as an example, a disinflation resulting from an announced change in the inflation target and a more aggressive response of monetary policy to deviations in inflation from steady-state.

Such situations appear in observed data. The Central Bank of Chile, for example, after gaining its independence in 1989, announced annual inflation targets from 1991 to 1999, with a commitment to reach low and single-digit inflation.<sup>1</sup> Figure 1 shows the evolution of inflation over this period, together with the announced target bands. As seen, the rate of inflation gradually moved towards the final structure. This is also a topical issue in a number of developed countries around the world,

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<sup>&</sup>lt;sup>1</sup> See Schmidt-Hebbel and Tapia (2002) for a discussion of the Chilean experience on the implementation of inflation targeting.



Fig. 1. Chilean inflation, 1980–2011

with central banks announcing a path for policy rates; for example, the Bank of Canada announced on 21 April 2009 that it would hold the policy rate at 0.25% until 'Autumn 2010' after which it would return to policy rule.

The remainder of this article is structured as follows: Section II briefly presents the structural model used to simulate data; Section III reports and discusses our results; Section IV concludes.

## II. The Model

The model we use is a modified version of Ireland (2007). We deviate from Ireland by using a constant rather than time-varying inflation target and a stationary rather than permanent technology process. The seven linearized equations in the seven variables output  $y_t$ , the nominal interest rate  $r_t$ , the inflation rate  $\pi_t$ , the growth rate of output  $g_t$ , and the exogenous shocks, demand  $a_t$ , mark-up  $e_t$  and stationary technology  $z_t$ , are

$$\hat{y}_{t} = -\frac{1}{\sigma}(r_{t} - E_{t}\pi_{t+1}) + E_{t}\hat{y}_{t+1} + \frac{(1 - \rho_{a})}{\sigma}\hat{a}_{t} - \frac{1}{\sigma}\ln\beta$$
(1)

$$(1 + \beta \alpha)\pi_t = (1 + \beta \alpha - \alpha - \beta)\pi^* + \alpha \pi_{t-1} + \psi \sigma \hat{y}_t - \psi \hat{z}_t + \beta E_t \pi_{t+1} - \hat{e}_t$$
(2)

$$\begin{aligned} \dot{r}_{t} &= (1 - \rho_{r})(\pi^{*} - \ln \beta) + \rho_{r} r_{t-1} \\ &+ \rho_{\pi}(\pi_{t} - \pi^{*}) + \rho_{v} \dot{y}_{t} + \rho_{g} \dot{g}_{t} + \varepsilon_{r,t} \end{aligned}$$
(3)

$$\hat{g}_t = \hat{y}_t - \hat{y}_{t-1}$$
 (4)

$$\hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_{a,t} \tag{5}$$

$$\hat{e}_t = \rho_e \hat{e}_{t-1} + \varepsilon_{e,t} \tag{6}$$

$$\ln z_t = (1 - \rho_z) \ln z + \rho_z \ln z_{t-1} + \varepsilon_{z,t} \tag{7}$$

Hat notation represents deviations from steady state. Equations 1–3 give the IS-curve, the Phillips curve and the Taylor rule, respectively.  $\pi^*$  is the inflation target. Details of the parameters and their calibrated values are presented in Table 1.

The solution method we use is that of Cagliarini and Kulish (2013) who provide conditions under which the path of an economy is unique when there is an anticipated change in the structural parameters of the economy.

#### **III. Results**

Empirical evaluation of structural breaks follows the procedures in Bai and Perron (1998) and Bai and Perron (2003) in which break dates can be endogenously determined when estimating multiple

Parameter	Description	Value	
$\pi^*$	Target rate of annual inflation (per cent)	10	
$\rho_r$	Persistence of the interest rate in the Taylor rule	0.65	
$\rho_{\pi}$	Reaction to deviations of inflation from $\pi^*$	0.5	
$\rho_v$	Reaction to deviation of output from steady state	0.1	
$\rho_{\sigma}$	Reaction to output growth	0.2	
Z	Steady-state level of TFP	1.1	
β	Discount factor	0.9925	
σ	Inverse of the intertemporal elasticity of substitution	1	
α	Degree to which pricing is backward looking	0.25	
$\psi$	Degree of nominal rigidities	0.1	
$\rho_a$	Persistence of demand shock	0.9	
$\rho_e$	Persistence of mark-up shock	0.9	
$\rho_z$	Persistence of technology shock	0.9	
$\sigma_a$	SD of demand shock	0.02	
$\sigma_e$	SD of mark-up shock	0.001	
$\sigma_z$	SD of technology shock	0.007	
$\sigma_r$	SD of policy shock	0.002	

 Table 1.
 Parameter calibration

structural change models. We apply these methods using Monte Carlo simulation to study what patterns, if any, might emerge when associating the locus of break date estimates to different policy transition periods.<sup>2</sup>

The simulated data has the following characteristic: the monetary authority announces a change in the parameters of the monetary policy rule for three different time periods between the announcement and the implementation 8 quarters, 20 quarters and 40 quarters after the announcement. The i.e. announced change is an increase in the persistence of the interest rate in the Taylor rule  $\rho_r$  to 1, an increase in the reaction of the policy rate to deviations of inflation from steady state  $\rho_{\pi}$  to 1 and a decrease in the annual inflation target  $\pi^*$  to 2%. These assumptions generate simulated series which approximate the experience of a number of disinflating economies (Fig. 2).

Reduced-form estimation of a regime shift in  $\pi$  is based on two parsimonious specifications widely used in the structural break literature:<sup>3</sup> an intercept plus an error term and a first-order autoregressive model. When all the dynamics are contained in the disturbance, as for the intercept plus error case, we use a heterscedasticity and autocorrelation consistent estimator along the lines of Andrews (1991) to allow for consistent estimation of the variance– covariance matrix. For the AR(1) model, we use a full break specification whereby breaks in the level of  $\pi$  can result from time variations in the intercept or inflation persistence (or a combination of both). For robustness, we also estimate breaks using the Phillips curve specification as shown in Equation 2.<sup>4</sup>

First, we investigate if changes in the policy rule of the kind discussed above can bring about breaks in  $\pi$  which are statistically significant. Table 2 summarizes the results of Monte Carlo simulations using the hypothesis tests presented in Bai and Perron (1998) for a transition period of 40 quarters.<sup>5</sup> For 5000 replications, the Udmax and Wdmax statistics (first and second column, respectively) report strong evidence of breaks across all sample sizes, with a rejection rate of the null hypothesis of no breaks close to 100% of total replications. These statistics, however, are silent about the specific number of breaks being tested. Therefore, in the third and fourth column, we also report rejection rates for the SupF(1|0) and Sup(2|1). High- and

<sup>&</sup>lt;sup>2</sup> Another way to address our question could be to treat simulated data as a Markov switching process along the lines of Diebold *et al.* (1994) and Filardo (1994). In this context, candidate break points could be associated to the time-varying transition probabilities between the policy announcement and implementation period. Such an approach, however, entails making assumptions about the stochastic behaviour of the states (see Kim and Nelson, 1999). In this article, we take an agnostic view leaving these extensions for future research.

<sup>&</sup>lt;sup>3</sup> See, for example, Stock and Watson (1996), Garcia and Perron (1996) and Bai and Perron (2003).

<sup>&</sup>lt;sup>4</sup> When using the Phillips curve, similar to the AR(1) case, we allow for breaks in both the intercept and the coefficient on lagged inflation. Allowing for shifts solely in the intercept (partial break) did not change our main findings. Results are available upon request.

<sup>&</sup>lt;sup>5</sup> Results for transition periods of 8 and 20 quarters are broadly similar and available upon request.



Fig. 2. Simulation of inflation

Table 2.	Bai–Perron	test:	rejection	rate <sup>*</sup>
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	Sample size	Udmax	Wdmax	SupF1 0	SupF2 1	
AR(0)	100	97	98	71	17	
	150	97	99	73	15	
	200	99	99	76	15	
AR(1)	100	96	96	73	18	
	150	95	95	74	17	
	200	97	98	77	16	
Phillips curve	100	97	98	75	16	
	150	97	96	74	15	
	200	98	99	78	13	

Note: <sup>a</sup>Denotes the percentage of total replications for which we obtain statistically significant evidence of structural breaks at the 5% level of confidence for a transition period of 40 quarters.

low-rejection rates for the SupF(1|0) and Sup(2|1), respectively, lend support to a single regime shift in the policy rule.<sup>6</sup>

Next, we estimate break dates using a singlebreak model for the three reduced-form specifications in Table 2. Figure 3 presents kernel smoothed densities for 5000 estimated break dates controlling for three different transition periods.<sup>7</sup> Noticeably, for regime shifts farther from the announcement date, the distribution mode moves closer to the implementation date. For example, when the transition period is set at 8 quarters, the most likely break

<sup>6</sup> These results can also be interpreted as test size distortions not leading to misalignments between the total number of breaks suggested by statistical evidence and the actual number of regime shifts simulated from a structural model. <sup>7</sup> Results in Fig. 3 are for a sample size of 200 data points and are robust for sample sizes of 100 and 150.



Fig. 3. Densities (break dates)

date coincides with the announcement date, whereas for a transition lasting 40 quarters the mode is estimated at around 30 quarters after the policy announcement. Similarly, lower densities around the announcement date (set at 100) are observed as the transition length increases.

These results indicate that in the context of anticipated regime changes, reduced-form evidence of structural breaks is broadly in line with an economy where agents have rational expectations. In this sense, smoother transitions to a new regime are captured by a gradual shift in estimated breaks towards the implementation date.

### **IV.** Conclusion

In this article, we investigated the interplay between a linear rational expectation model with predictable structural changes and reduced-form evidence of structural breaks. We studied this by varying the length of time between the announcement and the implementation of an inflation target change. Using the method to estimate break dates presented in Bai and Perron (1998, 2003), we found that standard parsimonious models, widely used in applied econometrics, provide plausible estimates of structural break dates in such environments. This follows from break estimates changing as a function of the policy transition period. Such changes are in line with the theoretical framework underpinning forward-looking rational expectations models.

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