Supply Chain Constraints and Inflation*

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Abstract

We develop a multisector, open economy, New Keynesian framework to evaluate how potentially binding capacity constraints, and shocks to them, shape inflation. We show that binding constraints for domestic and foreign producers shift domestic and import price Phillips Curves up, similar to reduced-form markup shocks. Further, data on prices and quantities together identify whether constraints bind due to increased demand or reductions in capacity. Applying the model to interpret recent US data, we find that binding constraints explain half of the increase in inflation during 2021-2022. In particular, tight capacity served to amplify the impact of loose monetary policy in 2021, fueling the inflation takeoff.

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In the later half of 2021 and into 2022, the United States experienced a burst of inflation as it emerged from the COVID-19 pandemic, led by a large increase in goods price inflation. Popular narratives suggest that strong consumer demand bumped up against constraints on the supply of goods, fueling inflation.[1] Further, in their public statements, policymakers frequently blamed disruptions in both domestic and foreign segments of supply chains for restraining the supply of goods.[2] Despite the plausibility of this narrative, it has been difficult to evaluate the quantitative importance of supply chain constraints for inflation, not least because we lack models that capture their impact.

In this paper, we investigate how potentially binding capacity constraints for domestic and foreign producers shape inflation in a multisector, open economy, New Keynesian (NK) model with imported inputs and input-output linkages across sectors. Solving for the model's non-linear equilibrium dynamics via piecewise linear approximations, we develop a Bayesian maximum likelihood procedure to estimate key parameters and infer when constraints bind. We then apply the model to quantify how constraints in the supply chain, and potential shocks to them, have influenced recent data outcomes. We find that binding constraints account for half (two percentage points) of the increase in inflation during 2021-2022. Interestingly, we also find that no single set of shocks can explain the inflation takeoff. Rather, shocks that tightened capacity set the stage for demand shocks – most importantly, monetary policy shocks – to trigger binding constraints and accelerate inflation in 2021.

The framework we develop features occasionally binding constraints in two different places. The first is a constraint that applies at the level of individual foreign firms, whereby foreign producers are able to supply output at constant marginal costs up to a predetermined level, at which point production is quantity-constrained. Motivated by evidence on disruptions in markets for imported inputs, we devote particular attention to binding constraints on foreign input supply. The second constraint is a similar limit on production capacity for domestic firms, which impacts both downstream firms and consumers. These dual constraints allow us to separately capture the role of domestic versus foreign supply chain disruptions on inflation.

Further, this framework features a distinction between supply-side versus demand-side explanations for binding constraints, with potentially important implications for policy. On the supply side, we assume the levels of the capacity constraints are exogenous and subject to stochastic

[1] See The Economist (2021), Rees and Rungcharoenkitkul (2021), and de Soyres, Santacreu and Young (2023).

[2] In International Monetary Fund (2021), Gita Gopinath writes: "Pandemic outbreaks in critical links of global supply chains have resulted in longer-than-expected supply disruptions, further feeding inflation in many countries." Smialek and Nelson (2021) characterize the views of the US Federal Reserve chair as follows: "[Jerome Powell] noted that while demand was strong in the United States, factory shutdowns and shipping problems were holding back supply, weighing on the economy and pushing inflation above the Fed's goal." See Lane (2022) for a discussion of views at the European Central Bank, and Goodman (2021) for a narrative of supply chain breakdowns.

shocks.[3] This formulation captures the type of time-varying input shortages that occurred during the COVID period, both in the United States and abroad.[4] On the demand side, an increase in demand may also exhaust excess capacity and induce capacity constraints to bind in the model. This alternative mechanism is salient, because the abrupt recovery of demand in 2021 seemed to stress existing supply chain capacity.

Separating these two mechanisms – that binding constraints may be the result of strong demand, or disruptions to capacity – represents a key quantitative challenge. Breaking the challenge into two pieces, we first must ascertain whether constraints bind, and then identify why they bind.

To shed light on how binding constraints may be detected, we note that binding constraints impact pricing decisions. In the model, constraints are internalized by each firm as it sets its price, such that the firm's optimal markup differs depending on whether the constraint is binding. Assuming that both exports and imports are invoiced in US Dollars, and prices are subject adjustment frictions, then domestic and import price inflation satisfy Phillips Curve type relationships. When the domestic constraint binds, we show that there is an additional term in sector-level, domestic price Phillips Curves that resembles a markup (equivalently, cost-push) shock. Similarly, there is a quasi-markup shock in the import price Phillips Curve when the import constraint binds.

Thus, our framework provides a structural interpretation for reduced-form markup/cost-push shocks, based on binding constraints.[5] This direct impact of binding constraints on inflation contrasts with other mechanisms, such as factor reallocation frictions or labor shortages, that work through marginal costs. It is also prima facie consistent with that fact that US profit margins increased as inflation took off in 2021. Further, from an empirical perspective, detecting whether constraints bind amounts to ascertaining whether inflation is 'too high' to be explained by marginal costs and expected inflation.

Turning to the second challenge, we demonstrate that data on quantities and prices together serve to identify the reasons why constraints bind – i.e., to disentangle whether demand shocks

[3] Pre-determined production capacity is shaped by past decisions about organization, installed capital, investments in worker capabilities, and the firm's stock of buyer/supplier relationships. Though we treat capacity as an exogenous stochastic variable, one could extend the model to allow for endogenous capacity investment decisions. We eschew this extension for now, because it distracts from our main focus on accounting for recent inflation dynamics.

[4] One source of these shocks would be pandemic-related factory shutdowns, as occurred in the US, China, Vietnam and elsewhere. They also capture shortages of inputs due other disruptions to global supply relationships (e.g., cancellation of supply contracts early in the pandemic led to shortages of foreign-supplied semiconductors that curtailed US auto production). Other historical shocks are also plausibly thought of as shocks to capacity – for example, the 2011 Tōhoku earthquake/tsunami took production capacity offline in Japan. We note here that disruptions in the global shipping industry (e.g., port congestion) and bottlenecks in distribution networks (e.g., trucking shortages) also made it difficult to deliver goods to buyers during the pandemic recovery. We focus on constraints on the supply of goods, rather than distribution of them, in our model.

[5] Del Negro et al. (2022) have argued that cost push shocks explain a large share of late-2021 inflation. They arrive at this conclusion based on analyzing data through the lens of a closed economy model without capacity constraints (the NYFed model), while we study an open economy in which purported markup shocks reflect structural capacity constraints.

or supply-side constraint shocks lead constraints to bind. While either a positive demand shock or negative constraint shock may trigger binding constraints and thus lead inflation to rise, these shocks have distinct implications for quantities. A demand shock pushes both inflation and output quantity up, while a negative constraint shock raises inflation whilst lowering output. Put differently, adverse constraint shocks lead to negative comovement between inflation and quantities (of output or imports) in the model. In contrast, there is positive comovement in these variables following a goods-biased demand shock. Implicitly, we use these quantitative patterns to identify shocks when apply the model to filter data. In particular, constraints help the model explain why both US goods output and imports of intermediate inputs did not respond to the surge in US demand.

To lay out the structure of the paper, we start by collecting data facts in Section 1, which both motivate elements of the framework and serve as inputs into quantification. Some are well known: headline consumer price inflation rose a lot, more for goods than services. And consumer expenditure shifted from services to goods, driving real goods expenditures above trend. On the import side, prices for imported industrial materials (inputs) rose rapidly in 2021, while prices for imported consumer goods were essentially flat. As for quantities, production of goods has recovered from its temporary pandemic downturn, but it has not increased in response to the surge in consumer demand for goods.[6] Stagnant domestic production in the face of surging demand (and the corresponding lack of imported inputs) hints at potentially binding constraints, whether domestic or foreign in nature.

In Section 2, we develop a model to organize our interpretation of these facts, in which we study the impact of constraints for domestic goods producers and foreign goods input suppliers. Using impulse responses from the model in Section 3, we describe how prices and quantities respond to demand shocks and shocks to constraints. In Section 4, we then apply the model to filter shocks from US national accounts data. To capture the rich data dynamics, we allow for a number of different shocks, including shocks to aggregate demand (time preference), consumers tastes for goods (as opposed to services), capacity levels at home and abroad, sector-specific productivity, and foreign production costs.

As a key intermediate step, we develop a Bayesian Maximum Likelihood estimation procedure to infer when constraints are binding and estimate structural parameters.[7] Our model presents several challenges for estimation. One challenge is that it features capacity shocks, and capacity is a latent variable that has no first order impact on other potentially observable equilibrium variables when constraints are slack. As a result, prior estimation routines (e.g., Guerrieri and Iacoviello (2017)) that use inversion filters to construct the likelihood function are not applicable in our

[6]Correspondingly, imports of final goods have risen by 40%, while imports of industrial materials and other intermediates have barely recovered to pre-pandemic levels.

[7]The structural parameters we estimate are substitution elasticities between home and foreign inputs, coefficients in the monetary policy rule, the mean level of capacity, and the stochastic processes for shocks.

context. Instead, our estimation procedure builds on prior work by Kulish, Morley and Robinson (2017), Kulish and Pagan (2017), and Jones, Kulish and Rees (2022), which treats the duration of binding constraints as a parameter to be estimated. In this, a second challenge is that the duration of binding constraints is an equilibrium outcome in our model, unlike prior applications of the duration-based estimation approach. Therefore, we adapt the maximum likelihood procedure to impose constraints on admissible duration parameter draws.[8] Overall, our estimated model fits the data well; most importantly, it captures the evolution of inflation for goods, services, and imports during the post-2020 period, making it a useful laboratory for analysis. Further, smoothed values for multipliers on the constraints imply that constraints bind during most of 2021-2022, and how tight they are fluctuates over time.

With the model and estimates in hand, we evaluate the role of binding constraints in explaining the evolution of inflation through a sequence of counterfactual exercises. The first counterfactual allows all shocks to be active, but exogenously relaxes the capacity constraints in all periods. Comparing this counterfactual to the data, we find that binding constraints explain about half of the increase in inflation in 2021-2022, about two percentage points of the four percentage point increase in overall inflation. Further, easing of constraints in the latter half of 2022 helps explain recent declines in goods and import price inflation.

Turning to individual shocks, we run a series of counterfactuals in which we introduce shocks individually and in combination to evaluate the role of individual shocks. We find that tight capacity, plausibly due to negative capacity shocks, set the stage for monetary policy shocks – looser policy than suggested by an extended Taylor rule – ignited inflation in 2021. By implication, neither aggregate nor goods-biased consumer demand shocks play an important role in 2021, though they do account for inflation dynamics in 2020.[9] As monetary policy was tightened in 2022, demand shocks then play a larger role in accounting for sustained inflation.

In addition to work cited above, our paper is related to two distinct strands of work. First, our approach to modeling capacity constraints is related to models developed in Álvarez-Lois (2006) and Boehm and Pandalai-Nayar (2022), which feature heterogeneous firms that differ in terms of their exogenous capacity constraints on output.[10] As Boehm and Pandalai-Nayar emphasize,

[8]In Kulish, Morley and Robinson (2017) and Jones, Kulish and Rees (2022), the binding constraint is the zero lower bound on interest rates, so the duration to be estimated reflects beliefs about how long the central bank will hold the interest rate at zero. Because this is a free policy variable, these papers treat durations as unconstrained in the estimation. In our application, the anticipated duration of binding capacity constraints is determined by the realized shock today and the state of the economy. Thus, we adapt the estimation procedure to this new environment; see Sections 4.1 and A.3.2 for full details.

[9]Though we do not directly account for fiscal policy, we note that tax and transfer policy changes enacted during the pandemic largely worked by supporting consumption. Thus, the consumption demand shocks that we infer from data partly capture the impact of these fiscal policies.

[10]Fagnart, Licandro and Portier (1999) studies the endogenous determination of capacity, in a model that provides a microfoundation for capacity constraints on output. Output-based constraints are related, but somewhat different than,

aggregating across these heterogeneous firms yields convex industry supply curves, in which industry price indexes increase with industry output, since it is related to the share of firms whose constraints are binding. In contrast to these papers, we employ a homogeneous firms framework, which has pedagogical advantages for comparability to textbook models. Further, we allow for binding aggregate constraints, which give rise to kinked convex supply (Phillips) curves with vertical segments where capacity is exhausted.

Second, our themes are related to recent work on how global value chains may have played a role in transmitting shocks during the pandemic crisis, including Bonadio et al. (2021), Lafrogne-Joussier, Martin and Mejean (2021), Gourinchas et al. (2021), Alessandria et al. (2022), and Lafrogne-Joussier, Martin and Mejean (2023).[11] Celasun et al. (2022) provide a comprehensive analysis of the global scope of disruptions and bottlenecks in supply chains during the pandemic, and attribute large output losses to them.

Several contributions specifically study the role of supply chain distributions in explaining price changes during the pandemic period. Amiti, Heise and Wang (2021), Young et al. (2021), and Santacreu and LaBelle (2022) demonstrate industry-level exposure to input price changes and/or supply chain disruptions are related to differences in output price changes across industries in the United States. Relatedly, Benigno et al. (2022) develop an index of global supply chain pressures from survey data and transportation indicators, and they find it has predictive power for inflation during the pandemic using a local projections empirical framework. di Giovanni et al. (2022) examine the role of disruptions to input markets and trade linkages on inflation during the pandemic, using a sufficient statistics approach in a two period, multi-country, multi-sector input-output framework.[12] Amiti et al. (2023) study how the combination of domestic labor market shocks and import supply chain disruptions contribute to inflation. Additional contributions focus on the impacts of fiscal policy on inflation, including di Giovanni et al. (2023) and de Soyres, Santacreu and Young (2023).

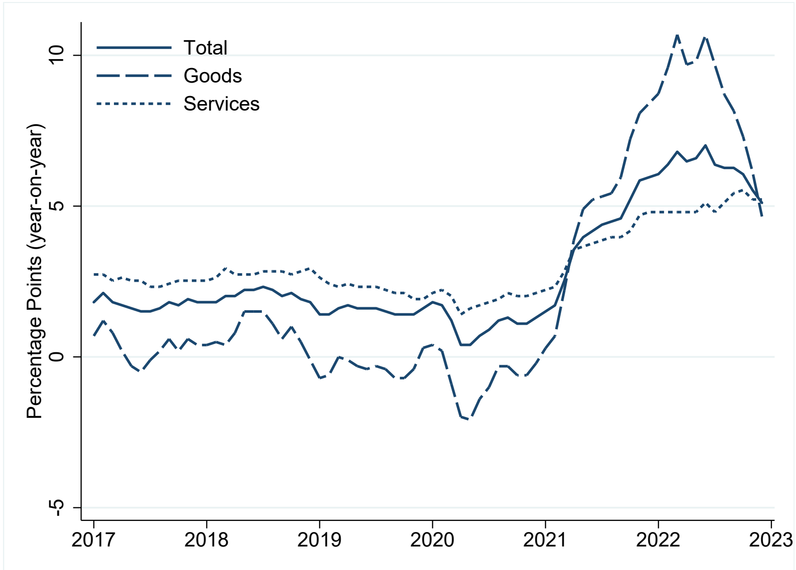
Relative to this literature, our paper is the first (to our knowledge) to analyze occasionally binding capacity constraints in the supply chain, within a complete DSGE model. In this, our paper extends the new literature on monetary policy in economies with production networks [Ozdagli and Weber (2021); La'o and Tahbaz-Salehi (2022)] to accommodate supply chain constraints. Thus,

prior work on capital utilization [Greenwood, Hercowitz and Huffman; Cooley, Hansen and Prescott (1995); Gilchrist and Williams (2000)], which we discuss further below.

[11] See also a discussion of the impact of Chinese shutdowns on US sourcing from China by Heise (2020).

[12] The sufficient statistics approach has previously been used to study macroeconomic shock propogation in Baqaee and Farhi (2019) in general, and the impacts of changes in trade on inflation by Comin and Johnson (2020). While well-suited to analyze shocks of foreign origin, the sufficient statistics approach is less useful for studying how trade mitigates/exacerbates the impacts of shocks that are domestic in origin, because domestic shocks have both direct effects and indirect effects through the import share. Further, as Comin and Johnson (2020) discuss, conclusions drawn about inflation from the sufficient statistics approach are sensitive to assumptions about the timing and persistence of changes in domestic sourcing shares.

Figure 1: Consumer Price Inflation



Note: Consumer prices are measured using the Personal Consumption Expenditure (PCE) price index, from the US Bureau of Economic Analysis (series identifiers DPCERGM, DGDSRGM, and DSERRGM.

we believe it opens the door to further study of the implications of supply chain bottlenecks for the conduct of policy.

1 Collecting Facts

We begin by collecting several key facts about recent inflation, consumer expenditure, production, and imports that motivate various elements of the framework we construct.

The first facts about consumer price inflation are well known: consumer price inflation rose substantially in 2021, led by inflation for goods. In Figure 1, we plot year-on-year growth in the price deflator for US personal consumption expenditure (PCE), as well as separate series for goods and services. The rise in headline inflation – from roughly 2 percent in 2021 to 6 percent as of early 2022 – is obviously startling. Importantly, this rise in inflation was led by goods price inflation, which rose from near zero to 10 percent in 2021. Further, note goods price inflation turns down rapidly in mid-2022, while services inflation persists (though it also turns down).

A second set of facts concerns import price inflation: prices for imported inputs rose dramatically in 2021, while price changes for imported consumer goods were modest. To illustrate this, we plot import price inflation by end use in Figure 2.[13] Inflation for imported industrial materials

[13]This data is compiled from the International Price Program of the Bureau of Labor Statistics. The source data consist primarily of free on board (FOB) price quotations, which correspond to prices received by foreign producers at foreign dock. During recent quarters, transport costs also increased dramatically, which then would be added to

rose substantially in 2021, peaking at 50% year on year. While the price of oil and derivative fuels (which doubled during this period) played a large role in driving this increase, the price of industrial materials excluding fuels also rose over 30% in 2021. In contrast, inflation for imported consumer goods was subdued.[14] This large difference between import price inflation for inputs versus consumer goods motivates our ensuing focus on disruptions impacting markets for imported inputs, rather than consumer goods.[15] In 2022, we further note that imported input price inflation evaporates, even excluding volatile fuels prices.

Tying the first and second set of facts together, goods production relies heavily on imported materials, relative production of services. Thus, the large increase in imported materials prices may play a role in explaining the surge of inflation in the goods sector discussed above. This observation is consistent with Amiti, Heise and Wang (2021), which documents that sectors that were more exposed to recent imported input price changes experienced higher producer price inflation in the U.S.[16] Our model framework will include this potential mechanism, alongside other competing drivers of inflation.

The third set of facts relate to consumer expenditures. While consumer expenditure collapsed during the lockdown phase of the pandemic, it rebounded and essentially returned to trend by the end of 2021. At the same time, the sector composition of consumer expenditures changed dramatically, as consumers reallocated away from services toward goods sectors. This is illustrated in terms of nominal expenditure shares in Figure 3a, and in terms of real quantities consumed for goods and services in Figure 3b. Further, note that the change in composition has proven remarkably persistent: real consumption of goods (correspondingly, the goods share in expenditure) remains high relative to pre-pandemic levels through 2023.

The final set of facts point to potential supply-side constraints. In Figure 4a, we plot real US gross output by broad sector. The key fact is that real production of goods (already stagnant before the pandemic) only just recovered and then trended slightly down in 2021-2022, which

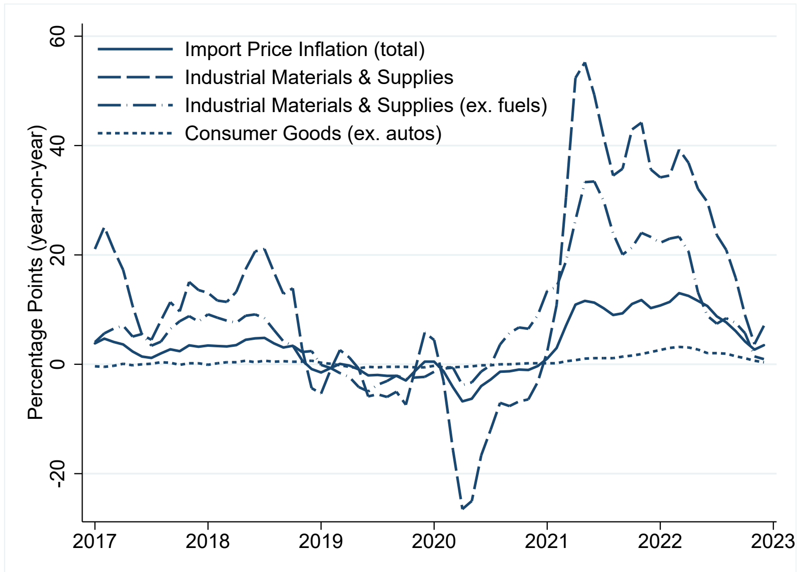
these FOB prices to arrive at CIF prices (inclusive of cost, insurance, and freight) paid by the importer. We abstract from these additional margins, in order to focus on changes in supply prices. Because international transport costs are generally low (CIF/FOB margins are typically in the range of 5-10% of the CIF value), even large changes in these margins have moderate direct effects on delivered prices.

[14] Because the scale of the axis obscures some detail, we refer the reader to Figure A.3 for a supplemental plot of quarterly inflation for imported consumer goods. The annualized value of quarterly inflation for consumer goods is below 2% in 2020 and into 2021. It rises thereafter, peaking at 6% in 2022:Q1, far below the levels of imported input price inflation.

[15] We have omitted several categories of imports from the figure for clarity, including capital goods imports (IR2), imports of automotive vehicles, parts, and engines (IR3), and foods, feeds, and beverages (IR0). To verbally summarize, inflation for capital goods imports was generally low, similar to imported consumer goods. Inflation for the automotive sector was also very low, and inflation for foods tracked total import price inflation closely. Thus, the behavior of imported materials prices stands out.

[16] In a related vein, Santacreu and LaBelle (2022) find that sectors more exposed to global supply chain disruptions (as measured an index of backlogs and delivery times) also experienced higher producer price inflation.

Figure 2: Import Price Inflation by End Use

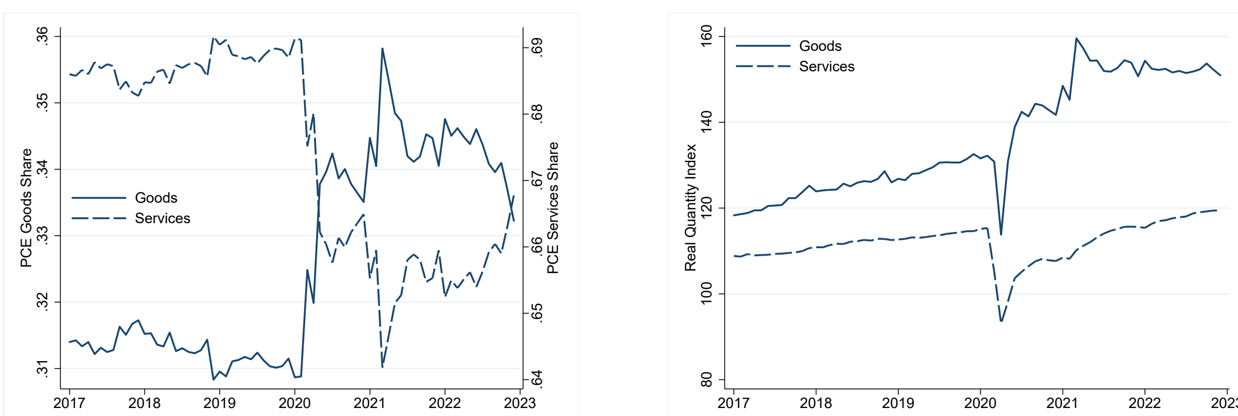


Note: Import price indexes are obtained from the US Bureau of Labor Statistics (series identifiers: IR for total imports, EIUIR1 for industrial materials, EUIIR1EXFUEL for industrial materials excluding fuels, and EIUIR4 for consumer goods).

Figure 3: Consumption by Sector

(a) Sector Shares in Expenditure

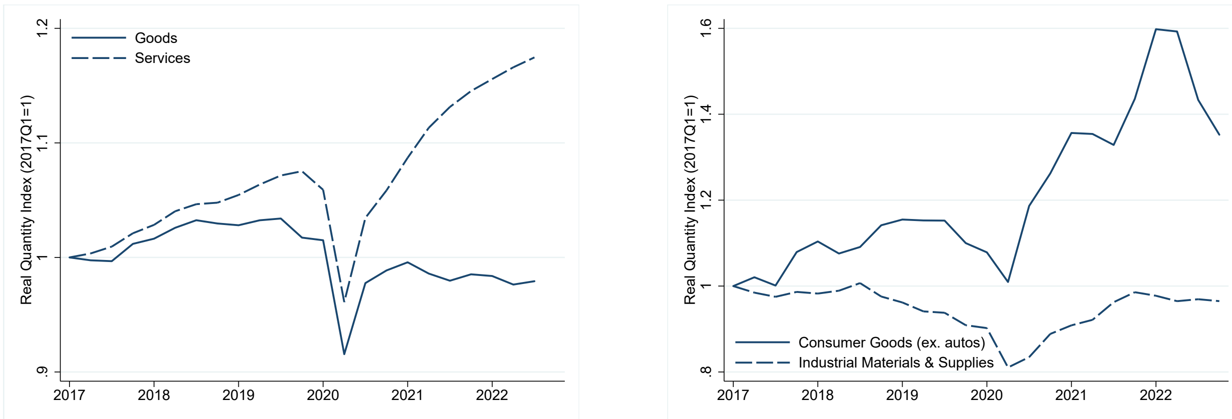
(b) Real Quantities Consumed by Sector



Note: Personal Consumption Expenditure shares and real quantity indexes by sector are obtained from the US Bureau of Economic Analysis (series identifiers: DPCERC, PCES, DGDSRA3, and DSERRA3).

Figure 4: Production and Import Quantities

(a) Real Gross Output by Sector (b) Import Quantities by End Use



Note: Real gross output constructed from underlying data from the US Bureau of Economic Analysis (GDP by Industry, Table 17). Real Quantity Indexes for imports obtained from the BEA (series identifiers: IB0000043,and B652RA3).

contrasts sharply with the observed rebound in services output. The confluence of stagnant goods production in the face of high domestic demand for goods immediately suggests that US producers may have faced binding constraints. Correspondingly, US demand for goods was filled in large part by imports: in Figure 4b, imported quantities for consumer goods (excluding autos) surge. In contrast, imports of industrial materials are flat, recovering only to its 2017 levels by the end of 2021 and plateauing there.

Deficient US goods production and stagnant imports of industrial materials are naturally connected, though the direction of causality is not immediately clear. Limited supplies of imported materials may have constrained domestic production, or distinct binding constraints of domestic origin may have curtailed production and indirectly depressed demand for imported inputs. Moreover, in principle, both these mechanisms might be active simultaneously. Below, we discuss how these quantity and price data together help distinguish between binding domestic versus foreign supply constraints in our model framework. With that aim in mind, we turn to details of the model.

2 Model

This section presents a small open economy model with many sectors $s \in \{1,\ldots,S\}$, which are are connected through input-output linkages. Within each sector, there is a continuum of monopolistically competitive firms, who set prices subject to Rotemberg-type adjustment costs. As in Gopinath et al. (2020), we assume that both exports and imports for the Home country are denominated in Home currency (i.e., US Dollars). Motivated by the data, we also allow import prices to differ for final goods and inputs.

The principal new features of the model are the output capacity constraints, for foreign and domestic firms. In writing down the model here, we allow these constraints to be potentially binding in any domestic sector, and we distinguish constraints that apply to foreign final versus input producing firms. Looking forward, we then restrict attention to particular constraints in quantitative analysis of the model for reasons of both tractability and empirical relevance, which we shall discuss further below. We also assume that the constraints are exogenously determined and (potentially) time varying, subject to stochastic shocks. This sets up a framework in which constraints may bind either due to negative shocks to capacity, or because other shocks lead firms to exhaust their excess capacity.

2.1 Consumers

There is a representative Home consumer, with preferences over labor supply L_t and consumption of sector composite goods $\{C_t(s)\}_{s\in S}$ represented by:

$$U\left(\{C_t, L_t\}_{t=0}^{\infty}\right) = \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \Theta_t \left[\frac{C_t^{1-\rho}}{1-\rho} - \chi \frac{L_t^{1+\psi}}{1+\psi}\right] \tag{1}$$

$$\text{with} \quad C_t = \left(\sum_s \zeta_t(s)^{1/\vartheta} C_t(s)^{(\vartheta-1)/\vartheta}\right)^{\vartheta/(\vartheta-1)}$$

$$\text{and} \quad C_t(s) = \left(\sum_s \gamma(s)^{1/\varepsilon(s)} C_{Ht}(s)^{(\varepsilon(s)-1)/\varepsilon(s)} + (1-\gamma(s))^{1/\varepsilon(s)} C_{Ft}(s)^{(\varepsilon(s)-1)/\varepsilon(s)}\right)^{\varepsilon(s)/(\varepsilon(s)-1)},$$

where $C_t(s)$ is consumption of a sector-composite good, which is comprised of domestic ($C_{Ht}(s)$) and foreign ($C_{Ft}(s)$) sub-composite goods. The parameter Θ_t is an aggregate preference (discount rate) shock at date t, and $\zeta_t(s)$ is a time-varying parameter that controls tastes for goods from sector s. We require that $\sum_s \zeta_t(s) = 1$ throughout, so $\zeta_t(s)$ is a relative sectoral demand shock. The parameter $\beta < 1$ is the usual time discount rate, $\rho \geq 0$ controls intertemporal substitution, $\psi > 0$ governs the elasticity of labor supply, $\vartheta \geq 0$ is the elasticity of substitution across sectors, and $\varepsilon(s) \geq 0$ is the elasticity of substitution between home and foreign consumption composites.

Financial markets are complete, and the agent's budget constraint is given by:

$$P_t C_t + \mathbf{E}_t\left[S_{t,t+1} B_{t+1}\right] \leq B_t + W_t L_t, \tag{2}$$

where $P_t C_t = \sum_s P_t(s) C_t(s)$, with P_t being the price for one unit of the composite consumption good and and $P_t(s)$ being the price of the sector composite good. B_t denotes the portfolio of Arrow-Debreu securities that pay off in domestic currency, and $S_{t,t+1}$ is the Home consumer's stochastic discount factor (defined below). Further, sectoral consumption expenditure is $P_t(s)C_t(s) = P_{Ht}(s)C_{Ht}(s) + P_{Ft}(s)C_{Ft}(s)$, where $P_{Ht}(s)$ and $P_{Ft}(s)$ are the prices of the home and foreign consumption composites.

Given prices $\{P_t, P_t(s), P_{Ht}(s), P_{Ft}(s), S_{t,t+1}, W_t\}$ and initial asset holdings B_0, the consumer chooses consumption, labor supply, and asset holdings to maximize Equation 1 subject to Equation 2 and the standard transversality condition. The optimal choices for consumption and labor supply satisfy:

$$C_t^{-\rho}\left(\frac{W_t}{P_t}\right) = \chi L_t^{\psi} \tag{3}$$

$$C_t(s) = \zeta_t(s)\left(\frac{P_t(s)}{P_t}\right)^{-\vartheta} C_t \tag{4}$$

$$C_{Ht}(s) = \gamma(s)\left(\frac{P_{Ht}(s)}{P_t(s)}\right)^{-\varepsilon(s)} C_t(s) \tag{5}$$

$$C_{Ft}(s) = (1-\gamma(s))\left(\frac{P_{Ft}(s)}{P_t(s)}\right)^{-\varepsilon(s)} C_t(s) \tag{6}$$

$$1 = \mathbf{E}_t\left[S_{t,t+1}\frac{P_t}{P_{t+1}}(1+i_t)\right] \tag{7}$$

where $S_{t,t+1} = \beta\frac{\Theta_{t+1}}{\Theta_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}$ is the stochastic discount factor, $P_t = \left(\sum_s \zeta_t(s)P_t(s)^{1-\vartheta}\right)^{1/(1-\vartheta)}$ is the aggregate price index, $P_t(s) = \left(\gamma(s)\left(P_{Ht}(s)\right)^{1-\varepsilon(s)} + (1-\gamma(s))\left(P_{Ft}(s)\right)^{1-\varepsilon(s)}\right)^{1/(1-\varepsilon(s))}$ is the sector-composite price index, and i_t is a one period, risk free nominal interest rate.

2.2 Domestic Producers

There is a continuum of firms within each sector in Home, each of which produces a differentiated good (indexed by ω). In addition, there exist competitive intermediary firms that aggregate these varieties into a domestic composite goods for each sector, which are then consumed, used as inputs, and exported. We start by describing these intermediaries, and then turn to behavior of individual firms.

2.2.1 Composite Domestic Good

Each competitive intermediary firm purchases output from domestic producers to form a domestic composite. The production function for the intermediary is: $Y_t(s) = \left(\int_0^1 Y_t(s,\omega)^{(\varepsilon-1)/\varepsilon} d\omega\right)^{\varepsilon/(\varepsilon-1)}$, where $Y_t(s,\omega)$ is the amount of output purchased from firm ω in sector s, and $\varepsilon > 1$ is the elasticity of substitution. Given prices $P_t(s,\omega)$ for individual domestic varieties, cost minimization implies demands $Y_t(s,\omega) = \left(\frac{P_t(s,\omega)}{P_{Ht}(s)}\right)^{-\varepsilon} Y_t(s)$, where $P_{Ht}(s) = \left[\int_0^1 P_t(s,\omega)^{1-\varepsilon} d\omega\right]^{1/(1-\varepsilon)}$ is the price of the sector composite good.

2.2.2 Domestic Firms

Each domestic producer in sector s is able to supply output up to a pre-determined capacity of $\bar{Y}_t(s)$, which we refer to as a firm-level capacity constraint. We assume this capacity level is exogenously determined and equal across firms within each sector.

The production function for domestic variety ω in sector s is:

$$Y_t(s,\omega) = Z_t(s,\omega) A(s) \left(L_t(s,\omega)\right)^{1-\alpha(s)} \left(M_t(s,\omega)\right)^{\alpha(s)} \tag{8}$$

$$M_t(s,\omega) = \left(\sum_{s'} \left(\alpha(s',s)/\alpha(s)\right)^{1/\kappa} M_t(s',s,\omega)^{(\kappa-1)/\kappa}\right)^{\kappa/(\kappa-1)} \tag{9}$$

$$M_t(s',s,\omega) = \left[\xi(s',s)^{\frac{1}{\eta(s')}} M_{Ht}(s',s,\omega)^{\frac{\eta(s')-1}{\eta(s')}} + (1-\xi(s',s))^{\frac{1}{\eta(s')}} M_{Ft}(s',s,\omega)^{\frac{\eta(s')-1}{\eta(s')}}\right]^{\frac{\eta(s')}{\eta(s')-1}}, \tag{10}$$

where $L_t(s,\omega)$ is the quantity of labor used by the firm, $M_t(s,\omega)$ is the firm's use of a composite input, $Z_t(\omega)$ is productivity, and $A(s) = \alpha(s)^{-\alpha(s)}(1-\alpha(s))^{-(1-\alpha(s))}$ is a normalization constant. The composite input combines inputs purchased from upstream sectors $M_t(s',s,\omega)$, with elasticity of substitution $\kappa \geq 0$. And those upstream inputs are themselves a CES composite of Home ($M_{Ht}(s',s,\omega)$) and Foreign ($M_{Ft}(s',s,\omega)$) composite inputs. The parameters $\eta(s) \geq 0$ are elasticities of substitution across country sources for inputs (conventionally termed the Armington elasticity), while $\xi(s',s) \in (0,1)$ controls relative demand for home inputs conditional on prices.

Producers set prices in domestic currency under monopolistic competition, and they select the input mix to satisfy the implied demand. These two problems can be analyzed separately. The firm chooses $\left\{L_t(s,\omega), M_t(s,\omega), M_t(s',s,\omega), M_{Ht}(s,\omega), M_{Ft}(s',s,\omega)\right\}$ to minimize the cost of producing $Y_t(s,\omega)$, which is $W_t L_t(s,\omega) + P_{Mt}(s) M_t(s,\omega)$, with $P_{Mt}(s) M_t(s,\omega) = \sum_{s'} P_t(s',s) M_t(s',s,\omega)$ and $P_t(s',s) M_t(s',s,\omega) = P_t(s') M_{Ht}(s',s,\omega) + P_{Ft}(s') M_{Ft}(s',s,\omega)$, where $P_{Ft}(s')$ is the (domestic currency) price of the foreign composite input from sector s' . The first order conditions to this problem can be written as follows:

$$W_t L_t(s,\omega) = \alpha(s) MC_t(s,\omega) Y_t(\omega) \tag{11}$$

$$P_{Mt}(s) M_t(s,\omega) = (1-\alpha(s)) MC_t(s,\omega) Y_t(s,\omega) \tag{12}$$

$$M_t(s',s,\omega) = \frac{\alpha(s',s)}{\alpha(s)} \left(\frac{P_t(s',s)/P_t}{P_{Mt}(s)/P_t} \right)^{-\kappa} M_t(s,\omega) \tag{13}$$

$$M_{Ht}(s',s,\omega) = \xi(s',s) \left(\frac{P_{Ht}(s')/P_t}{P_t(s',s)/P_t} \right)^{-\eta(s')} M_t(s',s,\omega) \tag{14}$$

$$M_{Ft}(s',s,\omega) = (1-\xi(s',s)) \left(\frac{P_{Ft}(s')/P_t}{P_t(s',s)/P_t} \right)^{-\eta(s')} M_t(s',s,\omega), \tag{15}$$

where $P_{Mt}(s) = \left(\sum_{s'} \left(\frac{\alpha(s',s)}{\alpha(s)} \right) P_t(s',s)^{1-\kappa} \right)^{1/(1-\kappa)}$ is the price of the composite input, $P_t(s',s)$ is the price of $M_t(s,\omega)$, and the firm's marginal cost is $MC_t(s,\omega) = (Z_t(s,\omega))^{-1} W_t^{1-\alpha(s)} (P_{Mt}(s))^{\alpha(s)}$.

Given this solution for marginal costs, the domestic firm chooses a sequence of prices to maximize profits, with knowledge of the demand curve for its output, and subject to quadratic adjustment cost for prices [Rotemberg (1982*a*,*b*)]. This pricing problem can be written as:

$$\max_{\{P_t(s,\omega)\}} \quad \mathbf{E}_0 \sum_{t=0}^{\infty} \frac{S_{0,t}}{P_t} \left[P_t(s,\omega) Y_t(s,\omega) - MC_t(s,\omega) Y_t(s,\omega) - \frac{\phi(s)}{2} \left(\frac{P_t(s,\omega)}{P_{t-1}(s,\omega)} - 1 \right)^2 P_{Ht}(s) Y_t(s) \right]$$
$$\text{s.t.} \quad Y_t(s,\omega) \leq \bar{Y}_t(s),$$

where the discount rate for profits reflects the domestic agent's stochastic discounting.[17] The final term in the first line captures the adjustment costs, where $\phi(s)$ governs the degree of price rigidity. Note also that the firm accounts for the potentially binding constraint in its pricing decisions. Denoting the Lagrange multiplier attached to the capacity constraint $\mu_t(s,\omega)$, optimal prices satisfy:

$$\begin{aligned} 0 = 1 - \varepsilon \left(1 - \frac{MC_t(s,\omega) + \mu_t(s,\omega)}{P_t(s,\omega)} \right) - \phi(s) \left(\frac{P_t(s,\omega)}{P_{t-1}(s,\omega)} - 1 \right) \frac{P_{Ht}(s) Y_t(s)}{P_{t-1}(s,\omega) Y_t(s,\omega)} \\ + E_t \left[\beta \frac{\Theta_{t+1}}{\Theta_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \frac{P_t}{P_{t+1}} \phi(s) \left(\frac{P_{t+1}(s,\omega)}{P_t(s,\omega)} - 1 \right) \frac{P_{Ht+1}(s) Y_{t+1}(s)}{P_t(s,\omega) Y_t(s,\omega)} \frac{P_{t+1}(s,\omega)}{P_t(s,\omega)} \right]. \end{aligned} \tag{16}$$

The corresponding complementary slackness condition is:

$$\mu_t(s,\omega) \left[Y_t(s,\omega) - \bar{Y}_t(s) \right] = 0. \tag{17}$$

And we require $\mu_t(s,\omega) \geq 0$ and the constraint to hold in equilibrium ($Y_t(s,\omega) \leq \bar{Y}_t(s)$) as usual.

[17] Of course, with complete markets, it is immaterial whether domestic or foreign agents own the firm.

When the constraint binds, then $\mu_t(s,\omega) > 0$. In Equation 16, we see this is equivalent to an increase in the marginal cost of the firm, which drives up the optimal price. When the capacity constraint is slack, such that $\mu_t(s,\omega) = 0$, then Equation 16 collapses to a standard intertemporal pricing equation. We discuss pricing in these alternative cases further in the symmetric equilibrium below.

2.3 Foreign Producers

Turning to foreign producers, we again describe aggregation of varieties by a competitive intermediary firm, and then present the foreign firm's pricing problem. Here we distinguish between producers of foreign consumption goods versus inputs, which allows us to to analyze disparate data on import prices by end use.

2.3.1 Composite Foreign Goods

For each end use $u \in \{C, M\}$, where C and M denote consumption and intermediate use respectively, there is a unit continuum of foreign firms that produce foreign inputs, indexed by ϖ. A competitive intermediary firm aggregates output produced by each foreign firm, and bundles it into the foreign composite according to the production function: $Y^*_{ut}(s) = \left(\int_0^1 Y^*_{ut}(s,\varpi)^{(\varepsilon-1)/\varepsilon} d\varpi\right)^{\varepsilon/(\varepsilon-1)}$. Demand for each variety then takes takes the standard CES form: $Y^*_{ut}(s,\varpi) = \left(\frac{P_{uFt}(s,\varpi)}{P_{uFt}(s)}\right)^{-\varepsilon} Y^*_{ut}(s)$,where $P_{uFt}(s,\varpi)$ is the price of variety ϖ and $P_{uFt}(s) = \left(\int_0^1 P_{uFt}(s,\varpi)^{1-\varepsilon} d\varpi\right)^{1/(1-\varepsilon)}$ is the price of the foreign composite, both denominated in Home currency.

2.3.2 Foreign Firms

Each foreign firm (in sector s, producing for end use u) is able to supply output up to a predetermined capacity of $\bar{Y}^*_{ut}(s)$, and this capacity is exogenous and equal across firms. As is standard in the small open economy, we directly assume foreign marginal costs are given by $MC^*(s,\varpi)$, where this cost is denominated in foreign currency and equal across end uses.

Each firm chooses a sequence for the price of its variety in Home currency $\{P_{uFt}(s,\varpi)\}$, subject to price adjustment frictions, to solve:

$$\max_{\{P_{Ft}(s,\varpi)\}} \quad \mathbf{E}_0 \sum_{t=0}^{\infty} \frac{S^*_{0,t}}{P^*_t E_t} \left[P_{uFt}(s,\varpi) Y^*_{ut}(s,\varpi) - E_t MC^*_t(s) Y^*_{ut}(s,\varpi) - \frac{\phi(s)}{2} \left(\frac{P_{uFt}(s,\varpi)}{P_{uFt-1}(s,\varpi)} - 1 \right)^2 P_{uFt}(s) Y^*_{ut}(s) \right]$$
$$\text{s.t.} \quad Y^*_{ut}(s,\varpi) \leq \bar{Y}^*_{ut}(s),$$

with knowledge of the demand curve for its output specified above. Here $S^*_{0,t} = \beta^t \left(\frac{C^*_t}{C^*_0}\right)^{-\rho}$ is the foreign stochastic discount factor (with C_t^*denoting foreign consumption), P_t^* is the foreign price level (in foreign currency), and E_t is a the nominal exchange rate (units of home currency to buy one unit of foreign currency).

Denoting the Lagrange multiplier attached to the capacity constraint $\mu_{ut}^*(s,\varpi)$, then the first order condition is:

$$1-\varepsilon\left(1-\frac{E_t\left(MC_t^*(s,\varpi)+\mu_{ut}^*(s,\varpi)\right)}{P_{uFt}(s,\varpi)}\right)-\phi(s)\left(\frac{P_{uFt}(s,\varpi)}{P_{uFt-1}(s,\varpi)}-1\right)\frac{P_{uFt}(s)Y_{ut}^*(s)}{P_{uFt-1}(s,\varpi)Y_{ut}^*(s,\varpi)}$$
$$+\mathbf{E}_t\left[\beta\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\rho}\left(\frac{E_tP_t^*}{E_{t+1}P_{t+1}^*}\right)\phi(s)\left(\frac{P_{uFt+1}(s,\varpi)}{P_{uFt}(s,\varpi)}-1\right)\frac{P_{uFt+1}(s)Y_{ut+1}^*(s)}{P_{uFt}(s,\varpi)Y_{ut}^*(s,\varpi)}\frac{P_{uFt+1}(s,\varpi)}{P_{uFt}(s,\varpi)}\right]=0. \quad (18)$$

The complementary slackness condition is:

$$\mu_{ut}^*(s,\varpi)\left[Y_{ut}^*(\varpi)-\bar{Y}_{ut}^*\right]=0. \quad (19)$$

And $\mu_{ut}^*(\varpi)\geq 0$ and the constraint $Y_{ut}^*(\varpi)\leq\bar{Y}_{ut}^*$ are required to hold in equilibrium.

2.4 Closing the Model

We assume that demand for exports of the home composite good takes the CES form:

$$X_t(s)=\left(\frac{P_{Ht}(s)}{P_tQ_t}\right)^{-\sigma(s)}X_t^*(s), \quad (20)$$

where $Q_t\equiv\frac{E_tP_t^*}{P_t}$ is the real exchange rate and $X_t^*(s)$ is an exogenous foreign sector-demand factor.

The market clearing condition for the home composite good is:

$$Y_t(s)=C_{Ht}(s)+\sum_{s'=1}^{S}\int_0^1 M_{Ht}(s,s',\omega)d\omega+X_t(s)+\int_0^1\left[\frac{\phi(s)}{2}\left(\frac{P_t(s,\omega)}{P_{t-1}(s,\omega)}-1\right)^2Y_t(s)\right]d\omega. \quad (21)$$

where the composite good is sold to consumers and domestic producers, exported, and used to cover price adjustment costs. For the foreign composite goods, we impose similar market clearing conditions:

$$Y_{Ct}^*(s)=C_{Ft}(s)+\frac{\phi(s)}{2}\left(\Pi_{CFt}(s)-1\right)^2Y_{Ct}^*(s) \quad (22)$$

$$Y_{Mt}^*(s)=\sum_{s'}M_{Ft}(s,s')+\frac{\phi(s)}{2}\left(\Pi_{MFt}(s)-1\right)^2Y_{Mt}^*(s). \quad (23)$$

Labor market clearing is then:

$$L_t = \sum_{s=1}^{S} L_t(s) \quad \text{with} \quad L_t(s) = \int_0^1 L_t(s,\omega)d\omega. \tag{24}$$

Trade in Arrow-Debreu securities implies that Home and Foreign consumers share risk, such that:

$$\Theta_t \left(\frac{C_t}{C_t^*}\right)^{-\rho} Q_t = \Xi, \tag{25}$$

where Ξ is a constant.

Turning to monetary policy, we specify a simple inflation-targeting rule for monetary policy. Since we allow for sector-specific preference shocks, we now distinguish measured price inflation from changes in the welfare-theoretic price index. We define an auxiliary price index under the assumption that preferences are (counterfactually) constant over time: $\bar{P}_t = \left(\sum_s \zeta_0(s) \left(P_t(s)\right)^{1-\vartheta}\right)^{1/(1-\vartheta)}$, where $\zeta_0(s)$ are steady-state CES weights. Then $\bar{\Pi}_t = \bar{P}_t / \bar{P}_{t-1}$ is the ratio of measured prices across periods, and the approximate inflation rate is given by $\bar{\pi}_t = \sum_s \left(\frac{P_0(s)C_0(s)}{P_0 C_0}\right) [\ln P_t(s) - \ln P_{t-1}(s)]$, which is a standard constant expenditure weight measure.[18] We then write the monetary policy rule in terms of measured prices:

$$1 + i_t = (1 + i_{t-1})^{\rho_i} \bar{\Pi}_t^{\omega(1-\rho_i)} \left(Y_t / Y_0\right)^{(1-\rho_i)\rho_y} \Psi_t \tag{26}$$

where $Y_t = \sum_s P_0(s) Y_t(s)$ is aggregate real gross output and Ψ_t is a monetary policy shock. The parameters ω and ρ_y determine how aggressively the central bank responds to inflation and the output gap (defined as the deviation of output from steady state), while the parameter ρ_i controls the degree of interest rate inertia.

2.5 Equilibrium with Symmetric Firms

We focus on an equilibrium with symmetric producers within each sector and country. The model parameters are $\left\{\rho, \psi, \chi, \vartheta, \beta, \kappa, \varepsilon, \Xi, i_0, \omega, \rho_i, \rho_y\right\}$, $\left\{\varepsilon(s), \alpha(s), \eta(s), \phi(s), \sigma(s), \phi(s), \gamma(s)\right\}_s$, and $\left\{\alpha(s', s), \xi(s', s)\right\}_{s,s'}$. Further, values for domestic variables $\left\{\Theta_t, \left\{\zeta_t(s), Z_t(s)\right\}_s, \Psi_t\right\}$, foreign variables $\left\{C_t^*, \left\{X_t^*(s), MC_t^*(s)/P_t^*\right\}_s\right\}$ and constraints $\left\{\bar{Y}_t(s), \bar{Y}_{Ct}^*(s), \bar{Y}_{Mt}^*(s)\right\}_s$ are exogenously given.

[18] The following relationship holds between the ratios of measured and welfare-based price indexes across periods: $\bar{\Pi}_t = \frac{\bar{P}_t / P_t}{\bar{P}_{t-1}/P_{t-1}} \Pi_t$, where $\frac{\bar{P}_t}{P_t} = \left(\sum_s \zeta_0(s) \left(\frac{P_t(s)}{P_t}\right)^{1-\vartheta}\right)^{1/(1-\vartheta)}$ and the ratio of aggregate prices across periods is $\Pi_t \equiv \frac{P_t}{P_{t-1}}$. We include these among auxiliary price definitions in the model equilibrium. With constant tastes, $\bar{P}_t = P_t$, so measured and welfare-based inflation coincide. Our approach to measured inflation mimics standard fixed expenditure weight indexes; see Gábor-Tóth and Vermeulen (2018) and Redding and Weinstein (2020) for discussion of measuring the cost of living with taste shocks.

We write all prices relative to the domestic price level, and we define $\Pi_t \equiv \frac{P_t}{P_{t-1}}$.

Given parameters and exogenous variables, an equilibrium is a sequence of aggregate quantities $\{C_t, L_t\}$, sector-level quantities $\left\{C_t(s), C_{Ht}(s), C_{Ft}(s), L_t(s), Y_t(s), M_t(s), X_t(s), Y^*_{Ct}(s), Y^*_{Mt}(s)\right\}_s$, input use $\{\{M_t(s',s), M_{Ht}(s',s), M_{Ft}(s',s)\}_{s'}\}_s$, aggregate prices $\left\{W_t/P_t, i_t, Q_t, \Pi_t, \bar{\Pi}_t, \bar{P}_t/P_t\right\}$, sector-level prices $\{\Pi_t(s), \Pi_{CFt}(s), \Pi_{MFt}(s), P_t(s)/P_t, MC_t(s)/P_t, P_{Mt}(s)/P_t, P_{Ht}(s)/P_t, P_{CFt}(s)/P_t, P_{MFt}(s)/P_t\}_s$, input prices $\{\{P_t(s',s)/P_t\}_{s'}\}_s$, and (normalized) multipliers $\left\{\mu_t(s)/P_t, \mu^*_{Ct}(s)/P^*_t, \mu^*_{Mt}(s)/P^*_t\right\}_s$ that satisfy the equilibrium conditions collected in Table 1. This system is $8 + 21S + 4S^2$ equations in the same number of unknowns.

Table 1: Equilibrium Conditions

Labor Supply	$C_t^{-\rho}\frac{W_t}{P_t} = \chi L_t^{\psi}$
Consumption Allocation	$C_t(s) = \zeta_t(s)\left(\frac{P_t(s)}{P_t}\right)^{-\vartheta} C_t$
	$C_{Ht}(s) = \gamma(s)\left(\frac{P_{Ht}(s)/P_t}{P_t(s)/P_t}\right)^{-\varepsilon(s)} C_t(s)$
	$C_{Ft}(s) = (1-\gamma(s))\left(\frac{P_{CFt}(s)/P_t}{P_t(s)/P_t}\right)^{-\varepsilon(s)} C_t(s)$
Euler Equation	$1 = E_t\left[\beta\frac{\Theta_{t+1}}{\Theta_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\left(\frac{1+i_t}{\Pi_{t+1}}\right)\right]$
Consumer Prices	$1 = \left(\sum_s \zeta_t(s)\left(\frac{P_t(s)}{P_t}\right)^{1-\vartheta}\right)^{1/(1-\vartheta)}$
	$\frac{P_t(s)}{P_t} = \left(\gamma(s)\left(\frac{P_{Ht}(s)}{P_t}\right)^{1-\varepsilon(s)} + (1-\gamma(s))\left(\frac{P_{CFt}(s)}{P_t}\right)^{1-\varepsilon(s)}\right)^{1/(1-\varepsilon(s))}$
Labor Demand	$\frac{W_t}{P_t}L_t(s) = (1-\alpha(s))\frac{MC_t(s)}{P_t}Y_t(s)$
Input Demand	$\frac{P_{Mt}(s)}{P_t}M_t(s) = \alpha(s)\frac{MC_t(s)}{P_t}Y_t(s)$
	$M_t(s',s) = \frac{\alpha(s',s)}{\alpha(s)}\left(\frac{P_t(s',s)/P_t}{P_{Mt}(s)/P_t}\right)^{-\kappa} M_t(s)$
	$M_{Ht}(s',s) = \xi(s',s)\left(\frac{P_{Ht}(s')/P_t}{P_t(s',s)/P_t}\right)^{-\eta(s')} M_t(s',s)$
	$M_{Ft}(s',s) = (1-\xi(s',s))\left(\frac{P_{MFt}(s')/P_t}{P_t(s',s)/P_t}\right)^{-\eta(s')} M_t(s',s)$
Marginal Cost	$\frac{MC_t(s)}{P_t} = \frac{1}{Z_t(s)}\left(\frac{W_t}{P_t}\right)^{1-\alpha(s)}\left(\frac{P_{Mt}(s)}{P_t}\right)^{\alpha(s)}$
Input Prices	$\frac{P_{Mt}(s)}{P_t} = \left(\sum_{s'}\left(\frac{\alpha(s',s)}{\alpha(s)}\right)\left(\frac{P_t(s',s)}{P_t}\right)^{1-\kappa}\right)^{1/(1-\kappa)}$
	$\frac{P_t(s',s)}{P_t} = \left[\xi(s',s)\left(\frac{P_{Ht}(s')}{P_t}\right)^{1-\eta(s')} + (1-\xi(s',s))\left(\frac{P_{MFt}(s')}{P_t}\right)^{1-\eta(s')}\right]^{1/(1-\eta(s'))}$
Domestic Pricing	$0 = 1-\varepsilon\left(1-\frac{MC_t(s)/P_t+\mu_t(s)/P_t}{P_{Ht}(s)/P_t}\right) - \phi(s)\left(\Pi_{Ht}(s)-1\right)\Pi_{Ht}(s)$
	$+E_t\left[\beta\frac{\Theta_{t+1}C_{t+1}^{-\rho}}{\Theta_t C_t^{-\rho}}\frac{1}{\Pi_{t+1}}\phi(s)\left(\Pi_{Ht+1}(s)-1\right)\Pi_{Ht+1}(s)^2\frac{Y_{t+1}(s)}{Y_t(s)}\right]$
Consumption Import Pricing	$0 = 1-\varepsilon\left(1-\frac{Q_t}{P_{CFt}(s)/P_t}\frac{MC^*_t(s)+\mu^*_{Ct}(s)}{P^*_t}\right) - \phi(s)\left(\Pi_{CFt}(s)-1\right)\Pi_{CFt}(s)$
	$+E_t\left[\beta\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\rho}\frac{Q_t}{Q_{t+1}}\frac{1}{\Pi_{t+1}}\phi(s)\left(\Pi_{CFt+1}(s)-1\right)\Pi_{CFt+1}(s)^2\frac{Y^*_{Ct+1}(s)}{Y^*_{Ct}(s)}\right]$

Table 1: Equilibrium Conditions

Input Import Pricing	$0 = 1-\varepsilon\left(1-\frac{Q_t}{P_{MFt}(s)/P_t}\frac{MC_t^*(s)+\mu_{Mt}^*(s)}{P_t^*}\right)-\phi(s)\left(\Pi_{MFt}(s)-1\right)\Pi_{MFt}(s)$
	$+E_t\left[\beta\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\rho}\frac{Q_t}{Q_{t+1}}\frac{1}{\Pi_{t+1}}\phi(s)\left(\Pi_{MFt+1}(s)-1\right)\Pi_{MFt+1}(s)^2\frac{Y^*_{Mt+1}(s)}{Y^*_{Mt}(s)}\right]$
Comp. Slackness and Firm Constraints	$\min\{\mu_t(s),\bar{Y}_t(s)-Y_t(s)\}=0$
	$\min\{\mu^*_{Ct}(s),\bar{Y}^*_{Ct}(s)-Y^*_{Ct}(s)\}=0$
	$\min\{\mu^*_{Mt}(s),\bar{Y}^*_{Mt}(s)-Y^*_{Mt}(s)\}=0$
Market Clearing	$Y_t(s)=C_{Ht}(s)+\sum_{s'}M_{Ht}(s,s')+X_t(s)+\frac{\phi(s)}{2}\left(\frac{P_t(s)}{P_{t-1}(s)}-1\right)^2Y_t(s)$
	$X_t(s)=\left(\frac{P_{Ht}(s)}{P_tQ_t}\right)^{-\sigma(s)}X^*_t(s)$
	$Y^*_{Ct}(s)=C_{Ft}(s)+\frac{\phi(s)}{2}\left(\Pi_{CFt}(s)-1\right)^2Y^*_{Ct}(s)$
	$Y^*_{Mt}(s)=\sum_{s'}M_{Ft}(s,s')+\frac{\phi(s)}{2}\left(\Pi_{MFt}(s)-1\right)^2Y^*_{Mt}(s)$
	$\Theta_t\left(\frac{C_t}{C^*_t}\right)^{-\rho}Q_t=\Xi$
	$\sum_s L_t(s)=L_t$
Monetary Policy Rule	$1+i_t=(1+i_{t-1})^{\rho_i}\bar{\Pi}_t^{\omega(1-\rho_i)}\left(Y_t/Y_0\right)^{(1-\rho_i)\rho_y}\Psi_t$ with
	$Y_t=\sum_s P_0(s)Y_t(s)$
Auxiliary Inflation and Price Definitions	$\Pi_{Ht}(s)=\left(\frac{P_{Ht}(s)/P_t}{P_{Ht-1}(s)/P_{t-1}}\right)\Pi_t$
	$\Pi_{CFt}(s)=\left(\frac{P_{CFt}(s)/P_t}{P_{CFt-1}(s)/P_{t-1}}\right)\Pi_t$
	$\Pi_{MFt}(s)=\left(\frac{P_{MFt}(s)/P_t}{P_{MFt-1}(s)/P_{t-1}}\right)\Pi_t$
	$\bar{\Pi}_t=\frac{\bar{P}_t/P_t}{\bar{P}_{t-1}/P_{t-1}}\Pi_t$
	$\frac{\bar{P}_t}{P_t}=\left(\sum_s\zeta_0(s)\left(\frac{P_t(s)}{P_t}\right)^{1-\vartheta}\right)^{1/(1-\vartheta)}$

2.6 Discussion

We briefly discuss some technicalities associated with solving the model. We then describe Phillips Curves in the model, which contain an important insight for interpreting simulation results.

2.6.1 Solving the Model

Because the model features occasionally binding constraints, we need to adopt an appropriate solution technique that captures the non-linearities induced by them. Among alternatives, we adopt the piecewise linear solution technique developed by Guerrieri and Iacoviello (2015). The perturbation-based solution algorithm combines first order approximations to the model equilibrium for both the unconstrained and constrained equilibria, where the point of approximation is the unconstrained equilibrium in all cases.[19]

To give some intuition for the solution procedure, suppose the model economy starts in an initial steady state in which constraints are slack. If a shock leaves the economy in the unbound equilibrium, then the model can be solved using standard procedures. If an initial shock leads constraints to bind, however, then the policy functions initially deviate from the unconstrained solution. Put differently, policy functions depend on whether the constraint is binding, as well as how long it is anticipated to continue to bind. To capture this, the solution algorithm forms a guess for the date (call it T) at which the economy returns to the unbound equilibrium (the guess is based on the dynamics of the unconstrained system). From T forward, the economy remains in the unconstrained equilibrium, so one can rapidly obtain policy functions for this interval. For dates 0 to T, the algorithm requires an initial guess for the sequence of bound/unbound equilibria in this interval. Given this guess, and the knowledge the economy remains in the unconstrained equilibrium after date T, one can solve backwards for policy functions applicable in the $(0,T)$ interval using first order approximations for both the bound and unbound equilibria, depending on which is applicable in a given period. Computing the implied paths for endogenous variables, the algorithm then verifies these satisfy model constraints. If not, then the guess is updated and the procedure iterates to convergence.

The log-linear approximation for the model used in our quantitative analysis is presented in Appendix A. Collecting log deviations from steady state for endogenous (both control and state) variables in the vector X_t, the general solution for the model can be written as:

$$X_t = \mathbf{J}\left(X_{t-1}, \varepsilon_t; \theta\right) + \mathbf{Q}\left(X_{t-1}, \varepsilon_t; \theta\right) X_{t-1} + \mathbf{G}\left(X_{t-1}, \varepsilon_t; \theta\right) \varepsilon_t, \tag{27}$$

where ε_t is the vector of exogenous shocks in period t, θ is a collection of structural parameters, and $\mathbf{J}(\cdot)$, $\mathbf{Q}(\cdot)$, and $\mathbf{G}(\cdot)$ are time-varying matrices (dependent on the state and current shocks) that describe the optimal policy function. Given parameters and the initial steady-state shares needed to parameterize the approximate model (described below), as well lagged value X_{t-1} and a realization for ε_t, we solve for the policy functions using the OccBin toolbox in Dynare.

2.6.2 Domestic and Import Price Phillips Curves

It is instructive to examine log-linear approximations for the dynamic pricing equations for domestic and imported goods. Noting that $\mu_t(s)/P_t$ and $\mu^*_{ut}(s)/P^*_t$ for $u \in \{C, M\}$ take on zero

[19]The solution procedures requires that the model satisfies two important conditions. First, it is assumed that the model returns to the unconstrained equilibrium in finite time after a once-off shock, if agents expect future shocks to be zero, regardless of whether constrains bind or not in the initial post-shock equilibrium. Second, the unconstrained equilibrium must be stable, in the Blanchard-Kahn sense. Both requirements are satisfied for the baseline model and parameter values.

values in the unconstrained equilibrium, we define auxiliary variables $\tilde{\mu}_t(s) \equiv \mu_t(s)/P_t + 1$ and $\tilde{\mu}^*_{ut}(s) \equiv \mu^*_{ut}(s)/P^*_t + 1$, and then we log-linearize the equilibrium with respect to these auxiliary variables. The resulting approximate pricing equations are:

$$\pi_{Ht}(s) = \left(\frac{\varepsilon - 1}{\phi(s)}\right)\left(\widehat{rmc}_t(s) - \widehat{rp}_{Ht}(s)\right) + \left(\frac{\varepsilon}{\phi(s)}\frac{P_0}{P_{H0}(s)}\right)\hat{\tilde{\mu}}_t(s) + \beta E_t\left[\pi_{Ht+1}(s)\right] \tag{28}$$

$$\pi_{uFt}(s) = \left(\frac{\varepsilon - 1}{\phi(s)}\right)\left(\widehat{rmc}^*_t(s) + \hat{q}_t - \widehat{rp}_{uFt}(s)\right) + \left(\frac{\varepsilon}{\phi(s)}\frac{P_0}{P_{uF0}(s)}\right)\hat{\tilde{\mu}}^*_{ut}(s) + \beta E_t\left[\pi_{uFt+1}(s)\right], \tag{29}$$

where hat-notation denotes deviations from steady state, $\pi_t(s) \equiv \ln P_t(s) - \ln P_{t-1}(s)$, $\pi_{Ft}(s) \equiv \ln P_{Ft}(s) - \ln P_{Ft-1}(s)$, $rmc_t(s) = \ln(MC_t(s)/P_t)$, $rmc^*_t(s) = \ln(MC^*_t(s)/P^*_t)$, $rp_{Ht}(s) = \ln(P_{Ht}(s)/P_t)$, $rp_{uFt}(s) = \ln(P_{uFt}(s)/P_t)$, and $q_t = \ln Q_t$. Equations 28-29 are sector-level domestic and import price Phillips curves.

Binding Constraints as Markup Shocks An important conceptual point is that binding constraints – when $\mu_t(s)$ or $\mu^*_{ut}(s)$ are strictly positive – appear as "markup shocks" in reduced form. That is, binding constraints lead inflation to be higher than can be accounted for given parameters, real marginal costs, and expected inflation. Thus, one can identify whether constraints bind in our model using the same approaches that would typically be used to identify exogenous, reduced-form markup shocks in standard New Keynesian models. The key difference is that these "markup shocks" have a concrete structural interpretation in our model.

This markup shock interpretation also serves to highlight how the mechanism we emphasize is distinct from alternatives. First, much attention has focused on the role of labor shortages. At the aggregate level, these may reflect changes in worker preferences, constraints on labor supply, or the persistent effects of temporary labor market displacements during the shutdown period. At the sector level, worker shortages may be explained by impediments to reallocating workers in response to differential changes in demand across sectors.[20] In either case, demand for workers outstripping supply ought to manifest as higher wages, which would then drive marginal costs higher. Thus, one would expect to see that changes in real marginal costs explain inflation outcomes, not markups (one might even expect markups to be compressed where labor shortages are tightest). To the extent that constraints masquerading as markup shocks explain inflation, this then limits the scope for these alternative labor market mechanisms.[21]

[20] We have assumed that factors are perfectly mobile across sectors in our model, with a common economy-wide wage. This contrasts with Ferrante, Graves and Iacoviello (2023), who analyze how asymmetric demand shocks lead to inflation when there are worker reallocation frictions.

[21] As an aside, data on prices and quantities also appear to be inconsistent with simple labor shortage explanations. Regarding prices, services are labor intensive relative to goods. To the extent that domestic labor shortages raise the cost of workers, then one would expect this to bite most sharply in the services sector – driving the price of services up relative to goods. This is evidently counterfactual. From a quantity perspective, services output largely recovered

In a related vein, the approach we adopt for modeling capacity differs from prior literature, which has emphasized variable capital utilization rather than output-based capacity constraints. Specifically, following Greenwood, Hercowitz and Huffman (1988), it is typically assumed that higher rates of capital utilization lead capital to depreciate faster. As a result, higher utilization raises the effective marginal cost for the firm (including wages, user costs of capital, and increased capital depreciation), so utilization affects inflation through marginal costs. Further, with the functional form assumptions in Greenwood, Hercowitz and Huffman, the standard log-linear Phillips Curve relationship between marginal costs and inflation (equivalently, utilization and inflation) holds. Thus, this alternative approach to capacity utilization will struggle to explain the highly non-linear response of inflation observed in recent data, as well as the role of reduced-form markup shocks in explaining it.

Profits Our model implies that price-cost margins (realized markups) are high when firms face binding constraints. Importantly, this is not because the competitive environment per se has changed – i.e., market structure is constant in our model – rather firm conduct changes when constraints bind. Firms cease to make price changes to target their ideal (flexible price, CES monopolistic competition) markups; rather, they start to "price to demand," based on willingness to pay for their constrained output.[22]

To examine the plausibility of this channel, we turn to data on profits per unit of output, which serves as an observable proxy for price-cost margins. To formalize this link, note that the absolute markup is equal to profits per unit of output in the steady state: $P_t(s) - MC_t(s) = \frac{\Xi_t(s)}{Y_t(s)}$, where $\Xi_t(s) \equiv P_t(s)Y_t(s) - MC_t(s)Y_t(s)$ is the profit of the representative producer in sector s.[23] Thus, tracking profits per unit over time sheds light on how markups are changing.

In Figure 5, we plot indexes of US corporate profits per unit of gross output for both the manufacturing sector and the aggregate private sector.[24] The takeaway is that profits per unit escalated sharply for manufacturing firms during the pandemic recovery, coinciding with the takeoff in goods price inflation and widespread complaints about binding (supply chain) constraints that limited production. Further, total profits (profits per unit times quantity sold) were at historically

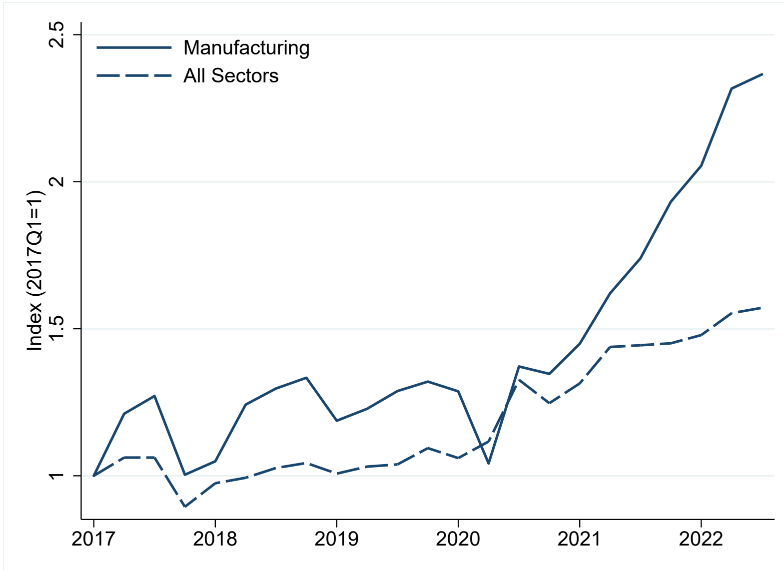
in the U.S. in 2021, while goods production did not rise to meet a surge in demand for goods. This is also hard to rationalize based on labor shortages, since services are more labor intensive than goods.

[22]The conclusion that capacity constraints influence firm conduct, holding market structure fixed, is not unique to our particular monopolistic competition model. For example, in oligopoly models with symmetric firms, it is well-known that Bertrand pricing leads to competitive pricing (equilibrium prices at marginal cost) when firms are unconstrained. In contrast, when firms are capacity constrained and they collectively cannot meet total market demand with prices set at marginal cost, then the Bertrand pricing equilibrium deviates from the perfectly competitive benchmark, with prices set above marginal cost.

[23]This holds exactly in the steady state, since adjustment costs are zero. It holds in the usual log-linear approximate sense near the steady state, since adjustment costs do not have first order effects on profits.

[24]This corporate profit measure omits profits attributable to non-corporate entities; We focus on corporate profits because data is available for manufacturing on a quarterly frequency in the national accounts.

Figure 5: Corporate Profits per unit of Gross Output



Note: Corporate profits (with inventory valuation adjustments) and gross output are obtained from the US Bureau of Economic Analysis (series identifiers N400RC and A390RC). The figure contains the ratio of corporate profits to gross output in each quarter, expressed as an index number (values are measured relative to the ratio in 2017Q1).

high levels in 2021. This pattern of high profitability alongside high inflation is a natural outcome of binding (domestic) constraints in our model. It is also remarkably consistent with emergent concerns about "greedflation" in the U.S., wherein corporations have been criticized for fueling inflation by gouging consumers [DePillis (2022)].[25] More recently, profit margins appear to be falling as inflation is falling in 2023 [Kerr (2023)].

3 Impulse Response Analysis

To illustrate how the model works, we turn to impulse response functions. We first discuss the quantitative setup, which puts restrictions on the general model presented above. We then analyze responses to particular demand and capacity constraints that play important roles in accounting for variation in the data below. Further, looking forward to the full quantitative analysis, we pay

[25]Greedflation is often attributed to the secular rise of market power, which has potentially increased the ability of corporations to pass through cost shocks (e.g., due to supply chain disruptions, labor shortages, etc.) to consumers. Our mechanism is different: it is not about pass-through of cost shocks, it is about how firms set prices conditional on costs when constraints bind. In this sense, our mechanism lines up well with anecdotes from the auto industry, where the constrained supply of new automobiles led to higher dealer markups and robust profitability [Smialek (2022)]. In this case, dealers can be thought of as having a Leontief production function, combining cars with dealer services to produce sold automobiles. In this special Leontief case, an input constraint is effectively a constraint on output. Our model allows primary factors to substitute for inputs, and allows domestic inputs to substitute for foreign inputs, so output constraints are distinct from input constraints, unlike this auto dealer example.

particular attention to illustrating how data may be useful in identifying shocks in this main text, leaving supplemental results to the appendix.

3.1 Quantitative Setup

While the model (as written above) allows for many sectors and many potentially binding constraints, we now hone in on a particular two sector structure with two potentially binding constraints, motivated by the stylized facts presented above. As for numbering sectors, let $s = 1$ be the goods sector and $s = 2$ be the services sector. We then focus on equilibria with potentially binding constraints for the goods sector, since the anomalous behavior of goods price inflation in recent data requires explanation.[26] Further, motivated by data that shows a surge of consumption goods imports during 2020-2021, we assume that the constraints for foreign consumer goods firms are slack as well. That is, we allow for potentially binding constraints for domestic firms and foreign input producing firms – i.e., constraints in the supply chain for goods.

With this setup, there are then four potential configurations of binding and slack constraints. The first equilibrium is the unconstrained equilibrium, in which both constraints are slack. This equilibrium corresponds to a multi-sector, open economy, New Keynesian model with dominant currency pricing. The second and third equilibria are situations where single constraints bind, whether the foreign firm capacity constraint or the domestic firm capacity constraint.[27] The fourth equilibrium is one where both the domestic and foreign constraints bind.

We set parameters in the model based on external calibration and an estimation procedure that we describe in Section 4.2 and Appendix A. Further, exogenous variables follow independent first-order autoregressive processes, with parameters estimated below.

3.2 Analyzing a Demand Shock

To start, we consider a temporary, but persistent, shock in the consumer discount rate (Θ_t), which raises the consumer's desire to consume in the current period. For concreteness, let $\hat{\Theta}_t = \lambda_\Theta \hat{\Theta}_{t-1} + \varepsilon_{\Theta t}$, with $\text{var}(\varepsilon_{\Theta t}) = \sigma_\theta^2$. To scale the shock, we assume that the initial innovation to Θ_t is $0.15\sigma_\theta$, and σ_θ and λ_Θ are estimated below. This is a relatively small shock, so our focus is on qualitative

[26] Taking the constraints for sector 2 to infinity – i.e., $\bar{Y}_t(2) \to \infty$, $\bar{Y}^*_{Ct}(2) \to \infty$, and $\bar{Y}^*_{Mt}(2) \to \infty$ – is sufficient to ensure constraints are only potentially binding for goods. One ought not over-interpret this assumption. This is a sufficient, but not necessary condition. Given any sequence of shocks, one can back out the level of finite constraints needed to prevents these constraints from binding.

[27] When the domestic firm constraint binds, then $Y_t(1) = \bar{Y}_t(1)$, so $\hat{y}_t(1) = \ln \bar{Y}_t(1) - \ln \bar{Y}_0(1)$, where the subscript zero denotes the steady state value. In this case, output is determined by the level of the constraint. This impinges on the availability of Home goods to downstream consumers, domestic firms, and the export market. It also impacts upstream demands for primary factors and inputs (domestic and imported) by the firm. When the foreign firm constraint binds, then $Y^*_{Mt}(1) = \bar{Y}^*_{Mt}(1)$. This implies that $\hat{\bar{y}}^*_{Mt}(1) = \ln \bar{Y}^*_{Mt}(1) - \ln Y^*_{M0}(1)$, so imported inputs are quantity constrained, and domestic firms must cope with input shortages.

results in this section, rather than magnitudes. We contrast the impacts of this shock in an unconstrained equilibrium to those in two alternative equilibria, in which only the domestic constraint binds, or only the foreign constraint binds on impact. In each case, we set the level of the constraint so that the constraint binds for only one period (i.e., on impact) in response to the shock, and is slack thereafter. While a special case, this serves to highlight the key impacts of hitting constraints.

3.2.1 Unconstrained Benchmark

Figure 6 collects impulse responses for key variables from the unconstrained model (i.e., assuming constraints do not bind after the shock). In Figure 6a, we see that the demand shock raises overall consumer price inflation, and inflation for services is about double that for goods after the shock. To break this down, sector-level inflation is a weighted average of domestic price inflation ($\pi_{Ht}(s)$) and import price inflation for consumption goods ($\pi_{CFt}(s)$), so we plot $\pi_{Ht}(s)$ and $\pi_{CFt}(s)$ in Figure 6b.[28]

First, we note that import price inflation for consumer goods is negative on impact in Figure 6b, so this also lowers overall consumption price inflation for goods relative to services, since the goods sector has a higher import share. Glancing forward, note that import price inflation for goods inputs is also negative on impact in Figure 6d, matching the dynamics for consumer import prices. The reason is that on impact the exchange rate appreciates in response to the demand shock, and this appreciation lowers import price inflation symmetrically across sectors and end uses ($\pi_{uFt}(s)$ coincide across u and s).

Second, domestic price inflation is also lower for goods than services. The reason is that real wages rise, and services use labor more intensively than do goods. To illustrate this, we iterate the Phillips Curves in Equation 28 forward to yield the decomposition:

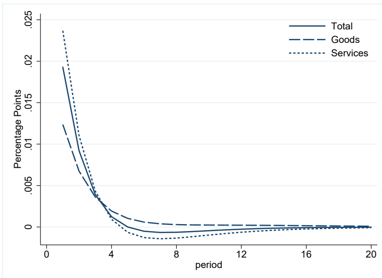
$$\pi_{Ht}(s) = \underbrace{(1-\alpha(s))\left(\frac{\varepsilon-1}{\phi(s)}\right)\sum_{r=0}^{\infty}\beta^r E_t\left[\hat{w}_{t+r}-\hat{p}_{Ht+r}(s)\right]}_{\text{real wage term}} + \underbrace{\alpha(s)\left(\frac{\varepsilon-1}{\phi(s)}\right)\sum_{r=0}^{\infty}\beta^r E_t\left[\hat{p}_{Mt+r}(s)-\hat{p}_{Ht+r}(s)\right]}_{\text{real input price term}}$$
$$+ \underbrace{\left(\frac{\varepsilon}{\phi(s)}\frac{P_0}{P_{H0}(s)}\right)\sum_{r=0}^{\infty}\beta^r E_t\left[\hat{\tilde{\mu}}_{t+r}(s)\right]}_{\text{binding constraint term}}. \quad (30)$$

where we have substituted for $\widehat{rmc}_{t+r}(s) - \widehat{rp}_{Ht+r}(s)$ and defined "real" values here in term of domestic output prices. The first term captures the role of real wages, where the labor share of gross output $(1-\alpha(s))$ is higher for services than goods. The second term then accounts for real input prices in costs. The third term captures the impact of binding constraints on markups, which

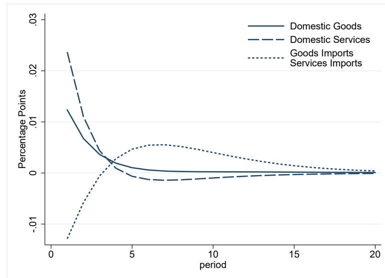
[28] Sector-level consumer price inflation is given by: $\pi_t(s) = \left(\frac{P_{H0}(s)C_{H0}(s)}{P_0(s)C_0(s)}\right)\pi_{Ht}(s) + \left(\frac{P_{F0}(s)C_{F0}(s)}{P_0(s)C_0(s)}\right)\pi_{CFt}(s)$, where $\pi_t(s) = \hat{p}_t(s) - \hat{p}_{t-1}(s)$ is inflation in sector s.

Figure 6: Demand Shock: Unconstrained Equilibrium

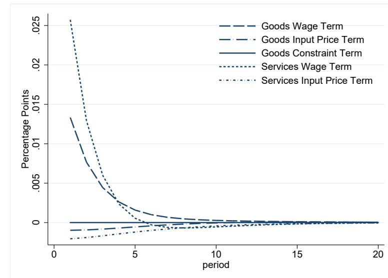
(a) Consumer Price Inflation



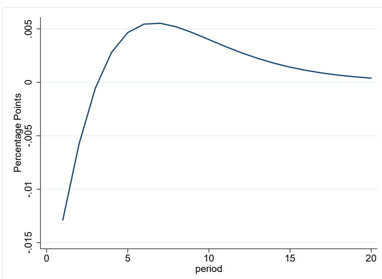
(b) Consumer Inflation Components



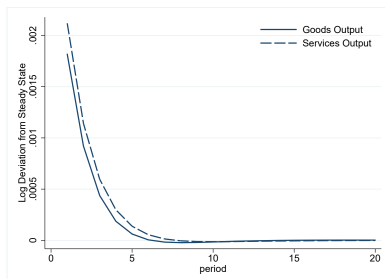
(c) Decompose Domestic Price Inflation



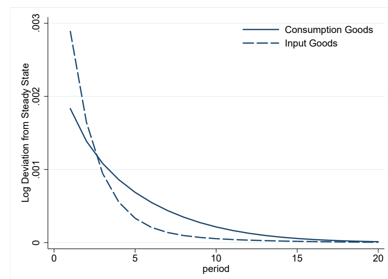
(d) Imported Input Price Inflation



(e) Gross Output Quantity



(f) Import Quantity



is identically zero in this simulation with slack constraints. We plot this decomposition by sector in Figure 6c. The real wage term for services clearly drives inflation for services beyond that for goods, reflecting the higher labor content of services. The second point to note in the figure is that input costs actually restrain inflation in both sectors, though these effects are small in magnitude.

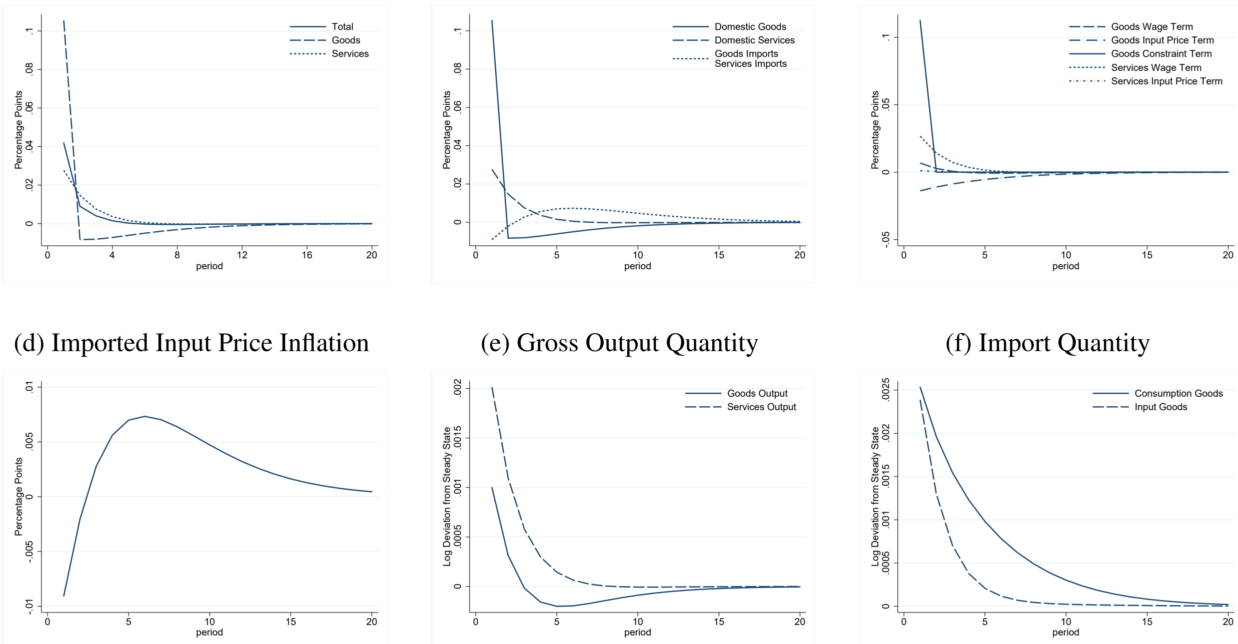
Looking at quantities, we plot real gross output by sector in Figure 6e and real goods imports in Figure 6f. The quantity of both domestic goods and services produced rises in response to increased demand in the unconstrained equilibrium. Further, both the quantity of imported consumer goods and inputs rise, with the increase in imported inputs outstripping consumption goods.

In all, while the demand shock raises overall inflation, it yields a mix of results that are inconsistent with recent data. Whereas goods price inflation exceeds services inflation in data, the opposite is true in the unconstrained model. Further, import price inflation is negative on impact, in contrast to data. Finally, both real goods output and imported inputs rise in the simulation, while they are largely flat in the data [see Figure 4]. With these puzzles in hand, we turn to versions of the model with binding constraints.

3.2.2 Binding Constraints

We now turn to discuss the impacts of the demand shock when constraints bind. We illustrate the impact of binding domestic constraints in Figure 7, and the impact of binding foreign constraints

Figure 7: Demand Shock: Domestic Constraint Binds



(a) Consumer Price Inflation (b) Consumer Inflation Components (c) Domestic Price Inflation

(d) Imported Input Price Inflation (e) Gross Output Quantity (f) Import Quantity

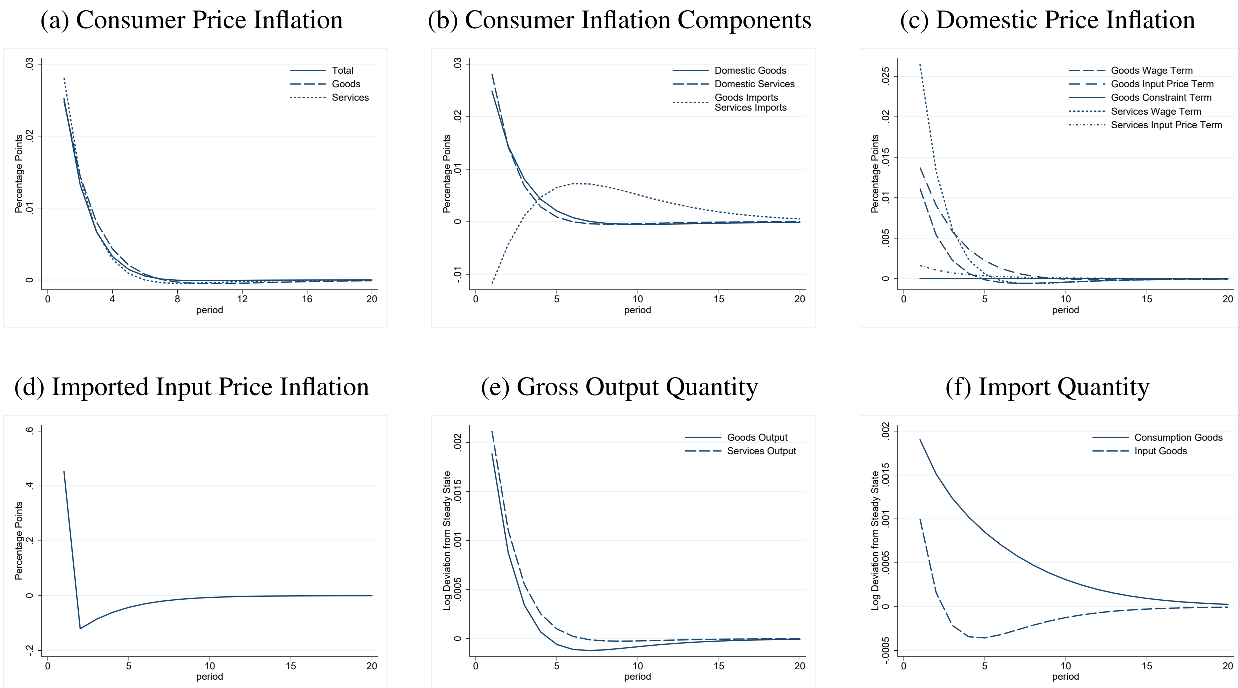
in Figure 8. For comparison to Figure 6, recall that we set the values of the constraints so that they bind only in the initial post-shock period, and are slack thereafter.

In 7a, we see that binding domestic constraints both lead to about twice as much inflation on impact. Importantly, goods price inflation now rises more than inflation in services, increasing about ten times as much as in the unconstrained equilibrium. This reflects high inflation in domestic goods prices, in Figure 7b.[29] Unpacking domestic prices using Equation 30 in Figure 7c, domestic price inflation surges due to the markup shocks induced by binding constraint, where $\hat{\tilde{\mu}}_1(1) > 0$ on impact. In Figures 7d and 7b, we again see that import price inflation falls on impact in response to the demand shock, reflecting an exchange rate appreciation and slack imported input constraints. Turning to quantities, goods output rises on impact (as there is surplus capacity in steady state), but its rise is capped at about half the unconstrained response, so goods output rises by significantly less than services output. Further, reflecting lower domestic goods output, the quantity of imported goods inputs is dampened as well, while imports of consumption goods imports increase by more than in the unconstrained equilibrium.

Turning to the case where only foreign constraints are binding in Figure 8, we note first that binding foreign constraints raise both consumer goods and domestic goods price inflation in Figures 8a and 8b. Looking at 8c, higher goods price inflation reflects the fact that prices for inputs in

[29] Like in the unconstrained equilibrium, low (positive) import price inflation for consumer goods actually attenuates overall goods price inflation; equivalently, domestic goods prices rise more than the consumer price index for goods.

Figure 8: Demand Shock: Foreign Constraint Binds



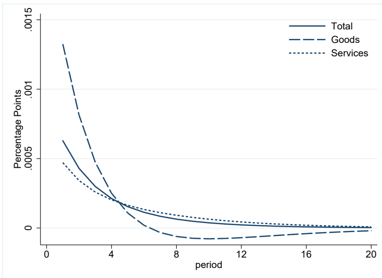
the goods sector increase on impact, and these are passed into domestic goods prices. In turn, the price of inputs for goods producers rises because imported goods inputs price inflation now spikes on impact in Figure 8d, due to the binding foreign constraints. The behavior of imported input price inflation differs from both the unconstrained equilibrium and the equilibrium with binding domestic constraints. It also differs from import price inflation for consumer goods in Figure 8b, where the effects of binding constraints on imported input prices dominates the impacts of the exchange rate appreciation, which leads imported consumer goods inflation to fall on impact. Turning to quantities, note that domestic goods output rises even though the foreign constraint binds, nearly the same amount as in the unconstrained equilibrium. Although foreign input constraints limited input availability for domestic producers, they also trigger substitution toward domestic goods producers, with offsetting effects for total production. Finally, imports of goods inputs are obviously constrained in Figure 8f, relative to the prior two cases.

3.3 Shocks to Constraints

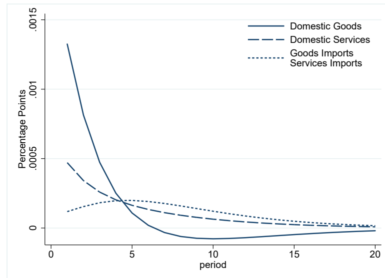
We now briefly summarize the impacts of shocks to the foreign and domestic firm-level constraints. To anchor the magnitudes, deviations in domestic and foreign capacity from steady state are given by: $\hat{\bar{y}}_t(1) = \lambda_{\bar{y}}\hat{\bar{y}}_{t-1}(1) + \varepsilon_{\bar{y}t}$, $\hat{\bar{y}}^*_{Mt}(1) = \lambda^*_{\bar{y}}\hat{\bar{y}}^*_{Mt-1}(1) + \varepsilon_{\bar{y}^*t}$with $\text{var}\left(\varepsilon_{\bar{y}t}\right) = \sigma^2_{\bar{y}}$ and $\text{var}\left(\varepsilon_{\bar{y}^*t}\right) = \sigma^2_{\bar{y}^*}$, with autocorrelation and shock variance parameters set based on estimates below.

Figure 9: Foreign Firm Capacity Shock

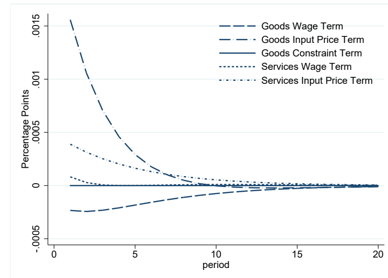
(a) Consumer Price Inflation



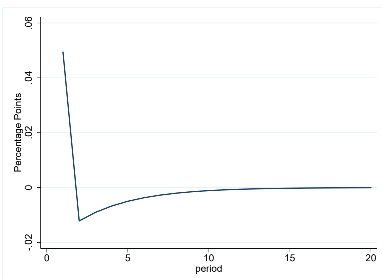
(b) Consumer Inflation Components



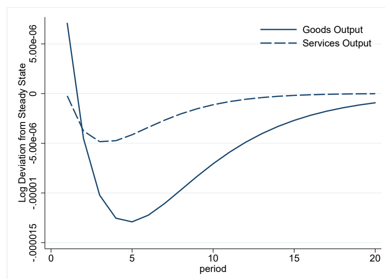
(c) Decompose Domestic Price Inflation



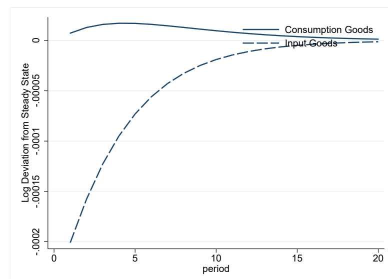
(d) Imported Input Price Inflation



(e) Gross Output Quantity



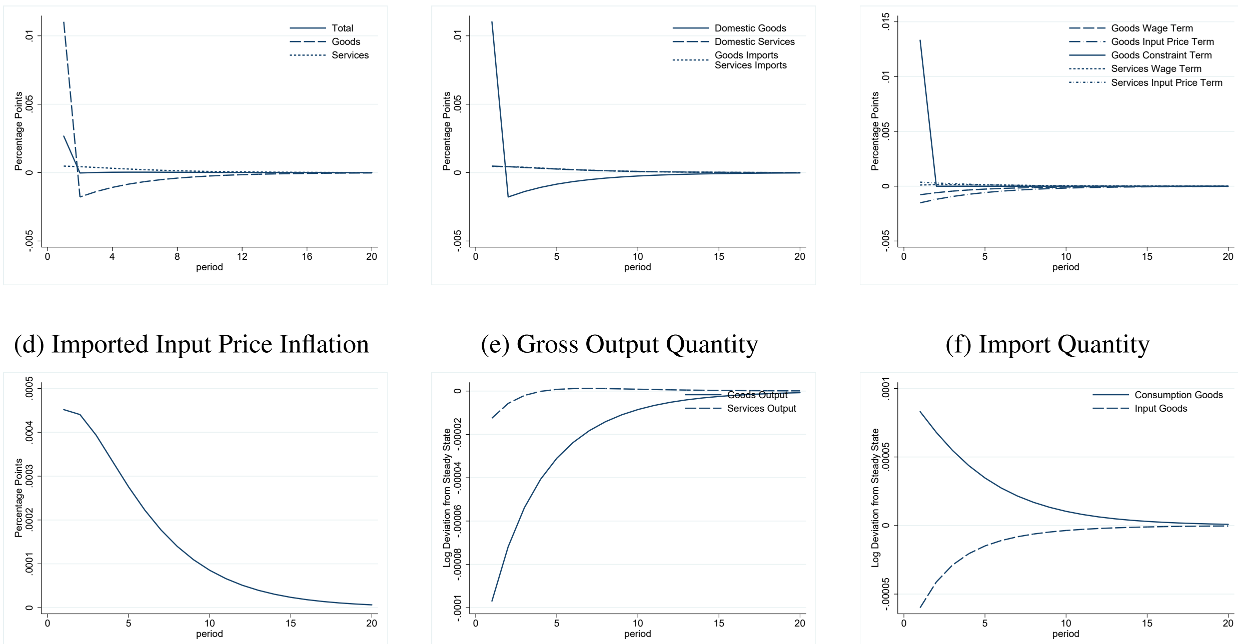
(f) Import Quantity



In Figure 9, we plot responses to a shock to the foreign capacity constraint, equal to $-0.15\sigma_{\bar{y}^*}$ in magnitude. As above, we set the initial steady state capacity level so that the constraint binds for only one period after the shock (0.01% excess capacity), and we assume that the domestic constraint is slack in all periods. Following the negative foreign capacity shocks, there is a sharp rise in imported input price inflation in Figure 9d This feeds through to domestic goods prices, and in turn to overall goods price inflation, which rises more than services price inflation in this scenario (Figures 9a and 9b). Nonetheless, the overall change in inflation is modest, reflecting the relatively small share of imported inputs in domestic input use, as well possibilities to substitute domestic for foreign inputs. Reflecting this substitution, domestic goods output actually rises slightly. Despite the fact that the real quantity of imported inputs falls on impact, due to reduced foreign capacity. Thus, this constraint shock leads to rising import price inflation together with falling quantities of imports.

In Figure 9d, we plot responses to a $-0.25\sigma_{\bar{y}}$ shock to the domestic capacity constraint, under the assumption that the foreign constraint is slack. There is again a rise in goods price inflation in Figure 10a, driven by an increase in domestic goods price inflation (Figure 10b). Again, this constraint shock leads the multiplier on the constraint to be greater than zero, and thus appears like a markup shock in the Phillips Curve, whose effects on domestic price inflation are captured in the goods constraint term in Figure 10c. The main difference again concerns quantity responses.

Figure 10: Domestic Firm Capacity Shock



(a) Consumer Price Inflation
(b) Consumer Inflation Components
(c) Decompose Domestic Price Inflation
(d) Imported Input Price Inflation
(e) Gross Output Quantity
(f) Import Quantity

Due to the fall in domestic goods capacity, actual realized goods output falls in this case (Figure 10e), and imports of intermediate goods fall on impact as well (Figure 10f). In contrast, imports of consumption goods increase, reflecting substitution from domestic to import sources. The negative comovement between inflation and real output/imports is a distinctive feature for this shock.

3.4 Summing Up

Stepping back, we collect a few summary results shed light on how the model will identify which constraints are binding and the structure of underlying shocks. First, the sector-composition of inflation depends on the configuration of shocks and constraints. With slack constraints, inflation for services outstrips that for goods. Binding foreign constraints raise goods price inflation to equal services price inflation, and binding domestic constraints lead goods price inflation to outstrip services inflation, which is a prominent feature of recent data. Second, imported goods input price inflation is high only when the foreign firm constraint binds, either following a demand shock or a shock to the constraint itself. Put differently, while a binding domestic constraint may explain excess inflation for goods, it cannot also generate a sharp increase in import price inflation on its own. Third, co-movement in prices and quantities differs depending on whether constraints are binding or slack, as well as the nature of the shocks. In the case of a demand shock, gross output rises along with inflation, as does the quantity of imported inputs. For constraint shocks, however, inflation and output comove negatively: a shock to the foreign firm constraint raises import price inflation while lowering imported input quantities, while a shock to the domestic constraint raises domestic goods price inflation while lowering quantities produced.

4 Accounting for Recent Inflation Experience

Tying together Sections 1 and 3, we now apply the model to parse recent data. We explain the procedure we use to estimate the model briefly in Sections 4.1, with additional details in Appendix A. Then, we discuss data, calibration, and estimated parameters in Section 4.2. In Section 4.3, we review model fit. We then discuss what the model tells us about recent inflation in Section 4.4.

, and then we discuss results.

4.1 Estimation Framework

Referring back to Section 2.6.1, the impact of a given structural shock in the model depends on whether constraints bind today following the shock, as well as the expected duration that constraints are expected to continue to bind into the future following that shock. To make this dependence explicit, let us define a set of regimes ($\mathbb{R}_t$), which record which constraints are binding at a given point in time: $\mathbb{R}_t = \{\mathbf{1}\left(Y_t(1) = \bar{Y}_t(1)\right), \mathbf{1}\left(Y^*_{Mt}(1) = \bar{Y}^*_{Mt}(1)\right)\}$, where the indicator functions switch on when individual constraints bind. Given a sequence $\left\{\mathbb{R}_{t+j}\right\}$ for $0 \leq j \leq J$, together with the assumption that $\left\{\mathbb{R}_{t+j}\right\} = \{0,0\}$ for $j > J$, we can solve for an equilibrium path for $\{X_t\}$, using the method described in Cagliarini and Kulish (2013) and Kulish and Pagan (2017) (see Appendix A for details).

Building on this idea, we re-parameterize the model solution in a convenient way. Specifically, let us define the duration that constraints are expected to bind from date t forward as $\mathbf{d}_t = [d_t, d^*_t]$, where each entry is a non-negative integer that records the number of periods that the domestic (d_t) or foreign constraint (d^*_t) binds. By convention, d_t and d^*_t take on zero values when constraints are slack today and expected to remain so in the absence of future shocks, and they are positive when they are binding today. As in Guerrieri and Iacoviello (2015), we construct policy matrices under the assumptions that agents know the state (X_{t-1}) and the current realization of the shocks (ε_t), but that they do not anticipate that future shocks will occur. Under these assumptions, $\mathbf{d}_t$ summarizes all the information about the anticipated sequence of regimes that is needed to solve for equilibrium responses to a one-time shock in our model. Specifically, constraints may switch on immediately in response to shock at date t, then bind for some (non-negative) number of consecutive periods, and switch off thereafter. In the absence of future shocks, constraints do not then switch on again in periods after they switch off (e.g., following a shock ε_t, constraints cannot be slack at date t and then binding at date $t+1$).[30] With these observations, we re-write the model solution directly in terms of durations:

$$X_t = \mathbf{J}(\mathbf{d}_t, \theta) + \mathbf{Q}(\mathbf{d}_t, \theta) X_{t-1} + \mathbf{G}(\mathbf{d}_t, \theta)\, \varepsilon_t, \tag{31}$$

where duration $\mathbf{d}_t$ implies a specific anticipated sequence of regimes over time. We refer the reader to Appendix A.3.2 for details on this result.

Following Kulish, Morley and Robinson (2017), Kulish and Pagan (2017), and Jones, Kulish and Rees (2022), our estimation framework exploits the fact that durations enter the policy function like parameters. As is standard, let us assume that observables (S_t) are linearly related to the unobserved state, as in $S_t = H_t X_t + \nu_t$, where ν_t is an i.i.d. vector of normally distributed measurement errors. Given $\mathbf{d} \equiv \{\mathbf{d}_t\}_{t=1}^{T}$ and θ, we can construct the piecewise linear solution with time-varying coefficients, and then apply the Kalman filter to construct the Likelihood function $\mathscr{L}(\theta, \mathbf{d} | \{S_t\}_{t=1}^{T})$. We put priors over structural parameters and independent priors over durations to construct the posterior, and then estimate the model via Bayesian Maximum Likelihood.

In implementing this approach to estimation, we are careful to account for the fact that the duration of binding constraints is an equilibrium object in the model – i.e., D_t depends on both the state $X_{t-1}$and current shock ε_t in our model. Thus, we impose a rational expectations equilibrium restriction on admissible durations, which requires that agents' forecasts about how long constraints bind following a given shock are consistent with equilibrium model responses. To impose this restriction, we proceed as follows. For each proposed duration and parameter draw, we filter the data for smoothed shocks. We then evaluate whether the equilibrium model response to those smoothed shocks is consistent with the proposed duration draw. We retain the proposed draw if this requirement is satisfied; otherwise, we reject it and draw again. Lastly, as a practical matter to restrict the size of the parameter space, we impose priors that allow capacity constraints to bind only for periods after the second quarter of 2020, thus focusing on the role of capacity in explaining the unusual post-pandemic inflation dynamics.[31]

[30] To be careful, this is not a general property of models with potentially binding constraints, but rather one that holds given the structural assumptions in our model about behavior and shock processes. While we lack a general proof of this property, we verify it holds numerically in the model in practice, and we can demonstrate that imposing this criterion in the estimation procedure is reasonable via Monte Carlo analysis. It is also the case that one could capture a more complex structure of potential regime changes via introduction of additional parameters (e.g., durations for binding constraints that start one period forward), at the cost of added computational complexity.

[31] As an extension, we intend to explore variants on these baseline results in which constraints are allowed to potentially bind prior to 2020. This extension expands the parameter space, so raises computational demands and requires careful treatment.

4.2 Data and Parameters

To populate Y_t, we collect standard macro variables together with particular series that serve to identify whether constraints are binding and shocks to them. Among standard macro variables, we include consumption price inflation and the growth rates of consumption expenditure for goods and services. We also use data on aggregate nominal GDP growth and the growth rate of (real) industrial production, which we treat as a proxy for output of the goods sector.[32] We also use data on labor productivity growth by sector, measured as real value added per worker.[33] On the international side, we use data on import price inflation for consumption goods, and we proxy input price inflation in the model using data on import price inflation for imported industrial materials (excluding fuels). We then also use data on the growth of total expenditure on imported consumption goods and imported materials inputs (again excluding fuels), which we associate with imported inputs of goods.[34]

These data are all obtained from quarterly US national accounts produced by the Bureau of Economic Analysis, with the exception of labor productivity data from the Bureau of Labor Statistics and industrial production from the Federal Reserve Board (G.17 program). Having constructed growth rates for individual variables from the first quarter of 1990 through the first quarter of 2022, we detrend the data by removing the mean growth rate from each series. Finally, because our estimation sample includes a significant period during which interest rates are at the zero lower bound, we use data on the "shadow Fed Funds rate" to estimate parameters in the monetary policy rule.[35]

We present the full set of parameters for the model in Appendix A, which we obtain through a mix of estimation and calibration. We calibrate key value shares in the model – e.g., consumer expenditure, input use, export and import shares, etc. – to match US national accounts and input-output data (see Table A.4). We set a subset of the structural parameters to standard values from

[32]In Figure 4, we plotted data on gross output by sector from the BEA. Though definitions of goods production and industrial production do not align exactly (industrial production includes manufacturing, mining, and electrical/gas utilities, while the BEA-defined goods sector excludes utilities and includes agriculture and construction), the dynamics of gross output for the goods sector and industrial production are similar. We opt for industrial production data here, because we require a sufficiently long time series to estimate model parameters. While quarterly gross output data is only available after 2005, industrial production data is available from 1947.

[33]We use data on labor productivity growth in manufacturing and total (private sector) labor productivity growth from the Bureau of Labor Statistics. We assume that labor productivity growth in manufacturing coincides with goods labor productivity (growth in real value added per worker) in the model, while also matching aggregate (economy-wide) labor productivity growth in the model.

[34]We use data for consumer goods (except food and automotive) to proxy for consumption imports, and we construct proxies for imported inputs (excluding fuels) by removing the subcategory of petroleum and products from industrial materials and supplies using standard chain index formulas and auxiliary NIPA data on the sub-categories of imports.

[35]During periods where the nominal Fed Funds rate is at zero, we replace it with the shadow rate from Wu and Xia (2016): `https://www.atlantafed.org/cqer/research/wu-xia-shadow-federal-funds-rate`. Changes in the shadow rate, which is not bounded below by zero, capture the consequences of unconventional policy actions taken by the Federal Reserve, such as forward guidance or quantitative easing policies. We have checked the results using an alternative shadow rate series from Jones, Kulish and Morley (2022) as well, which yields similar results.

the literature, including preference parameters and some elasticities of substitution (see Table A.3).

We then estimate four sets of parameters: (a) the elasticities of substitution between home and foreign goods, in consumption and production separately; (b) the parameters in the extended Taylor rule governing the response of interest rates to inflation and output, as well as interest rate inertia, (c) parameters governing the stochastic processes for exogenous variables, and (d) the variance of measurement errors. Regarding (c), we assume that exogenous variables evolve according to AR1 stochastic processes, so we need to estimate autocorrelation and shock variance parameters. We also estimate the levels for domestic and foreign capacity (i.e., $\ln \bar{Y}_0(1)$ and $\ln \bar{Y}^*_{M0}(1)$) around which capacity fluctuates as shocks hit. Interpreting these estimated parameters requires some care, in that they are identified by data for periods in which the constraints may potentially bind. Because we allow constraints only to bind from 2020:Q2 forward in the results below, the estimated value of this parameter is based only this subset of the data. As such, the resulting estimates are best thought of as the mean of capacity during this period, rather than steady state capacity per se.[36]

We obtain an estimated mean value for the elasticity of substitution between home and foreign goods of about 1.5 in consumption and 0.5 for inputs, so consumer goods are substitutes while inputs are complements. These values are not far from standard values estimated using aggregate time series variation in the macroeconomic literature, though there is limited prior work that distinguishes consumption and input elasticities. Reflecting the discussion above, the estimated values for excess capacity are low, at about 1% for both domestic firms and foreign firms. There is significant autoregressive persistence in most exogenous variables, and measurement error variances are plausible. See Table A.5 for the full set of estimated parameters.

4.3 Model Fit

Applying the quantitative model framework to the data, we construct Kalman-smoothed values for endogenous variables and observables. In Figure 11, we plot data and smoothed values for several key observables – goods, services, and aggregate price inflation for consumers, and imported input price inflation – over the 2017-2022 period, where each data point is the annualized value of quarterly inflation. To compute the smoothed inflation series, we take 1000 draws from the posterior distribution for model parameters, compute Kalman-smoothed inflation for each draw, and then plot statistics (the median, 5th, and 95% percentiles) for the distribution of smoothed values.

The model fits the dynamics of aggregate consumer price inflation well, accounting for essentially all of the four percentage point increase in headline inflation after 2020 (11a).[37] It also

[36] Below, we will explain more precisely how most of the results we obtain are insensitive to whether we estimate or calibrate capacity, by varying how we approach this issue. As noted in a prior footnote, we intend to allow constraints to potentially bind in pre-pandemic periods as well, which we anticipate will raise estimated capacity levels.

[37] Recall that aggregate consumer price inflation is treated as an unobserved variable. In the model, it is constructed

accounts well for the two percentage point rise in inflation for the services sectors (Figure 11b. Because goods price inflation is substantially more volatile than that for services, the model attributes more of its variation to measurement error. Nonetheless, smoothed values for goods price inflation also track the data well (11c). The model replicates the initial (roughly six percentage point) surge in goods price inflation in 2021, and goods price inflation then remains elevated into 2022. While the model captures its transitory (up/down) dynamics, it moderately undershoots the level of goods price inflation in 2022, meaning that the model attributes the gap to measurement error. The model also matches inflation for imported goods inputs well (11d), matching both levels and dynamics closely.

For brevity here, we present similar figures illustrating model fit for the remaining observables in Appendix A. Together with the inflation figures here, we assess that the model captures the behavior of economic variables well during the pandemic, so it is a useful laboratory for exploring the driving forces underlying the inflation surge.

4.4 Explaining the Inflation Surge

We provide three sets of results. The first two illustrate the role of constraints in explaining inflation. First, we examine the dynamics of the multipliers on the constraints. Second, we present counterfactuals in which we switch off the constraints, comparing model responses to the same set of shocks with and without constraints. The third set of results focuses on how individual shocks and constraints shape inflation outcomes, both individually and via interactions between them.

4.4.1 Multipliers on Constraints

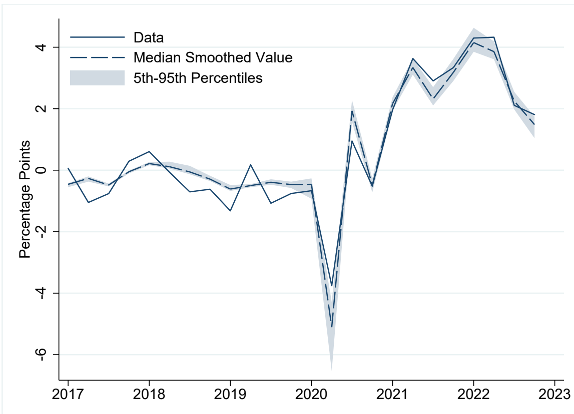
To start, we can directly illustrate the impact of constraints by examining the smoothed value of multipliers on the domestic and foreign constraints. Recall that these multipliers appear directly in the domestic price and import Phillips Curves, as quasi-markup shocks in reduced form. Thus, plotting them amounts to summarizing the latent changes in markups attributable to binding constraints. As is evident, the value of both the domestic and foreign multipliers escalates in 2021, coincident with the rise in headline inflation.[38] On the import side, constraints are slack in 2020,

by aggregating sector-level consumer price growth using fixed (steady-state) expenditure weights. In the data, however, the PCE deflator is a chain-weighted index, which features time-varying weights. Thus, part of the discrepancy between aggregate inflation in the model and data is likely due to differing index number concepts. Specifically, the dramatic increase in the goods expenditure share, combined with high goods price inflation, likely pushed measured inflation up relative to our fixed-weight index. Going forward, we focus entirely on decomposing model-based measures of inflation, so we do not belabor this point.

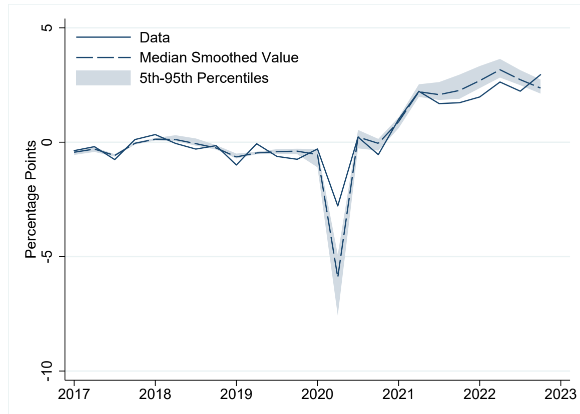
[38] As a technical note, recall we place positive mass on values $d_t > 0$ in our priors only starting in 2020:Q2 (i.e., we impose dogmatic priors that assign zero probably to binding constraints prior to this date). As such, the multiplier in the figure is identically zero up to that date. A second point worth discussion is that while multipliers are typically positive, they dip into negative territory at times in the simulations. This reflects two factors. First, there is approximation

Figure 11: Consumer Price Inflation in Model and Data

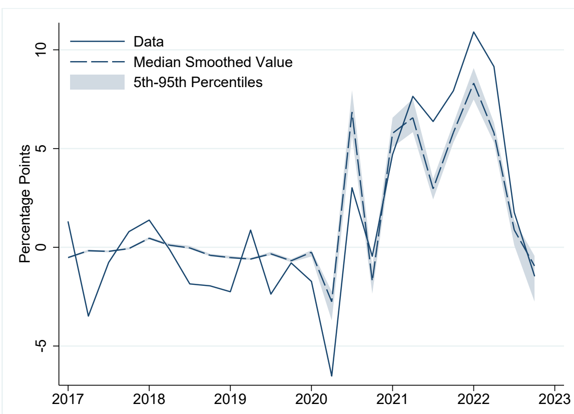
(a) Aggregate Consumer Inflation



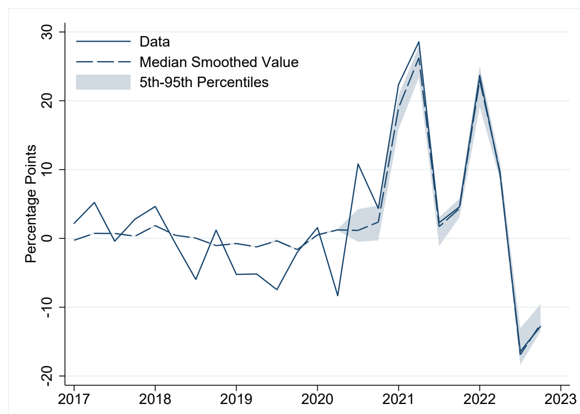
(b) Consumer Services Inflation



(c) Consumer Goods Inflation

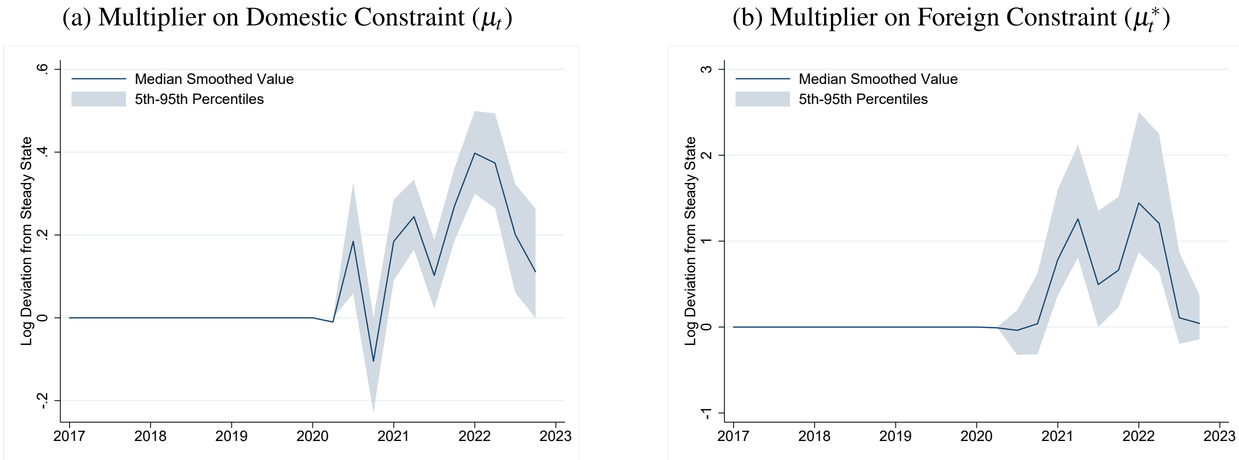


(d) Inflation for Imported Goods Inputs



Note: Inflation at each date is the annualized value for demeaned quarterly inflation, in percentage points. If demeaned quarterly inflation is $\pi_t(s) = \ln P_t(s) - \ln P_{t-1}(s)$ where t indexes quarters, then the annualized inflation rate is $4\pi_t(s)$. Data is raw data. We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, and then plot the median smoothed value as the dashed line. We shade the area covering the the 5% to 95% percentile for smoothed values (the interval is imperceptibly small prior to 2020).

Figure 12: Smoothed Values for Multipliers on Domestic and Foreign Constraints



Note: We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, and then plot the median smoothed value as the solid line. We shade the area covering the the 5% to 95% percentile for smoothed values.

then bind sharply at the start of 2021, relax somewhat, then bind sharply again into 2022, and ease in the latter half of 2022. Domestic multipliers fluctuate in 2020 with gyrations in the US economy, but are near zero heading into 2021. They rise steadily through 2021 into 2022, and then slacken (though still bind) through 2022:Q3.

For both multipliers, we note that the higher frequency dynamics correspond to fluctuations in goods price inflation and imported input price inflation in Figure 11, which foreshadows the quantitative role of the constraints in explaining inflation. We turn to model counterfactuals to parse the role of constraints further.

4.4.2 Relaxing Constraints

We now provide counterfactual analysis as to how inflation would have evolved in the absence of capacity constraints, given the path of realized shocks that we infer hit the US economic after 2020.

To describe this exercise more precisely, the mechanics of each iteration are as follows. We first draw model parameters from the estimated posterior distributions, including the durations for

error in the piecewise linear solution technique that we employ. In particular, it does not explicitly impose the zero lower bound on the multipliers, which is required in the full non-linear solution to the model. Instead, multipliers are implicitly calculated via residuals in the Phillips Curve – their value is pinned down by excess inflation over and above that which can be explained by real marginal costs and expected inflation. Because the Phillips Curve is approximated (along with other elements of the model), the residuals may give rise to negative multipliers. Second, our duration-based approach to solving the model is also subject to approximation errors, which also factors into the measured multipliers. Despite these approximation errors, we find that approximated multipliers are typically positive in simulations, consistent with the underlying theory.

binding constraints. Given these parameters, we apply the Kalman-filter to the data and construct smoothed model outcomes and shocks. Note that we construct smoothed shocks here assuming that constraints are potentially binding, in line with posterior duration estimates. Using these smoothed shocks, we then simulate the path of the economy under the counterfactual assumption that constraints are slack throughout, such that the solution conforms to the unconstrained equilibrium dynamics of the model. We repeat this procedure for one thousand posterior draws, and we plot statistics (means and percentiles) across these simulations in Figures 13 and 14.

Figure 13 presents results for consumer price inflation. The figures present raw data on annualized values of (de-meaned) quarterly inflation, along with data from counterfactual simulations in which we allow for measurement error in these observables.[39] In Figure 13a, we see that realized inflation for consumer goods is substantially higher than counterfactual inflation with slack capacity constraints during 2021 and into 2022, with the absolute gap peaking near six percentage points in early 2021. Put differently, given the shocks we infer from data, binding constraints account for about half of the acceleration in goods price inflation from 2020:Q2 through 2021:Q2.

Under the hood, these inflation outcomes are tied to the impact of binding constraints in holding back production of domestic goods and foreign goods inputs. In Figure 14a, we plot the path for smoothed domestic goods output along with counterfactual output. As is evident, in the absence of constraints, goods output would have risen significantly in 2021 relative to its pre-pandemic level, as a result of the other shocks (principally, demand shocks) that hit the economy. The fact that output did not rise in reality speaks directly to the role of constraints. Output of foreign goods inputs is similarly constrained in Figure 14b. Correspondingly, smoothed inflation for both domestically-produced goods and foreign-produced inputs is substantially higher than counterfactual inflation in Figures 14c and 14d.

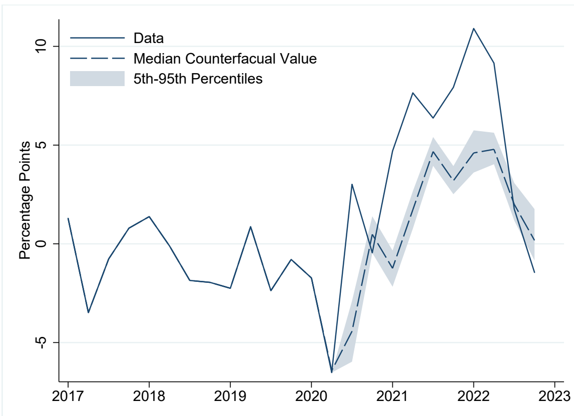
Interestingly, binding constraints also play an important role in driving price inflation for services in Figure 13b. While services price inflation initially accelerates due to the underlying shocks, it is between one and two percentage points higher in 2021 as a result of binding constraints. In the background, this reflects both the fact services use goods as inputs, so there is a direct inflation spillover from binding constraints in the goods sector via input-output linkages. Further, binding constraints serve to tighten the labor market as well, as the price increases they generate trigger substitution from goods inputs toward labor in production.

Adding up these results in Figure 13c, headline consumer price inflation is between one and two percentage points higher than counterfactual inflation during 2021-2022. And binding constraints

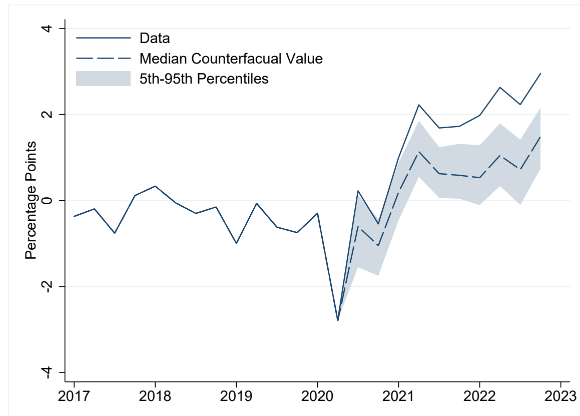
[39] In the procedure described above, we draw the variance of the measurement error from the posterior, and then filter the data given this draw. We then add a draw from the measurement error to the smoothed counterfactual endogenous variables to get counterfactual values for the observables that are comparable to data. An alternative approach to presenting the results would be to compare smoothed observables to model counterfactuals without measurement error; naturally, this alternative leads to similar conclusions.

Figure 13: Counterfactual Consumer Price Inflation without Capacity Constraints

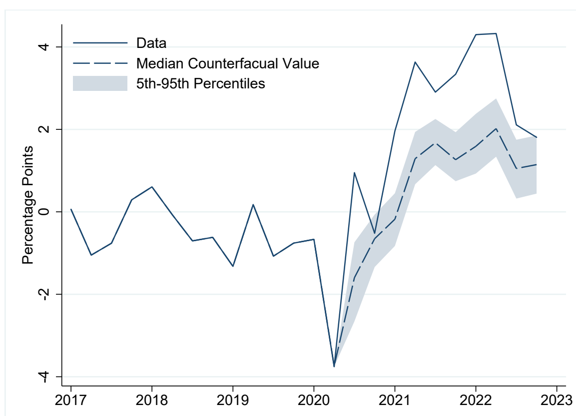
(a) Goods Inflation



(b) Services Inflation



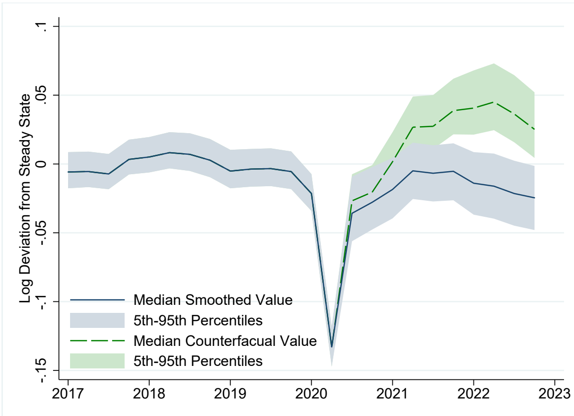
(c) Aggregate Inflation



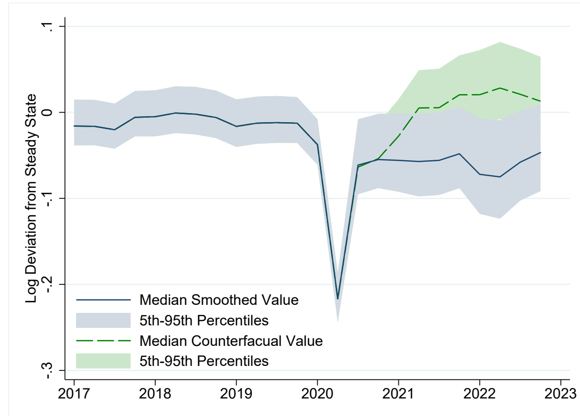
Note: We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, add measurement error to the observables, and then plot the median smoothed value as the solid line. We shade the area covering the the 5% to 95% percentile for smoothed values.

Figure 14: Counterfactual Quantities without Capacity Constraints

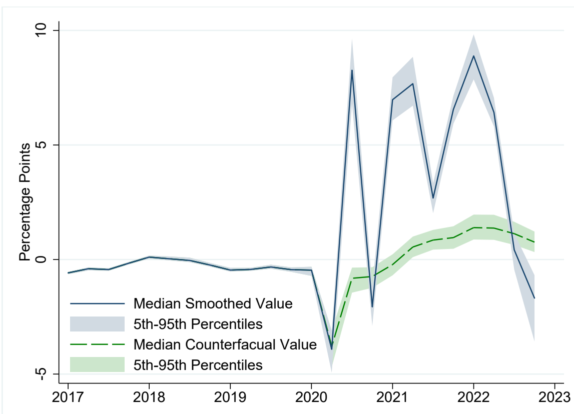
(a) Domestic Goods Output ($Y_t(1)$)



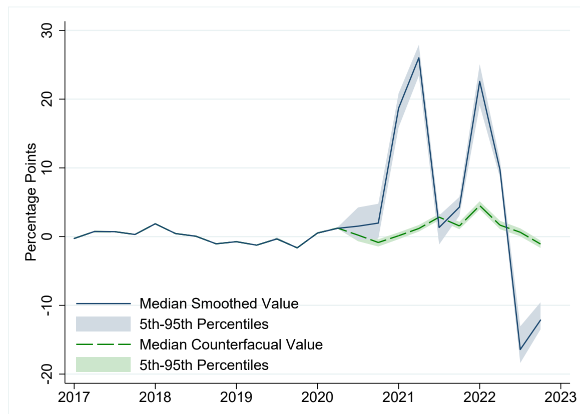
(b) Imported Goods Inputs $Y_{Mt}^*(1)$



(c) Domestic Goods Price Inflation ($\pi_t(1)$)



(d) Imported Goods Input Inflation ($\pi_{Mt}(1)$)



Note: We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, and then plot the median smoothed value as the solid line. We shade the area covering the the 5% to 95% percentile for smoothed values. Counterfactual assumes that constraints are slack in all periods.

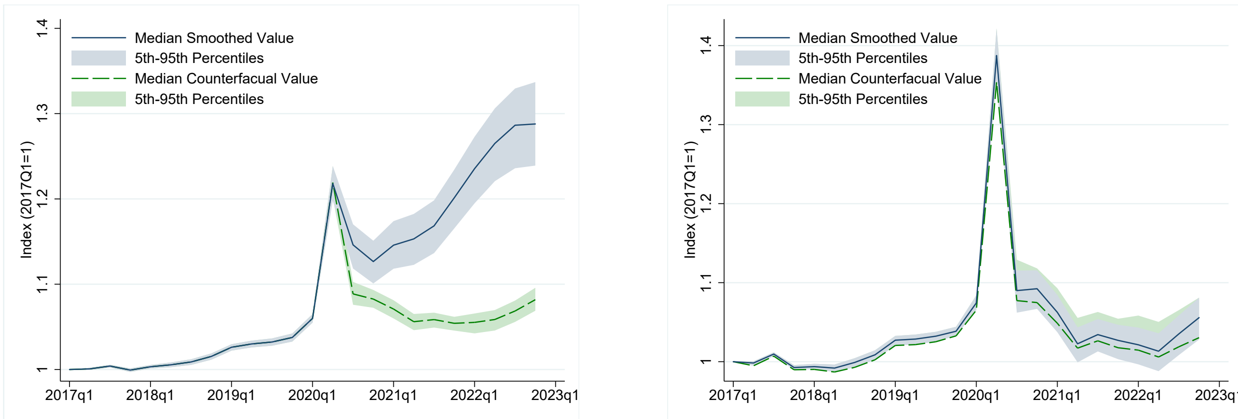
account for about one third of the acceleration in headline goods price inflation from 2020:Q2 through 2021:Q2. Note further that the effect of constraints is substantially diminished late in 2022, as actual and counterfactual inflation converge again.

Finally, we revisit the discussion surrounding profits per unit in this counterfactual exercise. In Figure 5, we presented an index of nominal profits per unit of gross output for manufacturing and the aggregate economy. In Figure 15 we present analogous results from the model for goods and services.[40] Similar to the data, the smoothed data from our model yields a sharp increase in profits

[40] In the model, the log change in nominal profits per unit of output from a given base period ($t=0$) is given by: $\left[\hat{\Xi}_t(s)-\hat{y}_t(s)\right]-\left[\hat{\Xi}_0(s)-\hat{y}_0(s)\right]=[\hat{p}_{Ct}-\hat{p}_{C0}]+\varepsilon\,[\widehat{rp}_t(s)-\widehat{rp}_0(s)]-(\varepsilon-1)\,[\widehat{rmc}_t(s)-\widehat{rmc}_0(s)]$, where $\hat{p}_{Ct}-\hat{p}_{C0}=\sum_{s=0}^{t}\pi_{Cs}$. We add trend inflation to these log changes, to make it comparable to the date in Figure 5, and then we convert the log change to levels to plot the index. While we discuss manufacturing and aggregate profits in Figure

Figure 15: Counterfactual Profits per Unit without Capacity Constraints

(a) Goods (b) Services



Note: We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, and then plot the median smoothed value as the solid line. We shade the area covering the the 5% to 95% percentile for smoothed values.

for the goods sector during the 2021-2022 period, even though this is not a targeted data moment. In contrast, the counterfactual economy with slack constraints yields no such goods profit surge. Moreover, profits per unit are essentially flat through the pandemic period (outside the 2020 spike), for both the economies with and without capacity constraints. We conclude that the model provides a plausible explanation for the run-up in profits for goods producers that occurred alongside the inflation takeoff, where both are explained in large measure by binding capacity constraints.

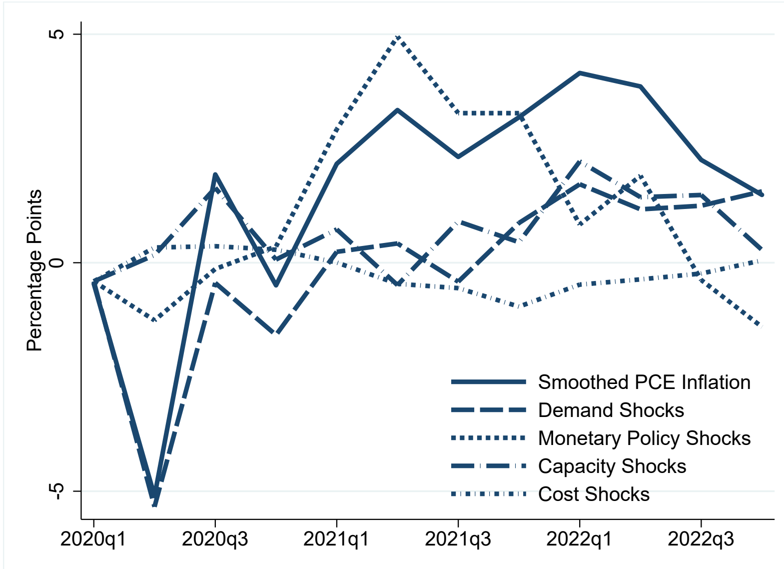
4.4.3 Decomposing the Role of Individual Shocks and Constraints

We now examine the role of individual shocks in explaining inflation outcomes. Parameterizing the state equation (Equation 31) using modal values for the structural parameters and durations, we Kalman filter the data to obtain smoothed shocks. We then feed the shocks into the structural model (summarized by Equation 27) one at a time. In each case, we solve for the simulated equilibrium path using Dynare's OccBin procedure, meaning that whether constraints bind at particular points in time is endogenous.

In Figure 16, we plot the path of aggregate consumer price inflation following four types of shocks: demand shocks (including both the discount rate and goods-biased preference shocks), monetary policy shocks, capacity shocks, and cost shocks (including domestic productivity and foreign cost shocks). The final line is the median value for smoothed PCE inflation, as presented

5 due data availability in the national accounts, the goods sector in our calibration includes manufacturing and other non-manufacturing goods sectors. Further, again for data reasons, we focused on corporate profits in Figure 5, while we have no distinction between corporate and non-corporate entities in the model. This implies that one ought to focus on qualitative comparisons between the figures, rather than a more precise quantitative comparison.

Figure 16: Counterfactual Consumer Price Inflation for Individual Shocks



Note: Smoothed PCE inflation is the median smoothed value reported in Figure 11a. The other series are the simulated path of consumer price inflation following the indicated subset of smoothed shocks in 2020-2022.

previously in Figure 11a.[41] At the outset, temporary negative demand shocks yield a decline then rebound of inflation in 2020. Into 2021, monetary policy shocks kick in: inflation rises by almost 8 percent from 2020:Q2 to 2021:Q2 in response to all shocks, and we attribute this about half to demand and half to monetary policy shocks. Later in the period, the effects of monetary policy wane, as the Fed raised interest rates and the implied monetary policy shocks shrink. The impact of the demand shocks is then larger in explaining sustained inflation through the end of 2022.

Turning to the remaining shocks, we find that cost shocks are unimportant in accounting for inflation dynamics throughout the period. We also find a relatively small role for capacity initially, though negative capacity shocks seem to be pushing inflation up through 2022. To interpret these results, we recall that the estimated "steady state" level of capacity is based only on data from the post-2020 period, so capacity shocks move relative to this level. Thus, the results indicate that negative capacity shocks tightened constraints over the course of the pandemic, leading to higher inflation.

Supplemental Results for Calibrated Capacity Levels In interpreting the role of capacity shocks in Figure 16, it is important to recall that we estimate the mean level of capacity using data only for the post-pandemic period. We find that there is a small amount of excess capacity during this period on average. At the same time, mean capacity in normal times is likely sub-

[41] We use this series as the benchmark here, so that it is the same across this figure and the next for comparison.

stantially higher. In this event, it is possible that capacity shocks lowered capacity during the pandemic period relative to its pre-pandemic level, an effect that we are not picking up in the prior simulations.

To investigate this, we now turn to an alternative set of simulations, in which we calibrate steady state capacity levels instead of estimating them. We set the level of excess capacity for domestic firms in steady state to 5% and that for foreign firms to 10%. These levels are sufficiently high to ensure that constraints are slack prior to 2020:Q2, consistent with the priors we impose on the constraint durations.[42] With this calibration, we re-estimate the remaining structural parameters of the model and constraint durations. We then replicate the analysis above, in which we set up the state equation using modal parameter values, filter shocks from data, and then simulate model responses to those shocks.

We present two sets of results in Figure 17. The first set replicates Figure 16, where we feed one set of shocks through the model at a time. Comparing Figure 17a to Figure 16, the interesting result is that no individual shock appears to have larger impacts on inflation here. The underlying reason is that no single shock is capable of causing capacity constraints to bind. Specifically, with high steady state capacity, monetary policy and demand shocks are too small to trigger the constraints, meaning that model outcomes conform closely to those observed in the prior counterfactuals in which we exogenously relax the constraints (see Section 4.4.2).

In Figure 17b, we plot a second set of simulation results in which we feed in combinations of the shocks. In all cases, we feed capacity shocks into the model, and then we add either demand, monetary policy, or cost shocks in addition to the capacity shocks. These results correspond more closely with the results in Figure 16, in that the takeoff in inflation is fueled by monetary policy, with an additional contribution of demand shocks. This means that negative capacity shocks seem to be important in explaining why capacity was tight during the pandemic periods – i.e., why capacity was lower than its prevailing level prior to the pandemic. These capacity shocks set the stage for the other demand-side shocks to trigger the constraints.

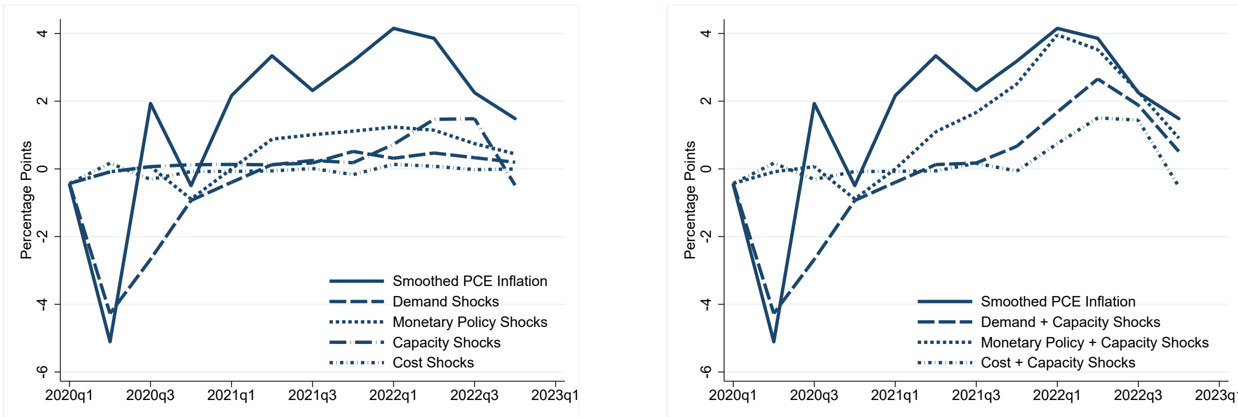
5 Concluding Remarks

We have developed a quantitative framework to study inflation that places potentially binding capacity constraints at center stage. We show that binding constraints alter the Phillips Curve relationship between inflation and real marginal costs, because firms take these constraints into account when setting prices. Specifically, when constraints bind, firms set prices to equate demand to their

[42]This amount of excess capacity is also roughly consistent with historical fluctuations in capacity utilization for the US, as measured by the Federal Reserve G.17 data, in which the maximal value for capital utilization about five percent higher than the minimum.

Figure 17: Counterfactual Consumer Price Inflation with Calibrated Capacity and Individual Shocks

(a) Individual Shocks (b) Capacity Shocks plus Individual Shocks



Note: Smoothed PCE inflation is the median smoothed value reported in Figure 11a. The other series are the simulated path of consumer price inflation following the indicated subset of smoothed shocks in 2020-2022.

constrained capacity, rather than targeting their optimal unconstrained markup over marginal costs. This implies that binding constraints introduce a term that looks like a markup shock in both domestic and import price Phillips Curves. Applying the quantitative framework to interpret recent US data, we find that binding constraints are quantitatively important drivers of inflation, explaining half of the rise in US inflation during 2021-2022. We also find that negative capacity shocks tightened constraints during the pandemic period, which set the stage for modestly sided demand shocks to have outsized impacts on inflation. In particular, monetary policy shocks loom large in driving inflation in 2021.

Going forward, there are various extensions of this framework that would be useful to consider. While we have focused on simple output price rigidities, it would be useful to extend the model to include wage rigidity to generate additional internal persistence. It would also be useful to extend the framework to consider additional types of shocks. Incorporating labor supply shocks would allow the model to address a richer set of labor market facts and assess the relative roles for capacity constraints versus labor shortages in explaining inflation outcomes. It would also be useful to extend the model to incorporate fiscal shocks, especially to study the tax policy instruments that supported consumption in the early stages of the pandemic. Finally, we have included capacity as an exogenous, stochastic variable in our framework. We also see high returns to extending the model to include endogenous capacity investment.

More generally, the framework can be deployed to study optimal policy, and by extension potential policy mistakes during the pandemic recovery. In our framework, binding constraints imply that demand shocks work through both the IS and Phillips Curves, appearing like a markup shock. This complicates policy design, relative to canonical frameworks in which shocks to the IS and Phillips Curves are unrelated. Further, when reduced-form markups may reflect either the influence of exogenous markup shocks, or the impact of binding constraints, optimal policy will depend on the central bank's ability to discriminate between them. Given the importance of monetary policy shocks in our quantitative analysis, a critical analysis of policy is warranted.

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A Quantitative Model

In this appendix, discuss the quantitative version of the general model described in Section 2. We start by discussing log-linear approximation of the model equilibrium conditions. We then describe the stochastic processes for exogenous variables and calibration details.

A.1 Log-Linearization of the Model Equilibrium Conditions

In Table 1, we wrote out the full nonlinear model equilibrium. In practice, we solve a piece-wise linear approximation to the model, as in Guerrieri and Iacoviello (2015). This entails log-linearizing the model equilibrium conditions for both the unconstrained and constrained equilibria around the steady state.

We normalize Home prices relative to the domestic price level, and we define "real" prices with the letter r attached to the price. Further, lower case variables with hats denote log deviations from steady state. For example, the log deviation in the real wage from steady state is given by $\widehat{rw}_t = \hat{w}_t - \hat{p}_t$, while the real price of home output in sector s is $\widehat{rp}_{Ht}(s) = \hat{p}_{Ht}(s) - \hat{p}_t$, and so on.[43] Foreign currency prices (denoted by stars) are normalized relative to the foreign price level; for example, foreign real marginal costs are $\widehat{rmc}_t^*(s) = \widehat{mc}_t^* - \hat{p}_t^*$. We also define deviations in the value of constraints from steady state: $\hat{\bar{y}}_t(1) = \ln \bar{Y}_t(1) - \ln \bar{Y}_0(1)$ and $\hat{\bar{y}}_t^*(1) = \ln \bar{Y}_t^*(1) - \ln \bar{Y}_0^*(1)$. Finally, to reduce the number of potential foreign shocks, we assume that foreign export demand is given by $X_t^*(s) = \varpi(s) \left(\frac{P_t^*}{P_t^*(s)}\right)^{-\sigma(s)} C_t^*$, where we treat $\frac{P_t^*}{P_t^*(s)}$ and $\varpi(s)$ as constants, so $\hat{x}_t^*(s) = \hat{c}_t^*$.

We present the log-linear equilibrium conditions in Tables A.1 and A.2. Table A.1 contains equilibrium conditions that hold in both unconstrained and constrained equilibria. Table A.2 collects equilibrium conditions that differ across equilibria, depending on which constraints are slack or binding.

[43] For completeness, $\widehat{rp}_t(s) = \hat{p}_t(s) - \hat{p}_t$, $\widehat{rp}_{Ft}(s) = \hat{p}_{Ft}(s) - \hat{p}_t$, $\widehat{rmc}_t(s) = \widehat{mc}_t(s) - \hat{p}_t$, $\widehat{rp}_{Mt}(s) = \hat{p}_{Mt}(s) - \hat{p}_t$, $\widehat{rp}_{Mt}(s', s) = \hat{p}_{Mt}(s', s) - \hat{p}_t$.

Table A.1: Common Equilibrium Conditions across Unconstrained and Constrained Equilibria

Labor Supply	$-\rho \hat{c}_t + \widehat{rw}_t = \psi \hat{l}_t$
Consumption Allocation	$\hat{c}_t(s) = \hat{\zeta}_t(s) - \vartheta \widehat{rp}_t(s) + \hat{c}_t$ with $\sum_s \zeta_0(s) \hat{\zeta}_t(s) = 0$
	$\hat{c}_{Ht}(s) = -\varepsilon(s)\left(\widehat{rp}_{Ht}(s) - \widehat{rp}_t(s)\right) + \hat{c}_t(s)$
	$\hat{c}_{Ft}(s) = -\varepsilon(s)\left(\widehat{rp}_{Ft}(s) - \widehat{rp}_t(s)\right) + \hat{c}_t(s)$
Euler Equation	$0 = \mathbf{E}_t \hat{\Theta}_{t+1} - \hat{\Theta}_t - \rho\left(\mathbf{E}_t \hat{c}_{t+1} - \hat{c}_t\right) + i_t - \mathbf{E}_t \pi_{t+1}$
Consumer Prices	$0 = \sum_s \left[\zeta_0(s)\left(\frac{P_0(s)}{P_0}\right)^{1-\vartheta}\right]\left[\widehat{rp}_t(s) + \frac{1}{1-\vartheta}\hat{\zeta}_t(s)\right]$
	$\widehat{rp}_t(s) = \gamma(s)\left(\frac{P_{H0}(s)}{P_0(s)}\right)^{1-\varepsilon(s)} \widehat{rp}_{Ht}(s) + (1-\gamma(s))\left(\frac{P_{CF0}(s)}{P_0(s)}\right)^{1-\varepsilon(s)} \widehat{rp}_{Ft}(s)$
Labor Demand	$\widehat{rw}_t + \hat{l}_t(s) = \widehat{rmc}_t(s) + \hat{y}_t(s)$
Input Demand	$\widehat{rp}_{Mt}(s) + \hat{m}_t(s) = \widehat{rmc}_t(s) + \hat{y}_t(s)$
	$\hat{m}_t(s',s) = -\kappa\left(\widehat{rp}_{Mt}(s',s) - \widehat{rp}_{Mt}(s)\right) + \hat{m}_t(s)$
	$\hat{m}_{Ht}(s',s) = -\eta(s')\left(\widehat{rp}_{Ht}(s') - \widehat{rp}_{Mt}(s',s)\right) + \hat{m}_t(s',s)$
	$\hat{m}_{Ft}(s',s) = -\eta(s')\left(\widehat{rp}_{FMt}(s') - \widehat{rp}_{Mt}(s',s)\right) + \hat{m}_t(s',s)$
Marginal Cost	$\widehat{rmc}_t(s) = -\hat{z}_t(s) + (1-\alpha(s))\widehat{rw}_t(s) + \alpha(s)\widehat{rp}_{Mt}(s)$
Input Prices	$\widehat{rp}_{Mt}(s) = \sum_{s'}\left(\frac{\alpha(s',s)}{\alpha(s)}\right)\left(\frac{P_0(s',s)}{P_{M0}(s)}\right)^{1-\kappa} \widehat{rp}_{Mt}(s',s)$
	$\widehat{rp}_{Mt}(s',s) =$
	$\xi(s',s)\left(\frac{P_{H0}(s')}{P_0(s',s)}\right)^{1-\eta(s')} \widehat{rp}_{Ht}(s') + (1-\xi(s',s))\left(\frac{P_{MFt}(s')}{P_0(s',s)}\right)^{1-\eta(s')} \widehat{rp}_{FMt}(s')$
Consumption Import Pricing	$\pi_{Ft}(s) = \frac{\varepsilon-1}{\phi(s)}\left(\widehat{rmc}^*_t(s) + \hat{q}_t - \widehat{rp}_{Ft}(s)\right) + \beta \mathbf{E}_t \pi_{Ft+1}(s)$
Domestic Pricing for Services	$\pi_{Ht}(2) = \frac{\varepsilon-1}{\phi(2)}\left(\widehat{rmc}_t(2) - \widehat{rp}_{Ht}(2)\right) + \beta \mathbf{E}_t \pi_{Ht+1}(2)$
Input Import Pricing for Services	$\pi_{MFt}(2) = \frac{\varepsilon-1}{\phi(2)}\left(\widehat{rmc}^*_t(2) + \hat{q}_t - \widehat{rp}_{FMt}(2)\right) + \beta \mathbf{E}_t \pi_{FMt+1}(2)$
Market Clearing	$\hat{y}_t(s) = \left(\frac{C_{H0}(s)}{Y_0(s)}\right)\hat{c}_{Ht}(s) + \sum_{s'}\left(\frac{M_{H0}(s,s')}{Y_0(s)}\right)\hat{m}_{Ht}(s,s') + \left(\frac{X_0(s)}{Y_0(s)}\right)\hat{x}_t(s)$
	$\hat{x}_t(s) = -\sigma(s)\left(\widehat{rp}_{Ht}(s) - \hat{q}_t\right) + \hat{c}^*_t$
	$\hat{y}^*_{Ct}(s) = \hat{c}_{Ft}(s)$
	$\hat{y}^*_{Mt}(s) = \sum_{s'}\left(\frac{M_{F0}(s,s')}{Y^*_{M0}(s)}\right)\hat{m}_{Ft}(s,s')$
	$\hat{\Theta}_t - \rho\left(\hat{c}_t - \hat{c}^*_t\right) + \hat{q}_t = 0$
	$\sum_s \left(\frac{L_0(s)}{L_0}\right)\hat{l}_t(s) = \hat{l}_t$
Monetary Policy Rule	$i_t = \rho_i i_{t-1} + \omega(1-\rho_i)\hat{\bar{\pi}}_t + (1-\rho_i)\rho_y \hat{y}_t + \hat{\Psi}_t$
	with $\hat{y}_t = \sum_s \left(\frac{P_0(s)Y_0(s)}{Y_0}\right)\hat{y}_t(s)$
Auxiliary Inflation Definitions	$\pi_{Ht}(s) = \widehat{rp}_{Ht}(s) - \widehat{rp}_{Ht-1}(s) + \pi_t$
	$\pi_{Ft}(s) = \widehat{rp}_{Ft}(s) - \widehat{rp}_{Ft-1}(s) + \pi_t$
	$\pi_{FMt}(s) = \widehat{rp}_{FMt}(s) - \widehat{rp}_{FMt-1}(s) + \pi_t$
	$\bar{\pi}_t = \pi_t + \sum_s \zeta_0(s)\left(\frac{P_0(s)}{P_0}\right)^{1-\vartheta}\left(\widehat{rp}_t(s) - \widehat{rp}_{t-1}(s)\right)$

Table A.2: Equilibrium Conditions with Binding Constraints for Goods

Panel A: Only Domestic Constraint Binds	
Domestic Pricing	$\pi_{Ht}(1) = \left(\frac{\varepsilon-1}{\phi(1)}\right)\left(\widehat{rmc}_t(1) - \widehat{rp}_{Ht}(1)\right) + \left(\frac{\varepsilon}{\phi(1)}\frac{P_0}{P_{H0}(1)}\right)\hat{\tilde{\mu}}_t(1) + \beta \mathbf{E}_t \pi_{Ht+1}(1)$
Input Import Pricing	$\pi_{MFt}(1) = \left(\frac{\varepsilon-1}{\phi(1)}\right)\left(\widehat{rmc}^*_t(1) + \hat{q}_t - \widehat{rp}_{MFt}(1)\right) + \beta \mathbf{E}_t \pi_{FMt+1}(1)$
Domestic Constraint	$\hat{y}_t(1) = \hat{\bar{y}}_t(1) + \ln(\bar{Y}_0(1)/Y_0(1))$
Panel B: Only Foreign Constraint Binds	
Domestic Pricing	$\pi_{Ht}(1) = \left(\frac{\varepsilon-1}{\phi(1)}\right)\left(\widehat{rmc}_t(1) - \widehat{rp}_{Ht}(1)\right) + \beta \mathbf{E}_t \pi_{Ht+1}(1)$
Input Import Pricing	$\pi_{MFt}(1) =$ $\left(\frac{\varepsilon-1}{\phi(1)}\right)\left(\widehat{rmc}^*_t(1) + \hat{q}_t - \widehat{rp}_{MFt}(1)\right) + \left(\frac{\varepsilon}{\phi(1)}\frac{P_0}{P_{MF0}(1)}\right)\hat{\tilde{\mu}}^*_t(1) + \beta \mathbf{E}_t \pi_{MFt+1}(1)$
Import Constraint	$\hat{y}^*_t(1) = \hat{\bar{y}}^*_t(1) + \ln(\bar{Y}^*_0(1)/Y^*_0(1))$
Panel C: Both Constraints Bind	
Domestic Pricing	$\pi_{Ht}(1) = \left(\frac{\varepsilon-1}{\phi(1)}\right)\left(\widehat{rmc}_t(1) - \widehat{rp}_{Ht}(1)\right) + \left(\frac{\varepsilon}{\phi(1)}\frac{P_0}{P_{H0}(1)}\right)\hat{\tilde{\mu}}_t(1) + \beta \mathbf{E}_t \pi_{Ht+1}(1)$
Input Import Pricing	$\pi_{MFt}(1) =$ $\left(\frac{\varepsilon-1}{\phi(1)}\right)\left(\widehat{rmc}^*_t(1) + \hat{q}_t - \widehat{rp}_{MFt}(1)\right) + \left(\frac{\varepsilon}{\phi(1)}\frac{P_0}{P_{MF0}(1)}\right)\hat{\tilde{\mu}}^*_t(1) + \beta \mathbf{E}_t \pi_{MFt+1}(1)$
Domestic Constraint	$\hat{y}_t(1) = \hat{\bar{y}}_t(1) + \ln(\bar{Y}_0(1)/Y_0(1))$
Import Constraint	$\hat{y}^*_t(1) = \hat{\bar{y}}^*_t(1) + \ln(\bar{Y}^*_0(1)/Y^*_0(1))$

A.2 Stochastic Processes

We collect log deviations in exogenous domestic and foreign variables – including $\hat{\Theta}_t$, $\hat{\zeta}_t(1)$, $\hat{c}^*_t$, and $\left\{\hat{z}_t(s), \widehat{rmc}^*_t\right\}_s$ – into vector $\hat{F}_t$, and we assume that $\hat{F}_t$ is a first-order vector autoregressive process, as in $\hat{F}_t = \Lambda \hat{F}_{t-1} + \varepsilon_t$, where Λ is a diagonal matrix that contains autoregressive coefficients for each series (denoted λ_x for variable x) and ε_t is a vector of shocks.[44] We assume these vector of shocks follows a multivariate normal distribution, with $var(\varepsilon_t) = \Sigma$ having diagonal elements σ_x^2 for each variable x and zeros off-diagonal, and $cov(\varepsilon_t, \varepsilon_{t+s}) = 0$ at all leads and lags ($s \neq 0$).

Turning to constraints, we assume that the constraint for imports of consumption goods is not binding in all periods. In the first order approximate model, a sufficient condition to guarantee the constraint is never binding is to take $\bar{Y}^*_{Ct}(s)$ to infinity.[45] Similarly, we assume that constraints are not binding for services, for which taking $\bar{Y}_t(2)$ and $\bar{Y}^*_{Mt}(2)$ to infinity would be sufficient. This leaves $\bar{Y}_t(1)$ and $\bar{Y}^*_{Mt}(1)$ as the remaining constraints. We specify a stochastic process for them here, consistent with how we treat them as exogenous in the model.[46] We assume they follow an

[44] Note that we have imposed the restriction that foreign real marginal costs are the same for goods and services: $\widehat{rmc}^*_t(s) = \widehat{rmc}^*_t$. We estimate the stochastic process for this variable using data for goods imports, roughly speaking. Because the services sector is relatively closed, this cross-sector restriction has little substantive import.

[45] In the first order approximation, no decisions depend on the distance between the an endogenous variable and its constrained value. Thus, taking $\bar{Y}^*_{Ct}(s)$ to infinity ensures the constraint is always slack, without further indirect consequences for the approximate equilibrium.

[46] Reliable data on capacity at high frequencies is generally not available, so we cannot include capacity among the observable variables. Existing data on capacity, such as the series compiled by the Federal Reserve Board to produce its G.17 series, are not well suited to our exercise. One problem concerns data frequency. The Federal Reserve relies

autoregressive process:

$$\hat{\bar{y}}_t(1) = \rho_{\bar{y}} \hat{\bar{y}}_{t-1}(1) + \varepsilon_{\bar{y}t}(1) \tag{A.1}$$

$$\hat{\bar{y}}^*_t(1) = \rho_{\bar{y}^*} \hat{\bar{y}}^*_{t-1}(1) + \varepsilon_{\bar{y}^*t}(1), \tag{A.2}$$

where $\gamma \in (0,1)$ and $\varepsilon_{\bar{y}t}(1)$ and $\varepsilon_{\bar{y}^*t}(1)$ denote capacity shocks. We assume the capacity shocks are independent, mean zero normal random variables, with variances $var(\varepsilon_{\bar{y}t}(1)) = \sigma^2_{\bar{y}}$ and $var(\varepsilon_{\bar{y}^*t}(1)) = \sigma^2_{\bar{y}^*}$, and $cov(\varepsilon_{\bar{y}t}(1), \varepsilon_{\bar{y},t+s}(1)) = cov(\varepsilon_{\bar{y}^*t}(1), \varepsilon_{\bar{y}^*,t+s}(1)) = 0$ at all leads and lags ($s \neq 0$).

A.3 Quantitative Implementation

We set parameters for quantitative analysis through a mix of calibration and estimation. We describe calibrate parameters first. We then provide details regarding the estimation procedure and discuss the results.

A.3.1 Calibration

We set values for a subset of the structural parameters based on standard values in the literature, which we collect in Table A.3. We use input-output data compiled by the US Bureau of Economic Analysis to pin down values for steady-state expenditure shares. We report these shares, which reflect mean values over the 1997-2018 period, in Table A.4, along with their corresponding definitions in the model.

A.3.2 Estimation Procedure

As described in the main text, we build on papers by Kulish, Morley and Robinson (2017), Kulish and Pagan (2017), and Jones, Kulish and Rees (2022) that estimate models with occasionally binding constraints by treating the duration of those binding constraints as an estimable parameter. To explain the method in more detail, we first describe how to solve the model for given durations for

on underlying survey data collected at an annual frequency, so this data sheds little direct light on the dynamics of capacity at higher frequencies (monthly or quarterly). A second problem concerns how capacity survey questions are posed to firms. Specifically, the survey instrument asks firms to report how much they could produce if they had access to all the labor and materials they need to produce. This survey question fails to capture key aspects of production that effectively limit true capacity. For example, firms make predetermined choices about essential labor, material inputs, and other aspects of the production process that limit their ability to produce today, but this would be not be picked up by the survey. Related to these concerns, we note two features of the actual G.17 capacity data. First, capacity utilization is well below 1 in historical data (typically near 0.75 in recent data) – taken literally, capacity constraints are never even close to binding, which seems implausible. Further, measures of capacity utilization have trended down over time, as if firms are carrying more excess capacity now than in the past. This is prima facie inconsistent with auxiliary evidence of decreased slack in other dimensions of the production process (e.g., the rising prevalence of "lean production" methods, such as just-in-time inventory management).

Table A.3: Calibration

Parameter	Value	Reference/Target
ψ	2	Labor supply elasticity of 0.5
ρ	2	Intertemporal elasticity of substitution of 0.5
β	.995	Annual risk-free real rate of 2%
ϑ	0.5	Elasticity of substitution across sectors in consumption
ε	4	Elasticity of substitution between varieties
κ	0.3	Elasticity of substitution for inputs across sectors
$\sigma(s)$	1.5	Export demand elasticity
ϕ	23.6453	To yield first order equivalence to Calvo pricing, with average price duration of 4 quarters [Sims and Wolff (2017)].

Table A.4: Steady State Shares

Model and Data	Description
$\begin{bmatrix} \zeta_0(1)\left(\frac{P_0(1)}{P_0}\right)^{1-\vartheta} \\ \zeta_0(2)\left(\frac{P_0(2)}{P_0}\right)^{1-\vartheta} \end{bmatrix} = \begin{bmatrix} 0.26 \\ 0.74 \end{bmatrix}$	Sector shares in consumption expenditure
$\begin{bmatrix} \gamma(1)\left(\frac{P_{H0}(1)}{P_0(1)}\right)^{1-\varepsilon} \\ \gamma(2)\left(\frac{P_{H0}(2)}{P_0(2)}\right)^{1-\varepsilon} \end{bmatrix} = \begin{bmatrix} 0.80 \\ 0.995 \end{bmatrix}$	Home shares in consumption expenditure by sector
$\begin{bmatrix} \alpha(1) \\ \alpha(2) \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$	Input expenditure share of gross output
$\begin{bmatrix} \left(\frac{\alpha(1,1)}{\alpha(1)}\right)\left(\frac{P_0(1,1)}{P_{M0}(1)}\right)^{1-\kappa} & \left(\frac{\alpha(1,2)}{\alpha(2)}\right)\left(\frac{P_0(1,2)}{P_{M0}(2)}\right)^{1-\kappa} \\ \left(\frac{\alpha(2,1)}{\alpha(1)}\right)\left(\frac{P_0(2,1)}{P_{M0}(1)}\right)^{1-\kappa} & \left(\frac{\alpha(2,2)}{\alpha(2)}\right)\left(\frac{P_0(2,2)}{P_{M0}(2)}\right)^{1-\kappa} \end{bmatrix} = \begin{bmatrix} 0.70 & 0.20 \\ 0.30 & 0.80 \end{bmatrix}$	Sector shares in input expenditure
$\begin{bmatrix} \xi(1,1)\left(\frac{P_{H0}(1)}{P_0(1,1)}\right)^{1-\eta} & \xi(1,2)\left(\frac{P_{H0}(1)}{P_0(1,2)}\right)^{1-\eta} \\ \xi(2,1)\left(\frac{P_{H0}(2)}{P_0(2,1)}\right)^{1-\eta} & \xi(2,2)\left(\frac{P_{H0}(2)}{P_0(2,2)}\right)^{1-\eta} \end{bmatrix} = \begin{bmatrix} 0.77 & 0.84 \\ 0.99 & 0.98 \end{bmatrix}$	Home shares in input expenditure
$\begin{bmatrix} \frac{C_{H0}(1)}{Y_0(1)} & \frac{M_{H0}(1,1)}{Y_0(1)} & \frac{M_{H0}(1,2)}{Y_0(1)} & \frac{X_0(1)}{Y_0(1)} \\ \frac{C_{H0}(2)}{Y_0(2)} & \frac{M_{H0}(2,1)}{Y_0(2)} & \frac{M_{H0}(2,2)}{Y_0(2)} & \frac{X_0(2)}{Y_0(2)} \end{bmatrix} = \begin{bmatrix} 0.41 & 0.32 & 0.16 & 0.11 \\ 0.61 & 0.07 & 0.29 & 0.03 \end{bmatrix}$	Domestic output allocation
$\begin{bmatrix} \frac{M_{F0}(1,1)}{Y^*_{M0}(1)} & \frac{M_{F0}(1,2)}{Y^*_{M0}(1)} \\ \frac{M_{F0}(2,1)}{Y^*_{M0}(2)} & \frac{M_{F0}(2,2)}{Y^*_{M0}(2)} \end{bmatrix} = \begin{bmatrix} 0.76 & 0.24 \\ 0.08 & 0.92 \end{bmatrix}$	Foreign output allocation for inputs
$\begin{bmatrix} \frac{P_{H0}(1)Y_0(1)}{P_{H0}(1)Y_0(1)+P_{H0}(2)Y_0(2)} \\ \frac{P_{H0}(2)Y_0(2)}{P_{H0}(1)Y_0(1)+P_{H0}(2)Y_0(2)} \end{bmatrix} = \begin{bmatrix} 0.29 \\ 0.71 \end{bmatrix}$	Sector shares in gross output

the binding constraints. We then describe the estimation procedure in greater detail, including how we constrain admissible values for durations to be consistent with model equilibrium constraints.

Solving the Model for Given Durations As in Guerrieri and Iacoviello (2015), we construct a piecewise linear approximation to the model. This consists of taking linear approximations of the model equilibrium for four regimes: the unconstrained regime, a second regime in which only domestic constraints bind, a third regime in which foreign constraints bind, and a fourth regime in which both constraints bind. Further, the linear approximations for all these regimes are taken around the non-stochastic (unconstrained) steady state of the model. The solution procedure then combines these local approximations to solve for the policy function.

The linear approximation to the unconstrained system can be written as:

$$\mathbf{A}X_t = \mathbf{C} + \mathbf{B}X_{t-1} + \mathbf{D}\mathbb{E}_t X_{t+1} + \mathbf{F}\varepsilon_t,$$

where x_t is an $n \times 1$ vector of model variables, ε_t is an $l \times 1$ vector of structural shocks, and $\mathbf{A}$, $\mathbf{C}$, $\mathbf{B}$, $\mathbf{D}$, and $\mathbf{F}$ are conformable matrices determined by the structural equations. If agents expect the economy to remain unconstrained from date t forward, then standard rational expectations solution procedures imply that the reduced-form solution is given by: $X_t = \mathbf{J} + \mathbf{Q}X_{t-1} + \mathbf{G}\varepsilon_t$, where $\mathbf{J}$, $\mathbf{Q}$, and $\mathbf{G}$ describe the policy function and model dynamics.

There are three regimes in which one or both constraints bind, and let us index these by $r \in \{1,2,3\}$. Then we can express the linear approximation to the model equilibrium in each case as:

$$\bar{\mathbf{A}}_r X_t = \bar{\mathbf{C}}_r + \bar{\mathbf{B}}_r X_{t-1} + \bar{\mathbf{D}}_r \mathbb{E}_t X_{t+1} + \bar{\mathbf{F}}_r \varepsilon_t,$$

where $\bar{\mathbf{A}}_r$, $\bar{\mathbf{C}}_r$, $\bar{\mathbf{B}}_r$, $\bar{\mathbf{D}}_r$, and $\bar{\mathbf{F}}_r$ are conformable matrices that correspond to the structural equations for each.

We summarize the expected evolution of regimes from a given date t forward by the durations that the individual constraints are expected to bind, as in $\mathbf{d}_t = [d_t, d_t^*]$. To fix ideas, suppose that $d_t = 1$, which means that the domestic constraint binds today, and then is expected to be slack in the future. Further, suppose that and $d_t^* = 0$, so the foreign constraint is slack today and in the future. This implies that the constrained system governs model responses in period t and then the unconstrained system applies thereafter. Working backwards from the unconstrained solution, then $\mathbb{E}_t X_{t+1} = \mathbf{J} + \mathbf{Q}X_t$, so then $\bar{\mathbf{A}}_1 X_t = \bar{\mathbf{C}}_1 + \bar{\mathbf{B}}_1 X_{t-1} + \bar{\mathbf{D}}_1 (\mathbf{J} + \mathbf{Q}X_t) + \bar{\mathbf{F}}_1 \varepsilon_t$, where $r = 1$ is the system that applies when the domestic constraint binds and the foreign constraint is slack. Solving this linear equation yields the reduced form solution for X_t.

Generalizing this idea, the system will evolve according to:

$$\mathbf{A}_t X_t = \mathbf{C}_t + \mathbf{B}_t X_{t-1} + \mathbf{D}_t \mathbb{E}_t X_{t+1} + \mathbf{F}_t \varepsilon_t, \tag{A.3}$$

where $\mathbf{A}_t$, $\mathbf{C}_t$, $\mathbf{B}_t$, $\mathbf{D}_t$, and $\mathbf{F}_t$ are the structural matrices that apply at date t. Then the piecewise linear solution is given by:

$$X_t = \mathbf{J}_t + \mathbf{Q}_t X_{t-1} + \mathbf{G}_t \varepsilon_t, \tag{A.4}$$

where $\mathbf{J}_t$, $\mathbf{Q}_t$, and $\mathbf{G}_t$ are determined via the following backward recursion, which is initialized as starting from the unconstrained solution:

$$\begin{aligned} \mathbf{Q}_t &= \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}\right]^{-1} \mathbf{B}_t \\ \mathbf{J}_t &= \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}\right]^{-1} \left(\mathbf{C}_t + \mathbf{D}_t \mathbf{J}_{t+1}\right) \\ \mathbf{G}_t &= \left[\mathbf{A}_t - \mathbf{D}_t \mathbf{Q}_{t+1}\right]^{-1} \mathbf{F}_t. \end{aligned} \tag{A.5}$$

At this point, it is useful to note that this recursive solution coincides with the recursion employed by the OccBin toolkit [Guerrieri and Iacoviello (2015)] to obtain policy functions for a given guess about the sequence of regimes. The Occbin toolkit then proceeds to verify whether the guess about the sequence of regimes is consistent with model equilibrium, given the current value of the shocks. Put differently, it solves for endogenous constraint durations given ε_t. While we do not discuss this second step here, we do solve for endogenous durations (using Occbin) when we analyze counterfactual responses to shocks in the model. We also take the dependence of $\mathbf{d}_t$ on ε_t into account in the estimation procedure, with details below.

While Equations A.4 and A.5 present the model solution for a given anticipated sequence of regimes, it is important to note that the anticipated sequence changes as durations evolve over time. The duration $\mathbf{d}_t$ implies a particular sequence of regimes anticipated at dates $t+1$, $t+2$, etc. Given this sequence and the maintained assumption that agents do not anticipate future shocks, one then uses the recursion above to solve for the associated policy matrices: $\mathbf{J}(\mathbf{d}_t, \theta), \mathbf{Q}(\mathbf{d}_t, \theta)$, and $\mathbf{G}(\mathbf{d}_t, \theta)$, where the notation captures the dependence of these matrices on $\mathbf{d}_t$. At date $t+1$, a new value for durations ($\mathbf{d}_{t+1}$) will be realized, and one then solves the recursion anew to obtain $\mathbf{J}(\mathbf{d}_{t+1}, \theta), \mathbf{Q}(\mathbf{d}_{t+1}, \theta)$, and $\mathbf{G}(\mathbf{d}_{t+1}, \theta)$. And so on. The state (transition) equation of the model then features time-varying coefficients:

$$X_t = \mathbf{J}(\mathbf{d}_t, \theta) + \mathbf{Q}(\mathbf{d}_t, \theta) X_{t-1} + \mathbf{G}(\mathbf{d}_t, \theta) \varepsilon_t. \tag{A.6}$$

When $\mathbf{d}_t = 0$, the unconstrained solution applies, so $\mathbf{J}(\mathbf{d}_t, \theta) = \mathbf{J}(\theta)$, $\mathbf{Q}(\mathbf{d}_t, \theta) = \mathbf{Q}(\theta)$, and $\mathbf{G}(\mathbf{d}_t, \theta) = \mathbf{G}(\theta)$ are time invariant.

Joint Estimation of Durations and Structural Parameters We assume that a vector of observables (S_t) are linked to underlying model states via the measurement equation: $S_t = \mathbf{H}_t X_t + \nu_t$, where ν_t is an i.i.d. vector of normally distributed measurement errors and $\mathbf{H}_t$ is a conformable (potentially time-varying) matrix linking states to observables. Using this state space representation of the model, we can apply the Kalman filter to construct the Likelihood function $\mathscr{L}(\theta, \mathbf{d}| \{S_t\}_{t=1}^{T})$, where $\mathbf{d} = \{\mathbf{d}\}_{t=1}^{T}$ is the sequence of durations.

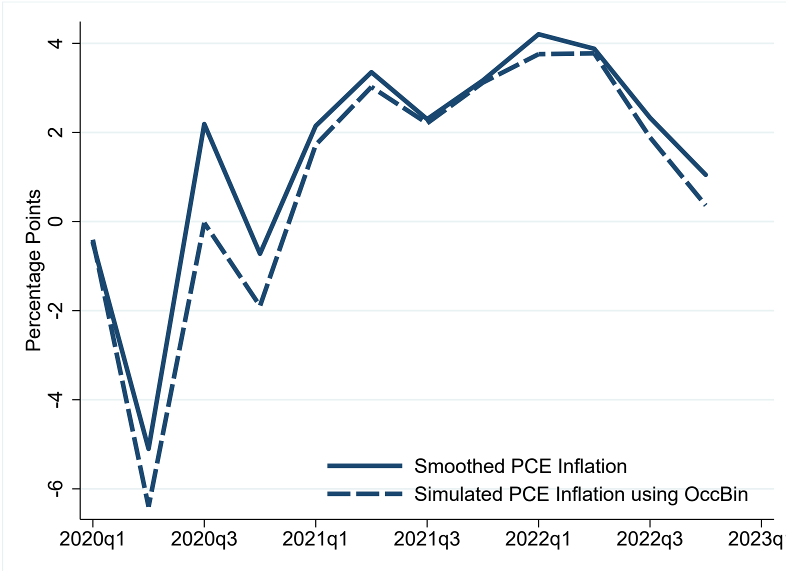
We put priors over structural parameters and independent priors over durations to construct the posterior, and then estimate the model via Bayesian Maximum Likelihood. We construct draws from the joint posterior distribution $p\left(\theta, \mathbf{d}| \{S_t\}_{t=1}^{T}\right)$ using a Metropolis-Hastings algorithm with two blocks – one for the structural parameters, which are continuous, and a second for the discrete duration parameters – as in Kulish, Morley and Robinson (2017). We use a uniform proposal density for the durations, between 0 (unconstrained) and a sufficiently large maximum duration. We discuss the priors in Section A.3.2 below.

In evaluating proposed parameter and durations draws, we recognize that it is desirable for posterior estimates of constraint durations to be consistent with agents' forecasts about how long constraints will endogenously bind given shocks. To this end, we constrain admissible draws to enforce this constraint, in an approximate sense. For a given proposed joint parameter (θ^i) and duration draw ($\mathbf{d}^i$), we construct the piecewise linear solution for the model and use the Kalman filter to obtain smoothed structural shocks $\{\tilde{\varepsilon}_t^i\}_{t=1}^{T}$ and equilibrium variables $\{\tilde{X}_t^i\}_{t=1}^{T}$ given the data. At each sample period $\tau \in [1, \ldots, T]$, we then use the piecewise linear solution to project model outcomes forward given the state and current shock – $\left(\tilde{X}_{\tau-1}^i, \tilde{\varepsilon}_\tau^i\right)$, assuming that there are no anticipated future shocks.[47] We then check for violations of the output capacity constraints. If projected home or foreign output violates the constraints, then we reject the proposed parameter draw as inconsistent with model equilibrium. Otherwise, we accept the parameter draw, evaluate the likelihood, and proceed through the estimation algorithm. Under this procedure, we accept about 25% of the proposed parameter/duration draws, so the estimation proceeds at reasonable computational pace.

In this procedure, note that we reject the proposed draw when it implies that constraints will be violated in expectation. In turn, we accept draws for which constraints are satisfied. Strictly speaking, we do not explicitly check whether the duration $\mathbf{d}_\tau$ is equal to the endogenous equilibrium duration consistent with $\left(\tilde{X}_{\tau-1}^i, \tilde{\varepsilon}_\tau^i\right)$ in the model. Nonetheless, our approach to estimation provides a good approximation to model outcomes with endogenously binding constraints. To see this, it is useful to compare smoothed inflation outcomes obtained via our estimation procedure

[47] Recall that in the absence of future shocks, agents anticipate that the model will return to the unconstrained state over time, where the duration of binding constraints ticks down toward zero in each passing period. We project model outcomes forward using this expected path for durations.

Figure A.1: Comparison Between Smoothed Inflation and OccBin Simulated Inflation



Note: Smoothed PCE Inflation is Kalman-smoothed consumer price inflation, where the filter is parameterized using the modal values of structural parameters and durations from the empirical estimation. Simulated PCE Inflation using OccBin is obtained by simulating model responses to smoothed shocks (see the text for further description).

with outcomes from the full structural model with endogenously binding constraints. Specifically, suppose we feed the structural shocks $\{\tilde{\varepsilon}_t^i\}_{t=1}^T$ obtained from our estimation procedure through the model, where structural parameters are set to their to their modal values and we use the OccBin procedure to solve for the endogenous duration of binding constraints in each period following the realization of shocks. We then plot this simulated inflation series to the smoothed inflation series from our estimation in Figure . As is evident, the two series track each other closely, so we conclude that our approach to capturing endogenously binding constraints in the estimation routine performs well.

Priors The full set of priors for structural parameters is included in Table A.5. We use standard priors on autoregressive persistence of exogenous variables, parameters in the monetary policy rule, elasticities, and the standard deviations of most structural shocks. We set priors on the persistences of the exogenous capacity shocks that are wider than the priors on the other exogenous variables, as well as wide (uniform) priors on the standard deviations of the capacity shocks, since these are nonstandard parameters.

We set uniform priors on measurement errors associated with three inflation series – consumer price inflation for goods, imported input price inflation, and imported consumption goods price inflation. Further, looking forward, we will report below that the posterior estimates are pushed toward the boundary of the allowed parameter space for these parameters. The logic for constraining the measurement error parameters in this way is twofold. First, because our focus is on inflation outcomes and the role of constraints in driving them, we want to lean heavily on the realized data here. Second, we estimate the model using both pre-2020 and post-2020 data. As is evident in raw data series, the post-2020 COVID period features extreme variability in outcomes relative to the pre-2020 data. One way for the model to make sense of this is to assign very high measurement errors to the data. This is unpalatable from our perspective, as we wish to parse the actual data for this period. Thus, we effectively constrain the model to treat the post-2020 inflation data as an accurate representation of latent unobserved model variables. We envision experimenting with alternatives to this approach (e.g., allowing for different measurement error or shock processes before and after 2020), as thinking about how to model the COVID period evolves.

As noted in the main text, we allow constraints to potentially bind only starting in the second quarter of 2020. That is, we put zero mass on positive durations at all dates at/before 2020:Q1, which can be thought of as a dogmatic prior that constraints were not substantively important prior to the pandemic. Thereafter in each period, we place equal mass on durations of 0 to 4 quarters, summing to 60% total (12% on each discrete duration). We place 30% mass on durations of 5, 6, 7, and 8 quarters, again equally spread (7.5% each). The remaining 10% mass is spread equally over durations 9 through 12, and we place zero mass on durations longer than 12 quarters.

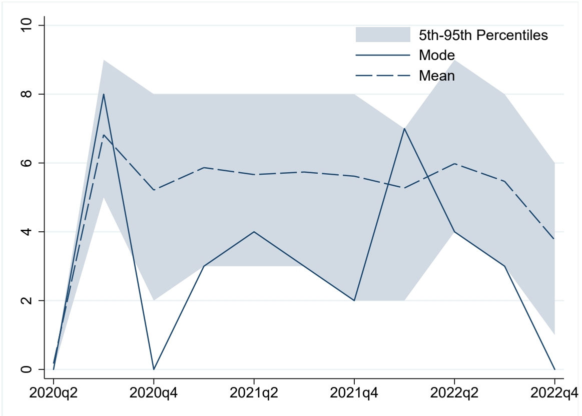
A.4 Estimation Results

In Table A.5, we provide the mode, mean, and 5th-95th percentiles for the posterior distributions of the structural parameters. As noted in the text, we find that domestic and foreign goods inputs are complements on the production side, while domestic and foreign goods are substitutes in consumption. The Taylor rule coefficient on inflation is near 1.5, which is standard. Interest rates also depend positively on deviations of output from steady state, and the policy rule features a significant degree of inertia. The stochastic processes for shocks generally feature persistence, with auto-regressive coefficients generally between 0.7 and 0.9. Building on the discussion of measurement error above, we note that posterior estimates for measurement errors on consumer goods price inflation and import price inflation are pushed toward the boundary of their prior distributions, reflecting tension in the model between fitting data before and during the COVID period. For all the other parameters, posterior distributions are generally well behaved, with single peaks well inside the allowable parameter space and reasonably tight distributions.

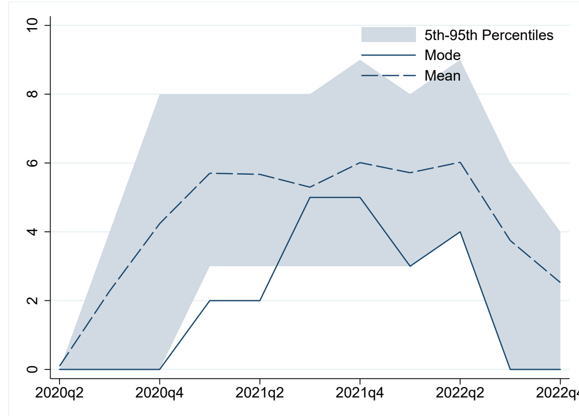
Turning to duration estimates, we plot statistics for the posterior distributions of domestic and foreign constraint durations in Figure A.2. Due to skewness in the distributions, modal values for the duration (our preferred approach to summarizing the posterior distribution) are below the

Figure A.2: Posterior Distributions for Constraint Durations

(a) Domestic Constraint Durations



(b) Foreign Constraint Durations



Note: At each date, there is a posterior distribution for constraint durations. Each figure presents the mean, mode, and interquartile range for this posterior distribution.

mean value in most periods. The time path for the duration estimates mimics the path of estimated multipliers on the constraints, as reported in Figure 12.

A.5 Model Fit

In the main text, we presented results on model fit for core inflation series in Figure 11. To evaluate model fit more broadly, we present data and smoothed values for the remaining observable variables in Figure A.3.[48] For legibility in the figures, we focus on the 2017-2022 period – the key period leading up to and through our analysis. The model fits most series well, even capturing the whiplash dynamics of the data in 2020. The model struggles to replicate data on US labor productivity, particularly in 2020 for services. Through the lens of the model, this implies that the data contains substantial measurement error during the pandemic period, which seems plausible to us. More broadly, a more sensitive treatment of the impact of lockdowns on the services sector would likely be needed to match data in the middle quarters of 2020. Nonetheless, referring back to the main text, the model is able to capture the dynamics of services inflation well overall, particularly in 2021-2022 when inflation escalates.

[48] While we treat the interest rate as an observable variable, we assume it is measured without error, so it is omitted here.

Table A.5: Prior and Posterior Distributions for Structural Parameters

Panel A: Elasticity and Taylor Rule Parameters

	Prior			Posterior			
Parameter	Dist	Mean	SD	Mode	Mean	5%	95%
Consumption Armington Elasticity: ι	G	1.5	0.25	1.439	1.445	1.098	1.816
Input Armington Elasticity: η	G	0.5	0.15	0.510	0.542	0.346	0.760
Taylor Rule Inflation: ω	N	1.5	0.12	1.537	1.556	1.359	1.754
Taylor Rule Inertia:α_i	B	0.75	0.1	0.877	0.873	0.851	0.894
Taylor Rule Output: α_y	G	0.12	0.05	0.243	0.240	0.135	0.342

Panel B: Stochastic Processes

	Prior			Posterior			
Parameter	Dist	Mean	SD	Mode	Mean	5%	95%
Preference for Goods: σ_ζ	IG	1	2	0.411	0.414	0.190	0.655
Discount Rate: σ_Θ	IG	1	2	3.254	3.354	2.914	3.807
Foreign Costs: σ_{rmc^*}	IG	1	2	2.150	2.265	1.944	2.631
Goods Productivity: $\sigma_{z(1)}$	IG	1	2	0.173	0.194	0.127	0.276
Services Productivity: $\sigma_{z(2)}$	IG	1	2	0.204	0.208	0.143	0.281
Foreign Constraint: $\sigma_{\bar{y}^*}$	U	1	0.58	0.028	0.043	0.008	0.109
Domestic Constraint: $\sigma_{\bar{y}}$	U	1	0.58	0.012	0.015	0.006	0.033
Monetary Policy Shock: σ_i	IG	1	2	0.152	0.159	0.138	0.184
Preference for Goods: ρ_ζ	B	0.5	0.15	0.823	0.726	0.431	0.929
Discount Rate: ρ_Θ	B	0.5	0.15	0.725	0.719	0.658	0.775
Foreign Costs: ρ_{rmc^*}	B	0.5	0.15	0.934	0.923	0.881	0.961
Goods Productivity: $\rho_{z(1)}$	B	0.5	0.10	0.549	0.536	0.370	0.701
Services Productivity: $\rho_{z(2)}$	B	0.5	0.15	0.923	0.865	0.505	0.963
Foreign Constraint: $\rho_{\bar{y}^*}$	B	0.5	0.20	0.702	0.665	0.358	0.886
Domestic Constraint: $\rho_{\bar{y}}$	B	0.5	0.20	0.865	0.726	0.447	0.922

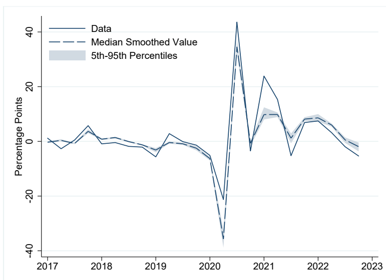
Panel C: Measurement Error

	Prior			Posterior			
Parameter	Dist	Mean	SD	Mode	Mean	5%	95%
Goods PCE: $\sigma^{\mathrm{me}}_{\mathrm{pceg}}$	IG	1	2	1.048	1.035	0.882	1.185
Services PCE: $\sigma^{\mathrm{me}}_{\mathrm{pces}}$	IG	1	2	0.661	0.669	0.563	0.783
Goods PCE Inflation: $\sigma^{\mathrm{me}}_{\pi(1)}$	U	0.25	.14	0.500	0.497	0.492	0.500
Services PCE Inflation: $\sigma^{\mathrm{me}}_{\pi(2)}$	IG	1	2	0.152	0.157	0.121	0.205
Imp. Input Goods Expenditure: $\sigma^{\mathrm{me}}_{\mathrm{inp}}$	IG	1	2	3.210	3.238	2.922	3.583
Imp. Consumption Goods Expenditure: $\sigma^{\mathrm{me}}_{\mathrm{finp}}$	IG	1	2	2.966	2.966	2.654	3.320
Imp. Input Goods Inflation: $\sigma^{\mathrm{me}}_{\mathrm{inpp}}$	U	0.75	.43	1.498	1.487	1.463	1.499
Imp. Consumption Goods Inflation: $\sigma^{\mathrm{me}}_{\mathrm{fimp}}$	U	0.075	0.043	0.146	0.114	0.037	0.148
Goods Productivity: $\sigma^{\mathrm{me}}_{\mathrm{prod1}}$	IG	1	2	1.142	1.165	1.014	1.321
Services Productivity: $\sigma^{\mathrm{me}}_{\mathrm{prod2}}$	IG	1	2	1.047	1.060	0.937	1.195
Industrial Production: $\sigma^{\mathrm{me}}_{\mathrm{ip}}$	IG	1	2	0.966	0.948	0.821	1.088
Aggregate Nominal GDP: $\sigma^{\mathrm{me}}_{\mathrm{nva}}$	IG	1	2	0.471	0.481	0.404	0.564

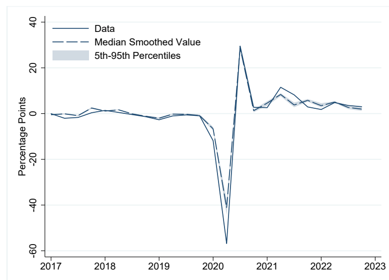
Note: G denotes the gamma distribution, IG denotes the inverse gamma distribution, U denotes the uniform distribution, B denotes the beta distribution, and N denotes the normal distribution.

Figure A.3: Data and Smoothed Model Observables

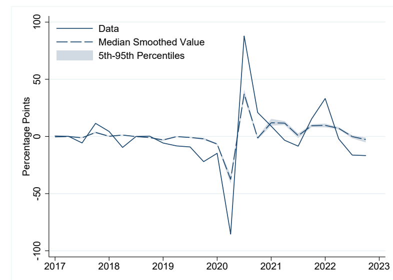
(a) Goods Cons. Expenditure



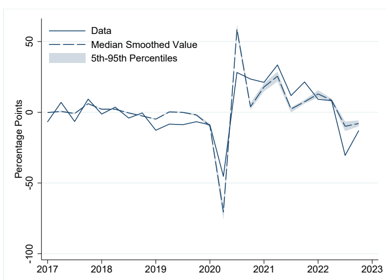
(b) Services Cons. Expenditure



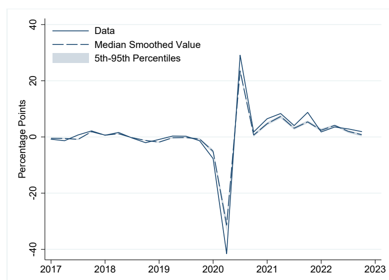
(c) Import Cons. Goods Expenditure



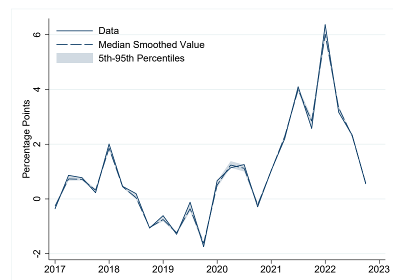
(d) Import Goods Input Expenditure



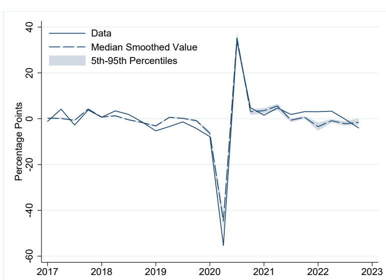
(e) Nominal GDP



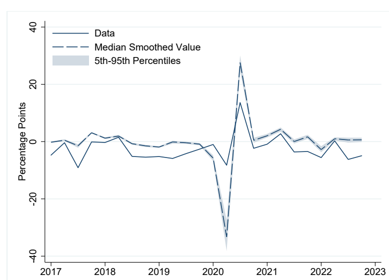
(f) Import Consumer Goods Inflation



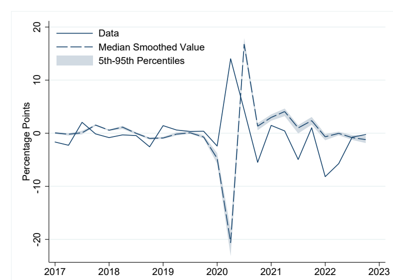
(g) Industrial Production



(h) Goods Productivity



(i) Aggregate Productivity



Note: All data and simulated series are annualized values for de-meaned quarterly growth rates in percentage points. Data is raw data. We take 1000 draws from the posterior distribution of model parameters, compute the Kalman-smoothed values for model variables for each draw, and then plot the median smoothed value as the dashed line. We shade the area covering the the 5% to 95% percentile for smoothed values.