The Inflation Accelerator*

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Abstract

We develop a tractable sticky price model in which the fraction of price changes evolves endogenously over time and, consistent with the evidence, increases with inflation. Because we assume that firms sell multiple products and choose how many, but not which, prices to adjust in every period, our model admits exact aggregation and reduces to a one-equation extension of the Calvo model. The model features a powerful *inflation accelerator*–a feedback loop between inflation and the fraction of price changes–which significantly increases the slope of the Phillips curve during periods of high inflation.

Keywords: Phillips curve, inflation, price rigidities.

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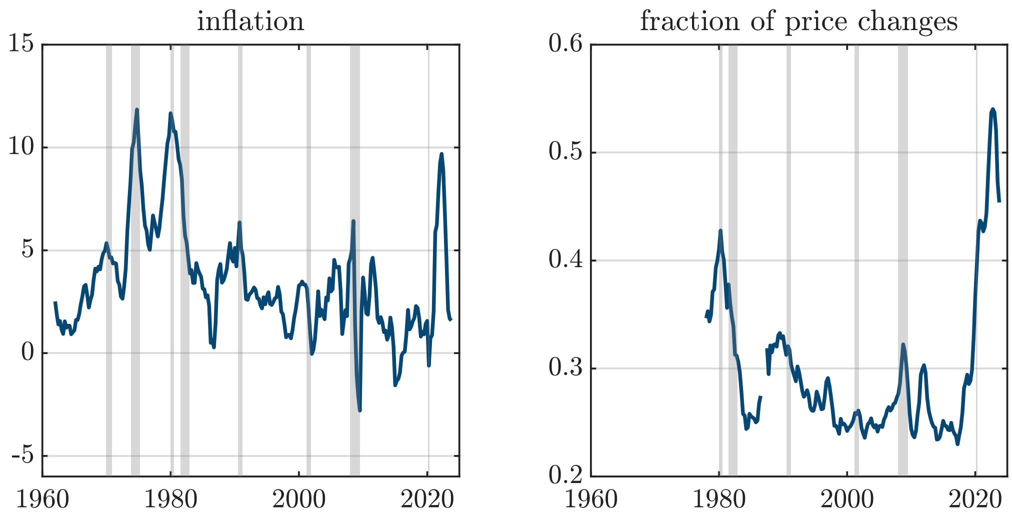
1 Introduction

The recent rise in inflation in many economies has spurred considerable interest in further understanding the dynamics of prices. Identifying the causes of high inflation hinges critically on the slope of the Phillips curve, which serves as a key summary of monetary non-neutrality and governs the tradeoff between inflation and output stabilization. Our goal in this paper is to study how the slope of the Phillips curve fluctuates over time. Since a key determinant of this slope is the fraction of price changes, it is imperative that we use a model that reproduces the widely-documented evidence that the fraction of price changes increases in times of high inflation. We illustrate this pattern for the U.S. in Figure 1.[1]

Figure 1: Inflation and the Fraction of Price Changes



Notes: The gray bars indicate NBER recessions. The CPI series excludes shelter.

Two classes of sticky price models are widely used in macroeconomics. The more popular time-dependent models[2] are tractable, but assume a time-invariant fraction of price changes, making them less suitable for analyzing episodes in which inflation and the fraction of price changes increase considerably. State-dependent (menu cost) models are a natural framework to endogenize the fraction of price changes,[3] but are considerably more difficult to use for

[1] We describe the data that underlies the figure in Section 3.1. See Gagnon (2009), Nakamura et al. (2018), Alvarez et al. (2018), Karadi and Reiff (2019), Montag and Villar (2023) and Blanco et al. (2024), who also document this pattern for the U.S. and other countries.

[2] See Calvo (1983) and Taylor (1980).

[3] See, for example, Dotsey et al. (1999), Golosov and Lucas (2007), Gertler and Leahy (2008), Midrigan (2011), Alvarez and Lippi (2014), Alvarez et al. (2016), Alvarez et al. (2018), Auclert et al. (2023). However, Blanco et al. (2024) show that the standard menu cost model has difficulties simultaneously reproducing the extent to which the fraction of price changes comoves with inflation and the distribution of price changes.

quantitative and policy analysis. The difficulty arises because characterizing the general equilibrium dynamics of a model subject to aggregate shocks requires keeping track of the distribution of prices across firms. This challenge is especially pronounced when the fraction of price changes responds to aggregate shocks, which gives rise to important non-linearities.

Our paper proposes a model in which the fraction of price changes evolves endogenously over time, as in state-dependent models, but which is as tractable as time-dependent models. The main challenge in endogenizing the fraction of price changes is that a firm's price adjustment decision depends on how far from the optimum its price is. Equilibrium outcomes are therefore a function of the entire distribution of price changes, an infinite-dimensional object. We circumvent this challenge by assuming that firms sell a continuum of products and choose how many, but not which, prices to adjust in every period, subject to an adjustment cost.[4] Because firms cannot choose which prices to adjust, the distribution of prices is no longer necessary to describe adjustment incentives, so the economy admits exact aggregation. We show that our model reduces to a one-equation extension of the Calvo model, with the additional equation pinning down the fraction of price changes. Our model nests the Calvo model in the limiting case when the adjustment cost goes to infinity.

We find that the Phillips curve is highly non-linear: its slope fluctuates considerably in the U.S. time series. In particular, the slope increases in times of high inflation due to a feedback loop between inflation and the fraction of price changes. On one hand, an increase in the fraction of price changes increases inflation, more so the higher inflation is to begin with. On the other hand, an increase in inflation increases the firms' incentives to adjust prices, further increasing the fraction of price changes. We refer to this feedback loop as the *inflation accelerator* and show that it is responsible for the bulk of the steepening of the Phillips curve in periods of high inflation. Our model therefore suggests that reducing inflation by a given amount is much less costly when inflation is high than when it is low.

The model we propose features multi-product firms that sell a continuum of goods and choose what fraction of their prices to change each period, subject to an adjustment cost that is increasing and convex in the number of prices that adjust. We assume decreasing returns to scale in production, which introduce strategic complementarities in price setting and dampen the slope of the Phillips curve. To best illustrate the mechanism of the model, we assume that monetary policy shocks–disturbances to a simple Taylor rule–are the only source of aggregate uncertainty.

[4]This assumption is reminiscent of that in Greenwald (2018) who uses a large family construct to endogenize refinancing decisions.

Relative to the Calvo model, endogenizing the fraction of price changes adds a single equation that balances the marginal cost and benefit of changing prices. Because the marginal benefit increases with inflation, so does the fraction of price changes. Endogenizing the fraction of price changes introduces non-linearities in the dynamics of output and inflation, so we solve the model using higher-order perturbations.

We start by building intuition for the workings of the model by studying impulse responses to expansionary monetary shocks in environments with low and high trend inflation. We show that output responds less and inflation responds more in an environment with high trend inflation for two reasons. First, the steady-state fraction of price changes is higher in environments with high inflation. Second, the fraction of price changes increases in response to shocks. Though this increase is relatively small, it has a considerable impact on the degree of aggregate price flexibility because adjusting firms respond to the underlying trend inflation and increase prices by large amounts, an effect reminiscent of Caplin and Spulber (1987).

Relative to menu cost models, an important advantage of our model is that we can derive closed-form expressions for the slope of the Phillips curve both around the steady state, as well as around any point in time. We use these expressions to build additional intuition for what shapes the dynamics of inflation and output in our model. We show that the slope of the Phillips curve is equal to the sum of two terms, one identical to the slope in the Calvo model, which increases mechanically with the fraction of price changes, and another which captures the inflation accelerator. Importantly, we show that this second term increases much more rapidly with trend inflation and thus accounts for the bulk of the increase in the slope of the Phillips curve in high-inflation environments.

We use our model to characterize how the slope of the Phillips curve evolves in the U.S. time series. We first identify the sequence of monetary shocks that allows the model to reproduce the path of inflation in the time series. We show that the model reproduces well the path of the fraction of price changes. We then consider log-linear perturbations around the equilibrium point at each date and derive the slope of the Phillips curve. We find that the slope of the Phillips curve varies considerably, ranging from 0.02 in low-inflation periods such as the 1990s to 0.13 in high-inflation periods such as the 1970s and the 1980s.[5] The

[5] That the slope of the Phillips curve increases with inflation is consistent with Costain et al. (2022) who use a model with state-dependent prices and wages, Morales-Jimenez and Stevens (2024), who use a model with menu costs and information frictions, and Flynn et al. (2024), who use a model with information frictions, to show that output responds less to monetary shocks when inflation is high. It is also broadly consistent with the empirical evidence in Ball and Mazumder (2011), Hazell et al. (2022) and Costain et al. (2022), where estimates of the slope are higher in samples that include periods of high inflation.

inflation accelerator accounts for the bulk of this increase: in its absence the higher fraction of price changes in this period would only increase the slope of the Phillips curve to 0.04.

That the slope of the Phillips curve varies over time has important implications for the tradeoff between inflation and output stabilization. We gauge how this tradeoff varies over time by calculating the sacrifice ratio: the fall in output required to reduce inflation by one percentage point. The sacrifice ratio varies considerably, from 2.5 in the low-inflation periods to 0.5 in high-inflation periods. Our model thus implies that if inflation is high to begin with, bringing it down requires a smaller drop in output than if inflation is low.

We use our model to revisit several themes central to monetary policy analysis. First, we enrich our baseline framework with additional sources of aggregate uncertainty and allow for time-varying monetary policy regimes. We use the estimated model to study the drivers of inflation in the historical data. We find that cost-push shocks were the main driver of inflation in the early 1970s, while a persistent increase in the inflation target was the main driver of inflation in the late 1970s. Importantly, we show that in our model, in contrast to the Calvo model, adverse shocks have much larger inflationary effects in the late 1970s, a period in which accommodative monetary policy elevated the slope of Phillips curve, compared to the 1990s. Thus, in the debate over whether the Great Inflation in the late 1970s reflects primarily bad policy or bad luck, our results point to an interaction: accommodative policy substantially amplifies the inflationary effects of adverse shocks.

Second, we study the determinacy properties of Taylor rules in environments with different levels of trend inflation. We show that the determinacy region is larger in our model compared to the Calvo model. Lastly, we characterize the welfare losses from price rigidity and derive a quadratic approximation to the welfare loss function. We show that the welfare losses in our model are smaller than those in the Calvo model and increase less with inflation owing to the endogenous response of the fraction of price changes.

The paper proceeds as follows. Section 2 presents the model. Section 3 describes the parameterization, analyzes the steady state and the time-series and discusses robustness. Section 4 performs the historical analysis, studies the determinacy of Taylor rules and characterizes the welfare losses from price rigidity. Section 5 concludes.

2 Model

We study an economy in which a continuum of multi-product firms choose what fraction of their prices to adjust in any given period. We circumvent the need to keep track of the distribution of prices by assuming that firms choose how many, but not which, prices to change. Owing to this assumption, our model reduces to a one-equation extension of the Calvo model, with the additional equation determining the fraction of price changes. Relative to a menu cost model, our model does not feature selection, consistent with the evidence.[6]

2.1 Consumers

A representative consumer has preferences over consumption c_t and hours worked h_t and maximizes life-time utility

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\log c_t - h_t\right),$$

subject to the budget constraint

$$P_t c_t + \frac{1}{1+i_t} B_{t+1} = W_t h_t + D_t + B_t,$$

where P_t is the nominal price level, B_t are holdings of a risk-free bond which pays nominal interest i_t, D_t are the firm dividends and W_t is the nominal wage rate.

2.2 Monetary Policy

Monetary policy follows an interest rate rule. As in Justiniano and Primiceri (2008), we assume that

$$\frac{1+i_t}{1+i} = \left(\frac{1+i_{t-1}}{1+i}\right)^{\phi_i} \left(\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{y_t}{y_{t-1}}\right)^{\phi_y}\right)^{1-\phi_i} \varepsilon_t, \tag{1}$$

where π is the inflation target, $1+i = \pi/\beta$ is the steady-state nominal interest rate, ϕ_i, ϕ_π and ϕ_y determine the inertia in the interest rate rule and the sensitivity of monetary policy to fluctuations in inflation and output growth, and the monetary policy shock $\log \varepsilon_t \sim N(0, \sigma_\varepsilon)$.

2.3 Technology

There is a continuum of intermediate goods firms indexed by i. Each firm produces a continuum of products with technology

$$y_{ikt} = \left(l_{ikt}\right)^{\eta},$$

[6] Both indirect evidence from the distribution of price changes (Midrigan, 2011; Alvarez et al., 2016), as well as more direct evidence on how adjustment hazards vary with individual price gaps (Carvalho and Kryvtsov, 2021; Karadi et al., 2024b; Gagliardone et al., 2025; Luo and Villar, 2021) suggests that selection is relatively weak. In Appendix A we use data from NielsenIQ to show that, consistent with our model, variation in the aggregate fraction of price changes over time is primarily driven by variation in the share of prices that individual firms adjust, not the share of firms that adjust a large fraction of their prices.

where y_{ikt} is the output of product k produced by firm i, l_{ikt} is the labor used in production and $\eta \leq 1$ is the span-of-control parameter which, as in Burstein and Hellwig (2008), introduces a micro-level strategic complementarity in price setting.

A perfectly competitive final goods sector aggregates the intermediate goods y_{ikt} into a composite final good using a CES aggregator

$$y_t = \left(\int_0^1 \int_0^1 (y_{ikt})^{\frac{\theta-1}{\theta}} \, \mathrm{d}k\mathrm{d}i \right)^{\frac{\theta}{\theta-1}},$$

where θ is the elasticity of substitution, which we assume to be the same across products and across firms. This implies that the demand for an individual product is

$$y_{ikt} = \left(\frac{P_{ikt}}{P_t} \right)^{-\theta} y_t, \tag{2}$$

where P_{ikt} is the price of an individual product and the aggregate price index P_t is

$$P_t = \left(\int_0^1 \int_0^1 (P_{ikt})^{1-\theta} \, \mathrm{d}k\mathrm{d}i \right)^{\frac{1}{1-\theta}}.$$

2.4 Problem of Intermediate Goods Producers

We next describe the profit maximization problem of intermediate goods producers.

2.4.1 Period Profits

The nominal profits of firm i from producing product k are

$$P_{ikt} y_{ikt} - \tau W_t l_{ikt},$$

where $\tau = 1 - 1/\theta$ is a subsidy that removes the markup distortion that would arise even in the absence of price rigidities. Using the demand function (2), we can express real profits as

$$\left(\frac{P_{ikt}}{P_t} \right)^{1-\theta} y_t - \tau \frac{W_t}{P_t} \left(\frac{P_{ikt}}{P_t} \right)^{-\frac{\theta}{\eta}} y_t^{\frac{1}{\eta}}. \tag{3}$$

2.4.2 Losses from Misallocation

Price dispersion within the firm generates losses from misallocation, reducing firm productivity. To see this, let

$$y_{it} = \left(\int (y_{ikt})^{\frac{\theta-1}{\theta}} \, \mathrm{d}k \right)^{\frac{\theta}{\theta-1}}$$

denote the composite output produced by firm i and let

$$l_{it} = \int l_{ikt} \mathrm{d}k$$

denote the total amount of labor it uses. We can then derive a firm-level production function

$$y_{it} = \left(\frac{X_{it}}{P_{it}} \right)^{\theta} l_{it}^{\eta},$$

where

$$P_{it} = \left(\int (P_{ikt})^{1-\theta} \, \mathrm{d}k \right)^{\frac{1}{1-\theta}} \tag{4}$$

denotes the price index of firm i and

$$X_{it} = \left(\int (P_{ikt})^{-\frac{\theta}{\eta}} \, \mathrm{d}k \right)^{-\frac{\eta}{\theta}} \tag{5}$$

determines the extent of misallocation. Absent dispersion in prices, $X_{it}/P_{it} = 1$ and productivity is maximized. With price dispersion, $X_{it}/P_{it} < 1$ and productivity is reduced.

2.4.3 Price Adjustment Cost

We assume that the firm has a convex cost of changing prices denominated in units of labor. This cost is increasing in the number of prices n_{it} the firm resets and is equal to

$$\frac{\xi}{2} (n_{it} - \bar{n})^2, \quad \text{if } n_{it} > \bar{n}$$

and zero otherwise. The parameter ξ determines the size of the adjustment cost and $\bar{n}$ is the fraction of free price changes. The key assumption we make is that although the firm can choose how many prices to change in a given period, it cannot choose which prices to change. That is, the products whose prices are changed are selected at random and with equal probability. When $\xi \to \infty$, the model collapses to the Calvo model with a constant fraction of price changes $\bar{n}$.

Our model shares similarities with the models in Romer (1990) and Gasteiger and Grimaud (2023), which also endogenize the fraction of price changes in the Calvo model.[7] In Romer (1990), firms choose a once-and-for-all price adjustment probability. Extending that model to allow for a time-varying adjustment probability would require keeping track of the distribution of prices because the gains from adjusting would be higher for prices further away from the optimum, just like in menu cost models. In Gasteiger and Grimaud (2023), each

[7] See also Kiley (2000), Devereux and Yetman (2002), Bakhshi et al. (2007) and Ascari et al. (2025).

firm's adjustment probability is imposed to be a function of the previous period's average price level, rather than the firm's own price. This sidesteps the need to track the distribution of prices, but it is not derived from an optimal adjustment decision. In contrast, our assumption that firms sell a continuum of products and choose how many, but not which, prices to change, implies that firms are ex-post symmetric and that a small number of state variables are sufficient to characterize a firm's incentives to adjust prices. This feature allows exact aggregation and renders our model very tractable.

2.4.4 Price Setting

We next describe the firm's problem in detail. The value of the firm is the present discounted sum of its flow profits (3) from all products

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \frac{1}{c_{t+s}} \left(\int \left[\left(\frac{P_{ikt+s}}{P_{t+s}} \right)^{1-\theta} y_{t+s} - \tau \frac{W_{t+s}}{P_{t+s}} \left(\frac{P_{ikt+s}}{P_{t+s}} \right)^{-\frac{\theta}{\eta}} y_{t+s}^{\frac{1}{\eta}} \right] \mathrm{d}k - \frac{\xi}{2} \left(n_{it+s} - \bar{n} \right)^2 \frac{W_{t+s}}{P_{t+s}} \right).$$

The log-linear specification of preferences implies that $c_t = \frac{W_t}{P_t} = y_t$ and, together with the definitions of P_{it} and X_{it} in equations (4) and (5), allows us to write the value of the firm as

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left[\left(\frac{P_{it+s}}{P_{t+s}} \right)^{1-\theta} - \tau \left(\frac{X_{it+s}}{P_{t+s}} \right)^{-\frac{\theta}{\eta}} y_{t+s}^{\frac{1}{\eta}} - \frac{\xi}{2} \left(n_{it+s} - \bar{n} \right)^2 \right]. \tag{6}$$

The firm chooses what fraction of prices n_{it} to reset every period and the reset price P_{it}^*. Because products are symmetric, $P_{ikt} = P_{it}^*$ for all products whose price is reset. To characterize these choices, we first describe how P_{it}^* and n_{it} affect the price and misallocation indices P_{it+s} and X_{it+s}.

Consider first the term $(P_{it+s})^{1-\theta}$. Using the definition of the firm's price index in equation (4) and the assumption that the firm chooses how many, but not which, prices to change, we can write it as a function of the history of reset prices and repricing probabilities

$$\begin{aligned} (P_{it+s})^{1-\theta} = {} & n_{it+s} \left(P_{it+s}^* \right)^{1-\theta} + \left(1 - n_{it+s} \right) n_{it+s-1} \left(P_{it+s-1}^* \right)^{1-\theta} + \\ & \left(1 - n_{it+s} \right) \left(1 - n_{it+s-1} \right) n_{it+s-2} \left(P_{it+s-2}^* \right)^{1-\theta} + \cdots + \\ & \prod_{j=1}^{s} (1 - n_{it+j}) n_{it} \left(P_{it}^* \right)^{1-\theta} + \prod_{j=1}^{s} \left(1 - n_{it+j} \right) \left(1 - n_{it} \right) \left(P_{it-1} \right)^{1-\theta}. \end{aligned} \tag{7}$$

The first term on the right hand side represents the contribution of the n_{it+s} newly reset prices in period $t+s$. The second term represents the contribution of the $\left(1 - n_{it+s}\right) n_{it+s-1}$ prices that were reset in period $t+s-1$ and were not reset in period $t+s$. This pattern continues with each subsequent term reflecting the contribution of prices reset in each period leading up to $t+s$, including those reset in period t, captured by the first term in the last line of the expression, as well as those reset prior to period t, captured by the last term of the expression. In writing this last term we used the definition of the price index in equation (4) to express the history of all reset prices prior to period t using a single state variable, P_{it-1}.

A similar argument allows us to rewrite the term $(X_{it+s})^{-\frac{\theta}{\eta}}$ as

$$\begin{aligned}(X_{it+s})^{-\frac{\theta}{\eta}} = n_{it+s}\left(P^*_{it+s}\right)^{-\frac{\theta}{\eta}} + (1-n_{it+s})\,n_{it+s-1}\left(P^*_{it+s-1}\right)^{-\frac{\theta}{\eta}} + \\ (1-n_{it+s})(1-n_{it+s-1})\,n_{it+s-2}\left(P^*_{it+s-2}\right)^{-\frac{\theta}{\eta}} + \cdots + \\ \prod_{j=1}^{s}(1-n_{it+j})n_{it}\left(P^*_{it}\right)^{-\frac{\theta}{\eta}} + \prod_{j=1}^{s}(1-n_{it+j})(1-n_{it})(X_{it-1})^{-\frac{\theta}{\eta}}.\end{aligned} \tag{8}$$

We next characterize the optimal choice of P^*_{it} and n_{it}.

Optimal reset price. To derive the optimality condition with respect to P^*_{it} we note that equations (7) and (8) imply that

$$\frac{\partial\left(P_{it+s}\right)^{1-\theta}}{\partial P^*_{it}} = (1-\theta)\left(P^*_{it}\right)^{-\theta}\prod_{j=1}^{s}(1-n_{it+j})n_{it}$$

and

$$\frac{\partial\left(X_{it+s}\right)^{-\frac{\theta}{\eta}}}{\partial P^*_{it}} = -\frac{\theta}{\eta}\left(P^*_{it}\right)^{-\frac{\theta}{\eta}-1}\prod_{j=1}^{s}(1-n_{it+j})n_{it}.$$

Therefore, the first order condition that determines the reset price P^*_{it} is

$$\left(\frac{P^*_{it}}{P_t}\right)^{1+\theta\left(\frac{1}{\eta}-1\right)} = \frac{1}{\eta}\frac{b_{2it}}{b_{1it}},$$

where

$$b_{1it} = \mathbb{E}_t\sum_{s=0}^{\infty}\beta^s\prod_{j=1}^{s}(1-n_{it+j})\left(\frac{P_{t+s}}{P_t}\right)^{\theta-1}$$

and

$$b_{2it} = \mathbb{E}_t\sum_{s=0}^{\infty}\beta^s\prod_{j=1}^{s}(1-n_{it+j})\left(\frac{P_{t+s}}{P_t}\right)^{\frac{\theta}{\eta}}(y_{t+s})^{\frac{1}{\eta}}.$$

The terms b_{1it} and b_{2it} capture the present value of revenue and marginal costs in future periods, weighted by the probability that a price reset today in still in effect in that future period. Strategic complementarities, captured by the term $1+\theta\left(\frac{1}{\eta}-1\right)$, dampen the extent to which the reset price responds to aggregate shocks, generating additional price stickiness. These expressions that determine the optimal reset price are analogous to those obtained in a Calvo model, with the only difference being that in the Calvo model n_{it} is constant.

Optimal Fraction of Price Changes. To derive the optimality condition with respect to n_{it}, we first note that equations (7) and (8) imply that

$$\frac{\partial\left(P_{it+s}\right)^{1-\theta}}{\partial n_{it}}=\prod_{j=1}^{s}\left(1-n_{it+j}\right)\left(\left(P_{it}^{*}\right)^{1-\theta}-\left(P_{it-1}\right)^{1-\theta}\right)$$

and

$$\frac{\partial\left(X_{it+s}\right)^{-\frac{\theta}{\eta}}}{\partial n_{it}}=\prod_{j=1}^{s}\left(1-n_{it+j}\right)\left(\left(P_{it}^{*}\right)^{-\frac{\theta}{\eta}}-\left(X_{it-1}\right)^{-\frac{\theta}{\eta}}\right).$$

Therefore, the first order condition that determines the fraction of price changes n_{it} is

$$\xi\left(n_{it}-\bar{n}\right) \;=\; b_{1it}\left(\left(\frac{P_{it}^{*}}{P_t}\right)^{1-\theta}-\left(\frac{P_{it-1}}{P_t}\right)^{1-\theta}\right)-\tau b_{2it}\left(\left(\frac{P_{it}^{*}}{P_t}\right)^{-\frac{\theta}{\eta}}-\left(\frac{X_{it-1}}{P_t}\right)^{-\frac{\theta}{\eta}}\right).$$

In choosing what fraction of prices to adjust, the firm balances the price adjustment costs on the left-hand side against the benefits resulting from changing its price index, captured by the first term on the right-hand side, and reducing misallocation, captured by the second term on the right-hand side. The terms b_{1it} and b_{2it} also determine the firm's incentive to adjust prices. For example, the more likely the firm is to adjust its prices in the future, the lower are b_{1it} and b_{2it} and therefore the lower are the benefits from adjusting prices today.

Importantly, the incentives to adjust are shaped by only two state variables, the price and misallocation indices, P_{it-1} and X_{it-1}, as in Blanco et al. (2024). In contrast to that paper, here we do not need to keep track of the joint distribution of these two state variables.

2.5 Equilibrium

Since firms are symmetric, $n_{it}=n_t$ and $P_{it}^*=P_t^*$, so all firms have the same price indices and losses from misallocation. Let $w_t=W_t/P_t$, $p_t^*=P_t^*/P_t$, $x_t=X_t/P_t$ and $\pi_t=P_t/P_{t-1}$. The pricing block of the model is characterized by the following system of equations:

1. the definition of the price index

$$1=n_t\left(p_t^*\right)^{1-\theta}+\left(1-n_t\right)\pi_t^{\theta-1}, \tag{9}$$

2. the optimal reset price

$$\left(p_t^*\right)^{1+\theta\left(\frac{1}{\eta}-1\right)}=\frac{1}{\eta}\frac{b_{2t}}{b_{1t}}, \tag{10}$$

where b_{1t} and b_{2t} are determined by

$$b_{1t}=1+\beta\mathbb{E}_t\left(1-n_{t+1}\right)\pi_{t+1}^{\theta-1}b_{1t+1} \tag{11}$$

$$b_{2t}=y_t^{\frac{1}{\eta}}+\beta\mathbb{E}_t\left(1-n_{t+1}\right)\pi_{t+1}^{\frac{\theta}{\eta}}b_{2t+1}, \tag{12}$$

3. the optimal choice of the fraction of price changes

$$\xi\left(n_t - \bar{n}\right) = b_{1t}\left(\left(p_t^*\right)^{1-\theta} - \pi_t^{\theta-1}\right) - \tau b_{2t}\left(\left(p_t^*\right)^{-\frac{\theta}{\eta}} - x_{t-1}^{-\frac{\theta}{\eta}}\pi_t^{\frac{\theta}{\eta}}\right), \tag{13}$$

4. the productivity term that captures the losses from misallocation

$$x_t^{-\frac{\theta}{\eta}} = n_t\left(p_t^*\right)^{-\frac{\theta}{\eta}} + \left(1 - n_t\right)x_{t-1}^{-\frac{\theta}{\eta}}\pi_t^{\frac{\theta}{\eta}}.$$

Relative to the Calvo model, the only new equation is equation (13) which determines the fraction of price changes. The Calvo model is a special case of our model that can be obtained by setting $\xi \to \infty$, in which case $n_t = \bar{n}$ is constant. In contrast to the Calvo model, our model exhibits important non-linearities, so we solve it using a third-order perturbation.

3 Inspecting the Mechanism

In this section, we explain the mechanism of the model. We first characterize how the steady state varies with trend inflation and how the economy responds to monetary shocks. We then study how the slope of the Phillips curve evolves in the U.S. time series.

3.1 Parameterization

We first discuss the parameters we assign values to and then those we calibrate endogenously.

3.1.1 Assigned Parameters

A period in the model is a quarter. We set several parameters to values conventional in the literature: the quarterly discount factor β is 0.99, the demand elasticity θ is 6, the returns to scale parameter η is 2/3, and the Taylor rule coefficients are $\phi_i = 0.75$, $\phi_\pi = 2$ and $\phi_y = 0.5$.

3.1.2 Calibrated Parameters

The parameters we calibrate endogenously are those determining the average level and volatility of inflation, as well as the average fraction of price changes and its comovement with inflation. Specifically, we set the inflation target, π, the volatility of the monetary policy shocks, σ_ε, the fraction of free price changes, $\bar{n}$, and the price adjustment cost, ξ, to reproduce the mean and standard deviation of inflation, the mean fraction of price changes, and the slope coefficient from regressing the fraction of price changes on the absolute value of inflation. This last statistic summarizes the extent to which the fraction of price changes and inflation comove in the time series.

Data. Our measure of inflation is the growth rate of the U.S. CPI, available from 1962:Q1 to 2023:Q4. We follow Nakamura et al. (2018) in using the series excluding shelter to ensure that the inflation data is compatible with the data on the fraction of price changes. The fraction of price changes is computed from the price quotes collected by the BLS that underlie the construction of the CPI.[8] Specifically, we use the monthly median fraction of price changes, excluding sales, available between 1978 and 2023, and convert it to a quarterly series.

Figure 1 plots the year-to-year percent change in the price level and the average quarterly fraction of price changes in the preceding year. On average, 25% of prices change in a given quarter in periods of low inflation. As documented by Nakamura et al. (2018), the fraction of price changes was relatively high, approximately 40% per quarter, in the high-inflation episode in the early 1980s. As documented by Montag and Villar (2023), the fraction of price changes spiked once again, to approximately 50%, during the post-Covid inflation episode.

Parameter Values. For many of our subsequent exercises, we will contrast the predictions of our model to those of a Calvo model. We calibrate the Calvo model using the same strategy, but discard the adjustment cost parameter and choose the fraction of price changes, $\bar{n}$, to reproduce the average fraction of price changes.

Table 1 reports the results of the calibration. As Panel A shows, both models reproduce the targeted moments exactly: the average inflation rate is 3.5%, the standard deviation of inflation is 2.7% and the average fraction of price change is 29.7%. In our model, as in the data, the fraction of price changes comoves systematically with inflation: the slope coefficient of a regression of the fraction of price changes on the absolute value of inflation is 0.016.

Panel B reports the parameter values. The volatility of monetary policy shocks is slightly lower in our model compared to the Calvo model because in our model endogenous movements in the fraction of price changes contribute to inflation fluctuations. The fraction of free price changes is 24.2% in our model and the adjustment cost parameter, ξ, implies that on average 0.6% of all labor is used in adjusting prices, in line with the evidence in Levy et al. (1997).

3.2 Steady State and the Transmission of Monetary Shocks

We next illustrate the mechanism of the model in the non-stochastic steady state. First, we explain how the steady-state fraction of price changes varies with trend inflation. Second, we

[8] We are grateful to Hugh Montag and Daniel Villar for kindly sharing these data with us. See Nakamura et al. (2018) for a detailed description of how the data was constructed.

Table 1: Endogenously Calibrated Parameters

A. Targeted Moments

	Data	Our model	Calvo
mean inflation	3.517	3.517	3.517
s.d. inflation	2.739	2.739	2.739
mean frequency	0.297	0.297	0.297
slope of n_t on $\lvert\pi_t\rvert$	0.016	0.016	–

B. Calibrated Parameter Values

		Our model	Calvo
$\log \pi$	inflation target	0.039	0.041
σ_ε	s.d. monetary shocks	0.015	0.018
$\bar{n}$	fraction of free price changes	0.242	0.297
ξ	adjustment cost	1.699	–

Notes: The inflation target is annualized.

use a first-order approximation around the non-stochastic steady state to provide intuition for how the economy responds to monetary shocks in environments with high and low inflation.

3.2.1 Steady-State Fraction of Price Changes

Letting variables without t subscripts denote the value of a variable in the non-stochastic steady state, we note that the fraction of price changes is pinned down by

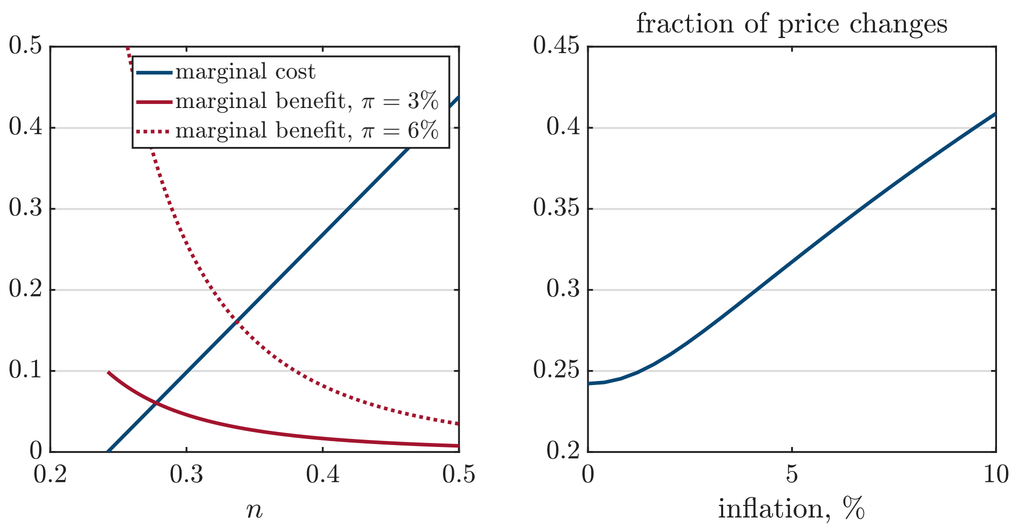
$$\xi\left(n-\bar{n}\right)=\frac{1}{1-\beta\left(1-n\right)\pi^{\theta-1}}\frac{1}{n}\left(1-\pi^{\theta-1}-\tau\eta\frac{1-\left(1-n\right)\pi^{\theta-1}}{1-\left(1-n\right)\pi^{\frac{\theta}{\eta}}}\left(1-\pi^{\frac{\theta}{\eta}}\right)\right), \qquad (14)$$

where the left-hand side of the equation is the marginal cost of increasing n and the right-hand side captures the marginal benefit to increasing n.[9]

The marginal cost is linearly increasing in n. Absent trend inflation, $\pi=1$, the marginal benefit of increasing n is equal to 0, implying that $n=\bar{n}$. Thus, absent trend inflation, the steady state of our model is identical to that of the Calvo model. More generally, with positive trend inflation, $\pi>1$, the marginal benefit of changing prices is positive and decreases with n, as illustrated in the left panel of Figure 2. The intersection of the marginal benefit and cost curves pins down the steady-state fraction of price changes. As the figure shows, higher trend inflation increases the marginal benefit of adjusting prices, thus increasing the fraction

[9] See Appendix B for all the derivations.

Figure 2: Steady-State Fraction of Price Changes



of price changes, as illustrated in the right panel. We note that in deriving equation (14) we have assumed an interior solution for n. Since the marginal benefit of changing prices is positive at $n = 1$ whenever $\pi \neq 1$, as $\xi \to 0$ the solution hits the corner $n = 1$ and the economy converges to the flexible-price equilibrium.

We build additional intuition for how the fraction of price changes varies with trend inflation by considering a second-order Taylor expansion of equation (14) around $\pi = 1$ and $n = \bar{n}$, as $\beta \to 1$. We show that, in this case, the steady-state fraction of price changes is

$$n = \frac{\bar{n} + \sqrt{\bar{n}^2 + 4\Gamma \frac{2-\bar{n}}{\bar{n}^2} (\pi - 1)^2}}{2},$$

where $\Gamma = \frac{\theta - 1}{2} \left(1 + \theta \left(\frac{1}{\eta} - 1\right)\right) \frac{1}{\xi}$ depends on the demand elasticity, the strength of strategic complementarities and the adjustment cost. The larger the demand elasticity, or the stronger the strategic complementarities, the greater are the losses from suboptimal prices, so the more sensitive is the fraction of price changes to trend inflation. Conversely, the higher the adjustment cost, the less sensitive is the fraction of price changes to trend inflation.

3.2.2 The Real Effects of Monetary Shocks

We next study the effects of monetary shocks by considering impulse responses to both small and large shocks and discussing how they depend on trend inflation.

Impulse Responses to Small Shocks. We first contrast how our economy responds to small monetary shocks in environments with low and high trend inflation. We consider two economies, with zero and 8% trend inflation, and report the responses of output, inflation and the fraction of price changes to a 0.25 standard deviations expansionary monetary shock. To build intuition, we consider a log-linear approximation of the model around the steady state of each economy. We compare the responses in our model to those in an otherwise identical model in which the fraction of price changes is equal to that in the steady state of our model, but $\xi = \infty$ so the fraction of price changes does not respond to shocks. We note that the steady-state fraction of price changes is approximately 1.5 times larger in the economy with 8% inflation compared to the economy with zero inflation (0.37 vs. 0.24).

The left panels of Figure 3 show the impulse responses in the economy with zero trend inflation. Note that, up to a first-order approximation, the fraction of price changes does not respond to the monetary shock. Hence, our model with an endogenous adjustment frequency has identical responses to the economy with a time-invariant frequency.

The right panels of Figure 3 depict the responses in an environment with 8% trend inflation. We make two observations. First, even when the fraction of price changes does not respond to the shock ($\xi = \infty$), the response of output is weaker and that of inflation is stronger compared to the economy without trend inflation. This is due to the larger steady-state fraction of price changes. Second, the response of output is smaller and shorter lived in our model relative to the $\xi = \infty$ economy. This is due to the increase in the fraction of price changes from 0.37 in steady state to 0.38 after the shock.

Although this increase in the fraction of price changes appears small, it leads to considerable aggregate price flexibility. To see why, consider the log-linearized system of equilibrium conditions characterizing the evolution of inflation. Letting hats denote log-deviations from the steady state, a first-order Taylor expansion of equation (9) implies that

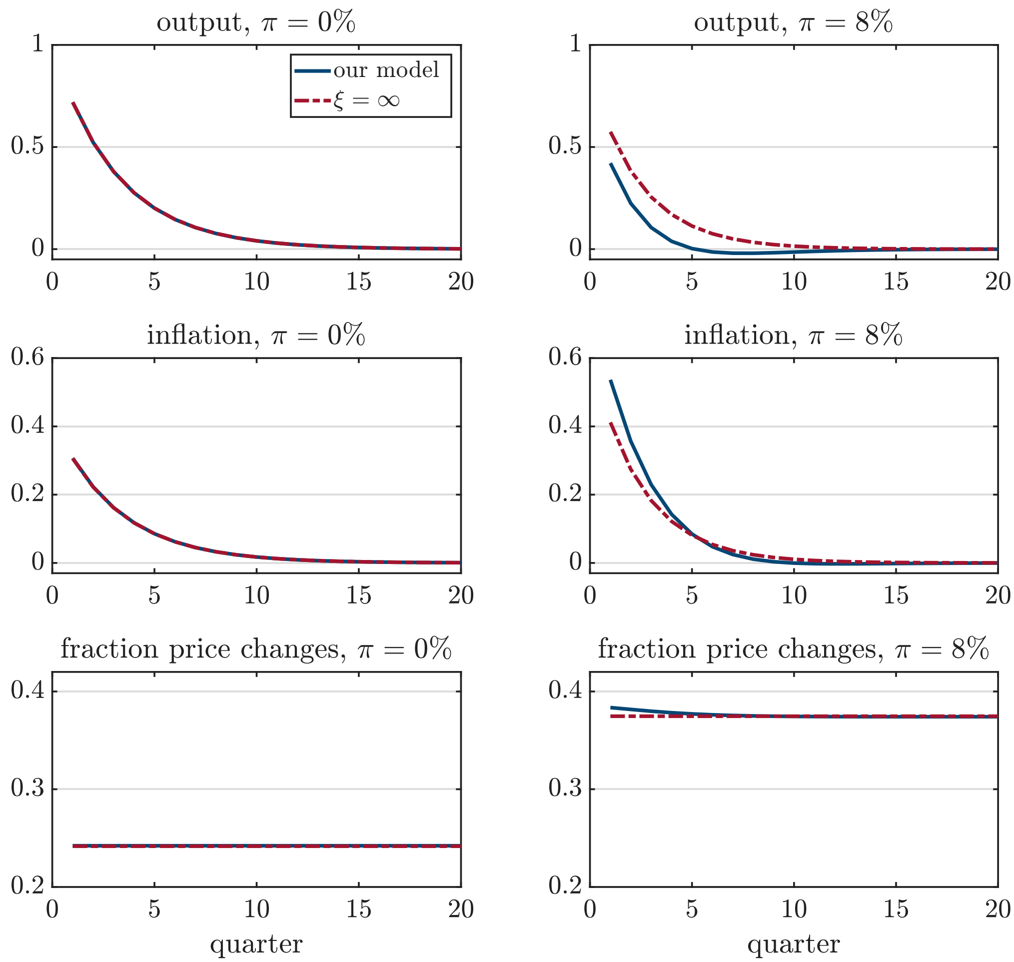
$$\hat{\pi}_t = \underbrace{\frac{1}{(1-n)\,\pi^{\theta-1}} \frac{\pi^{\theta-1}-1}{\theta-1}}_{\mathcal{M}} \hat{n}_t + \underbrace{\frac{1-(1-n)\,\pi^{\theta-1}}{(1-n)\,\pi^{\theta-1}}}_{\mathcal{N}} \hat{p}^*_t. \tag{15}$$

The second term on the right-hand side is familiar from the Calvo model and describes how inflation responds to an increase in the relative reset price. The elasticity

$$\mathcal{N} = \frac{1-(1-n)\,\pi^{\theta-1}}{(1-n)\,\pi^{\theta-1}}$$

increases with the fraction of price changes, n, and decreases with trend inflation, π. As Coibion et al. (2012) point out, a higher trend inflation reduces the sensitivity of inflation

Figure 3: Impulse Response to a Small Monetary Shock



to reset price changes because newly reset prices are higher and therefore have smaller consumption weights in the ideal price index.

The first term on the right-hand side of equation (15) is new to our model and captures the impact of changes in the fraction of price changes on inflation. The elasticity

$$\mathcal{M} = \frac{1}{(1-n)\,\pi^{\theta-1}} \frac{\pi^{\theta-1}-1}{\theta-1}$$

is zero absent trend inflation and increases with trend inflation. To understand why, note that inflation is approximately equal to the fraction of price changes times the average price change conditional on adjustment. If the average price change is zero, as is the case absent trend inflation, an increase in the fraction of price changes does not affect inflation. In contrast, if the average price change is large, inflation greatly responds to changes in the fraction of price changes. This effect is reminiscent of the mechanism in the menu cost model of Caplin and

Spulber (1987) in which small changes in the fraction of price changes render the aggregate price level flexible. Because newly-adjusting prices increase by a large amount, even small changes in the fraction of price changes add considerably to the response of inflation.

Impulse Responses to Large Shocks. So far we considered the responses to relatively small monetary policy shocks using a first-order approximation. To a first order, the increase in the fraction of price changes only contributes to aggregate price flexibility because adjusting firms respond to the underlying trend inflation, but not to the monetary shock. We next consider larger shocks and compute impulse responses non-linearly, under perfect foresight.

Figure 4 plots the responses to a 0.25 (small) and a 2.5 (large) standard deviations expansionary shock starting from the steady state of our baseline model.[10] Note that even in response to a small shock the aggregate price level is more flexible in our model, as evidenced by a smaller output and a larger inflation response. As explained above, the increase in the fraction of price changes increases the flexibility of the aggregate price level because these additional price changes incorporate the positive trend inflation in addition to the shock itself. Importantly, the response of output to a large shock is considerably smaller in our model, owing to the sharp increase in the fraction of price changes: more than 40% of prices change on impact. Consequently, inflation responds much more than in the Calvo model.

We thus conclude that our economy has many of the features of menu cost models, such as the large responsiveness of the price level to movements in the fraction of price changes in periods of high inflation (as in Caplin and Spulber, 1987) and considerable non-linearity in responses to shocks of different sizes (as in Blanco et al., 2024).[11]

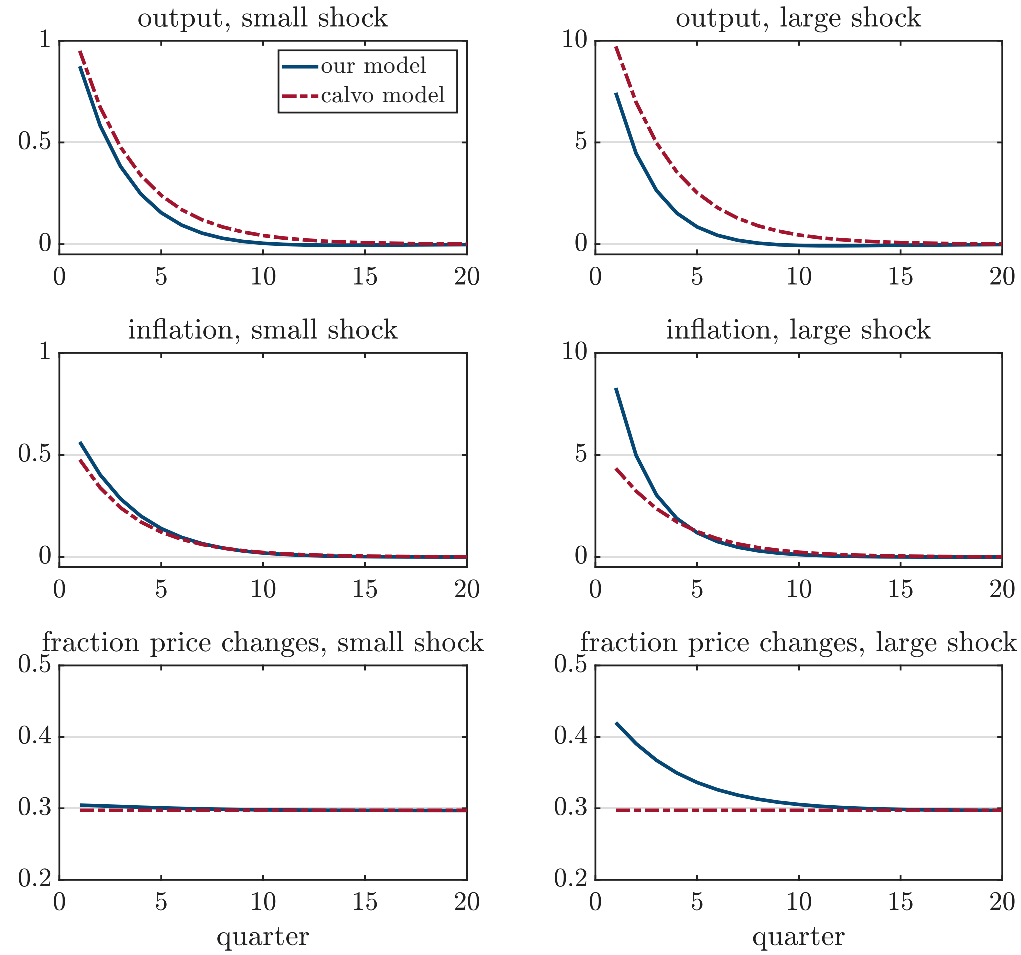
3.2.3 The Phillips Curve and the Inflation Accelerator

We next derive the Phillips curve in our model. We show that the slope of the Phillips curve increases rapidly with trend inflation due to a feedback loop between inflation and the fraction of price changes. On the one hand, an increase in the fraction of price changes increases inflation, more so when trend inflation is high. On the other hand, an increase in inflation increases the firms' incentive to change prices, raising the fraction of price changes. We refer to this feedback loop as the *inflation accelerator*.

[10]In Appendix C we also plot these responses in economies with zero and 8% trend inflation and contrast the responses to large expansionary and contractionary monetary shocks. We also show that the responses obtained using perfect foresight are similar to those obtained using the third-order perturbation.

[11]As Reiter and Wende (2024) show, a Rotemberg (1982) model with a suitably modified adjustment cost function can also generate non-linearities.

Figure 4: Impulse Response to Small and Large Monetary Shocks



Log-linearizing the expression determining the optimal fraction of price changes (13) around the non-stochastic steady state, we have

$$\hat{n}_t = \mathcal{A}\hat{\pi}_t + \mathcal{B}\hat{p}_t^* - \mathcal{C}\hat{x}_{t-1} + \frac{n-\bar{n}}{n}\hat{b}_{1t}, \tag{16}$$

where

$$\mathcal{A} = \frac{\theta-1}{\xi n}\frac{1}{1-\beta\left(1-n\right)\pi^{\theta-1}}\frac{\pi^{\frac{\theta}{\eta}}-\pi^{\theta-1}}{1-\left(1-n\right)\pi^{\frac{\theta}{\eta}}}$$

determines how sensitive the fraction of price changes is to inflation,

$$\mathcal{B} = \left(1-\tau\eta\right)\frac{\theta-1}{\xi n}\frac{1-\left(1-n\right)\pi^{\theta-1}}{1-\beta\left(1-n\right)\pi^{\theta-1}}\frac{1}{n}\frac{\pi^{\frac{\theta}{\eta}}-1}{1-\left(1-n\right)\pi^{\frac{\theta}{\eta}}}$$

determines how sensitive the fraction of price changes is to the relative reset price, and

$$\mathcal{C} = \frac{\theta-1}{\xi n}\frac{1-\left(1-n\right)\pi^{\theta-1}}{1-\beta\left(1-n\right)\pi^{\theta-1}}\frac{\pi^{\frac{\theta}{\eta}}}{1-\left(1-n\right)\pi^{\frac{\theta}{\eta}}}$$

determines how sensitive the fraction of price changes is to past misallocation. We note that both $\mathcal{A}$ and $\mathcal{B}$ are zero absent trend inflation, so the fraction of price changes is, to a first-order, irresponsive to monetary shocks, as in Figure 3. In the presence of trend inflation these elasticities are positive and decreasing in the adjustment cost parameter ξ.

Combining the log-linearized expression for the price index (15) with equation (16) implies

$$\hat{\pi}_t = \frac{\mathcal{M}\mathcal{B}+\mathcal{N}}{1-\mathcal{M}\mathcal{A}}\hat{p}_t^* - \frac{\mathcal{M}\mathcal{C}}{1-\mathcal{M}\mathcal{A}}\hat{x}_{t-1} + \frac{\mathcal{M}}{1-\mathcal{M}\mathcal{A}}\frac{n-\bar{n}}{n}\hat{b}_{1t}.$$

The elasticity of inflation to relative reset prices, $\hat{p}_t^*$, is equal to $\frac{\mathcal{M}\mathcal{B}+\mathcal{N}}{1-\mathcal{M}\mathcal{A}}$ and is amplified relative to the Calvo model whenever $\pi > 1$ and thus $\mathcal{M}$, $\mathcal{A}$ and $\mathcal{B}$ are positive. Intuitively, a higher desired reset price not only directly affects inflation with an elasticity $\mathcal{N}$, but also leads to more frequent price changes with an elasticity $\mathcal{B}$, which then increases inflation with an elasticity $\mathcal{M}$ and further increases the incentives to adjust prices with an elasticity $\mathcal{A}$, triggering a feedback loop. We refer to this feedback loop between the fraction of price changes, the optimal reset price and inflation as the *inflation accelerator*.

Finally, as we show in Appendix B where we derive the Phillips curve, the slope of the Phillips curve, namely the elasticity of inflation with respect to real aggregate marginal cost, $mc_t = \frac{1}{\eta}\frac{W_t}{P_t}y_t^{\frac{1}{\eta}-1}$, is equal to

$$\mathcal{K} = \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\left(1-\beta\left(1-n\right)\pi^{\frac{\theta}{\eta}}\right)\frac{\mathcal{M}\mathcal{B}+\mathcal{N}}{1-\mathcal{M}\mathcal{A}}. \tag{17}$$

The first term captures the effect of strategic complementarities which are stronger the more elastic demand is and the lower the returns to scale. The second term captures the horizon effect: a transitory increase in marginal costs in period t only increases the optimal reset price by a factor $1-\beta\left(1-n\right)\pi^{\frac{\theta}{\eta}}$, which reflects the discount factor and the probability that the current price will still be in effect in future periods. Finally, as discussed above, the last term captures the impact of reset prices on inflation.

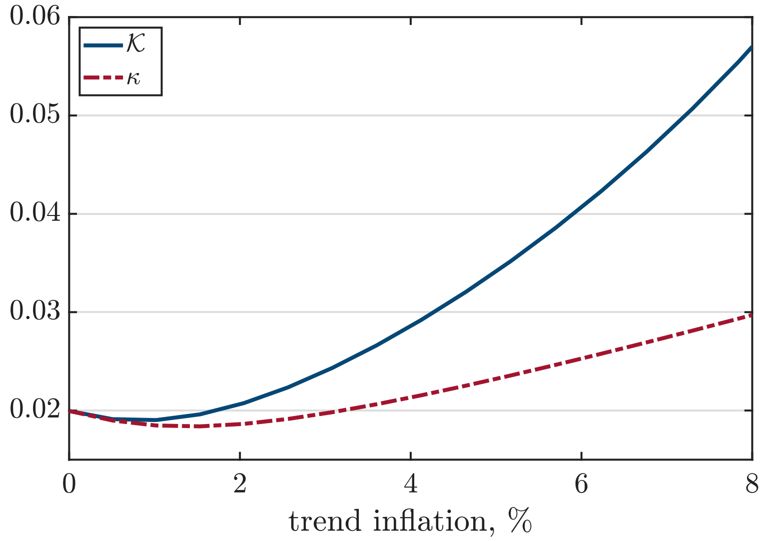
Absent the feedback effect of the fraction of price changes on inflation, i.e. when $\mathcal{M} = 0$, this reduces to the familiar slope of the Phillips curve in a Calvo model with trend inflation

$$\kappa = \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\left(1-\beta\left(1-n\right)\pi^{\frac{\theta}{\eta}}\right)\frac{1-\left(1-n\right)\pi^{\theta-1}}{\left(1-n\right)\pi^{\theta-1}}.$$

The difference between these two slopes, $\mathcal{K}-\kappa$, reflects the inflation accelerator which is positive when $\pi > 0$ and increases with π.

Figure 5 shows how the two slope, $\mathcal{K}$ and κ, vary with trend inflation. In computing these two objects, we let the fraction of price changes n to optimally increase with trend inflation

Figure 5: Trend Inflation and the Slope of Phillips Curve



as in equation (14). The slope of the Phillips curve increases considerably with inflation: as trend inflation increases from 0 to 8%, $\mathcal{K}$ increases from 0.02 to nearly 0.06. Most of this increase is due to the inflation accelerator: as trend inflation increases from 0 to 8%, the slope κ only increases from 0.02 to 0.03. The inflation accelerator thus considerably magnifies the slope of the Phillips curve.

3.3 The Phillips Curve in the Time-Series

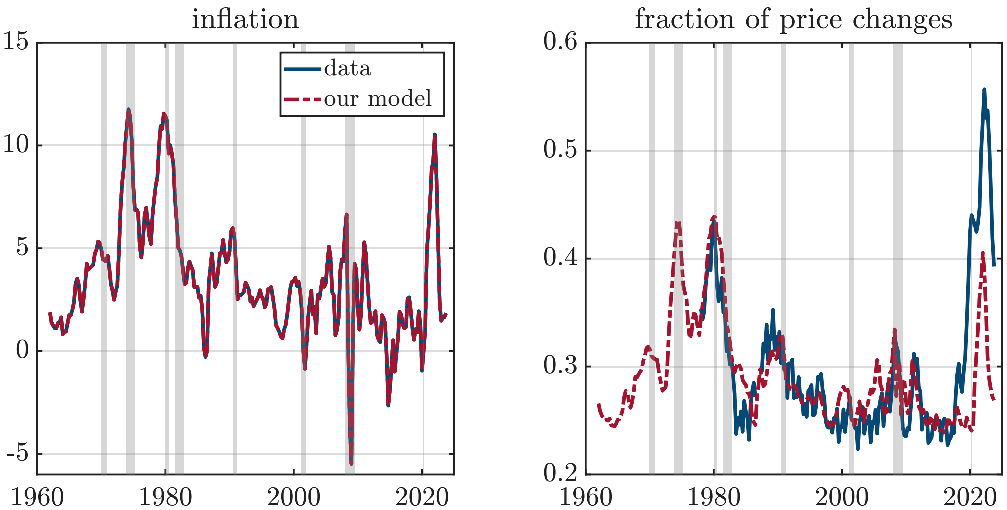
We next investigate how the slope of the Phillips curve evolves in the time series. To that end, we first identify the sequence of monetary shocks that allows the model to exactly reproduce the path of inflation in the data. We then log-linearize around the equilibrium point at each date and show that the slope of the Phillips curve varies considerably, ranging from 0.02 in relatively low-inflation periods to 0.13 in the high-inflation periods of the 1970s and 1980s.

3.3.1 Inflation and the Fraction of Price Changes in the Time Series

We initialize the economy in the stochastic steady state in 1962 and use the non-linear solution of our model, parameterized as described in Section 3.1, and a non-linear solver to back out the monetary policy shocks that reproduce the observed U.S. inflation series. For visual clarity, we target an inflation series that removes high-frequency fluctuations using a 3-quarter centered moving average.[12]

[12] In Appendix C, we show that our results hold if we target instead the raw inflation series.

Figure 6: Inflation and the Fraction of Price Changes



Notes: The gray bars indicate NBER recessions. The left panel plots the annualized quarterly inflation and the right panel plots the quarterly fraction of price changes. Both data series are smoothed with a 3-quarter centered moving average.

Figure 6 shows the path of inflation, which the model matches by construction, and the fraction of price changes in both the model and the data. The model reproduces well the relatively high fraction of price changes in the 1980s and its subsequent decline following the Volcker disinflation. Though the fraction of price changes also increases in our model during the post-Covid spike in inflation, the increase is not as large as in the data. Intuitively, our model predicts a stable relationship between inflation and the fraction of price changes. Since the post-Covid increase in inflation was not as large as that in the 1980s, the model predicts a smaller frequency response.[13] As we show in Appendix C, our model also reproduces the Klenow and Kryvtsov (2008) decomposition of inflation into intensive and extensive margin components, suggesting that it provides a reasonable account of the role of fluctuations in the fraction of price changes for inflation dynamics.

3.3.2 The Slope of the Phillips Curve

We next discuss how the slope of the Phillips curve varies over time. To this end, we log-linearize the equilibrium conditions of the model at each date t and trace out how an incremental increase in marginal costs affects inflation at every point in time. We use hats to

[13] We conjecture that allowing for sectoral shocks that trigger price increases in some sectors and price decreases in others would improve the model's fit because both the fraction of price increases and that of price decreases increased post-Covid (see Montag and Villar, 2023 and Morales-Jimenez and Stevens, 2024).

denote the log-deviation of a variable from the original equilibrium. For example, $\hat{\pi}_t$ denotes the response of inflation to the incremental increase in marginal costs.

As we show in Appendix B, the expression relating inflation to the fraction of price changes and the relative reset price is now

$$\hat{\pi}_t = \underbrace{\frac{1}{(1-n_t)\,\pi_t^{\theta-1}}\frac{\pi_t^{\theta-1}-1}{\theta-1}}_{\mathcal{M}_t}\hat{n}_t + \underbrace{\frac{1-(1-n_t)\,\pi_t^{\theta-1}}{(1-n_t)\,\pi_t^{\theta-1}}}_{\mathcal{N}_t}\hat{p}_t^*.$$

This expression is similar to that in equation (15), which perturbed the economy around the non-stochastic steady state, except that now the actual values of inflation, π_t, and fraction of price changes, n_t, determine how inflation reacts to an increase in the optimal reset price and the fraction of price changes. Once again, if inflation is high in a given period, the elasticity $\mathcal{M}_t$ that determines how inflation responds to an additional increase in the fraction of price changes is high as well. In times of high inflation firms desire larger price changes, so even a small increase in the fraction of price changes leads to a large increase in inflation. Similarly, the expression describing how the fraction of price changes responds to shocks is now

$$\hat{n}_t = \mathcal{A}_t\hat{\pi}_t + \mathcal{B}_t\hat{p}_t^* - \mathcal{C}_t\hat{x}_{t-1} + \frac{n_t-\bar{n}}{n_t}\hat{b}_{1t},$$

where once again the elasticities $\mathcal{A}_t$, $\mathcal{B}_t$ and $\mathcal{C}_t$ vary over time and increase with inflation.

Finally, the slope of the Phillips curve is equal to

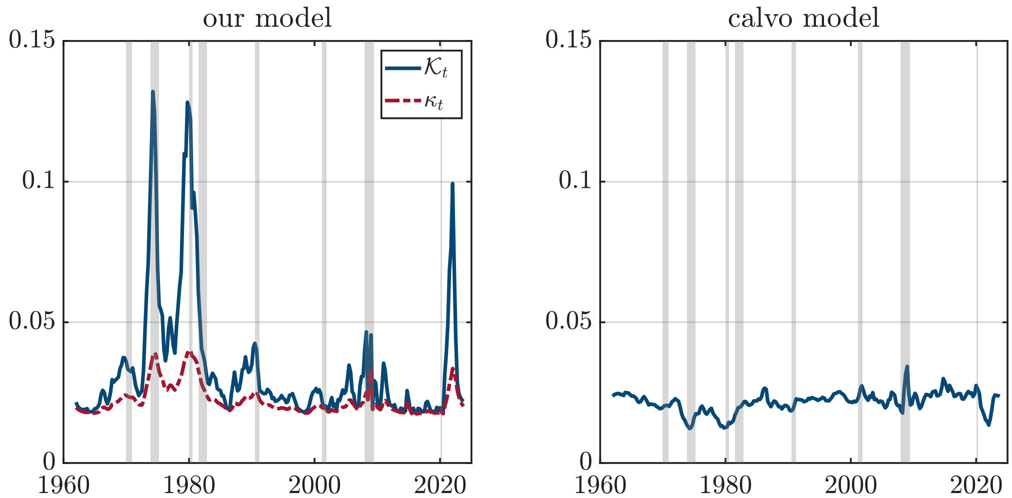
$$\mathcal{K}_t = \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\frac{y_t^{\frac{1}{\eta}}}{b_{2t}}\frac{\mathcal{M}_t\mathcal{B}_t+\mathcal{N}_t}{1-\mathcal{M}_t\mathcal{A}_t}$$

and is similar to that derived in equation (17). We find it useful to compare the slope of the Phillips curve in our model to that in a model with a time-varying fraction of price changes that is constrained not to respond to additional shocks. In this case, $\hat{n}_t = 0$ and the slope is

$$\kappa_t = \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\frac{y_t^{\frac{1}{\eta}}}{b_{2t}}\mathcal{N}_t.$$

This expression captures the fact that an elevated fraction of price changes mechanically increases the slope of the Phillips curve. Thus, the difference between the two slopes, $\mathcal{K}_t - \kappa_t$, captures the inflation accelerator, which now varies over time and reflects the endogenous response of the fraction of price changes as well as its disproportionately larger contribution to aggregate price flexibility in periods of high inflation.

Figure 7: The Slope of the Phillips Curve



Notes: The gray bars indicate NBER recessions.

The left panel of Figure 7 depicts the slope of the Phillips curve $\mathcal{K}_t$ in our model and contrasts it to κ_t. The slope $\mathcal{K}_t$ fluctuates significantly over time, from a low of 0.02 in low-inflation periods and to a high of 0.13 in times of high inflation. Crucially, the inflation accelerator is largely responsible for the steeper slope in high-inflation periods: in its absence, the slope κ_t peaks at just 0.04, only a third of the overall effect. Even though our model does not fully reproduce the sharp increase in the fraction of price changes post-Covid, the slope of the Phillips curve increased by a factor of five, from 0.02 in the first quarter of 2019 to 0.10 in the first quarter of 2022, once again largely due to the inflation accelerator.

For comparison, the right panel of Figure 7 reports the slope predicted by the Calvo model discussed in Section 3.1 in which the fraction of price changes is constant

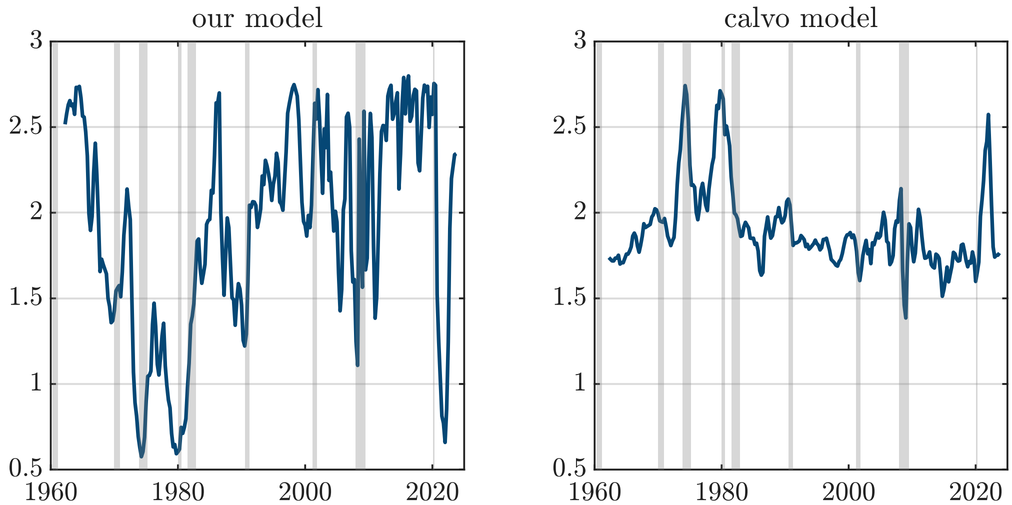
$$\kappa_t^{\text{calvo}} = \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)} \frac{y_t^{\frac{1}{\eta}}}{b_{2t}} \frac{1-(1-\bar{n})\,\pi_t^{\theta-1}}{(1-\bar{n})\,\pi_t^{\theta-1}}.$$

This slope fluctuates much less than in our model and, importantly, decreases in periods of high inflation. This is because the fraction of price changes does not increase with inflation and, as discussed in Section 3.2.2, the price level is less responsive to changes in the reset price when inflation is high, a mechanism that reduces the slope of the Phillips curve.

3.3.3 The Sacrifice Ratio

As in menu cost models, our model predicts state-dependent responses to aggregate shocks, in that both the history of shocks up to a given date and the size of shocks matter for the

Figure 8: Sacrifice Ratio



Notes: The gray bars indicate NBER recessions.

dynamics of prices and output. We illustrate this state-dependence by computing how the tradeoff between stabilizing inflation and output, namely the *sacrifice ratio*, varies over time. Specifically, we ask: what is the drop in output required to reduce inflation by one percentage point during the course of one year? We use the non-linear solution of the model to back out the change in the nominal interest rate necessary to achieve this reduction in inflation and then calculate the average decline in output during the course of the four quarters of that year. We repeat this experiment for every date and report the results in Figure 8.[14]

As the left panel of the figure shows, in periods of low inflation the sacrifice ratio is approximately 2.5. That is, output would have to fall by 2.5% on average over the course of a year for the monetary authority to reduce inflation by one percentage point. When inflation is at its peak, in the 1970s and 1980s, the sacrifice ratio is approximately 0.5. Thus, reducing inflation by one percentage point in that period would have been a lot less costly. Our model also predicts a sharp decline in the sacrifice ratio during the post-Covid inflation surge, from 2.7 prior to the pandemic to approximately 0.6 when inflation was at its peak in 2022.[15] In contrast, the sacrifice ratio fluctuates much less in the Calvo model and, in fact, increases in times of high inflation. Our model therefore implies that if inflation is high to begin with, bringing it down requires a smaller drop in output than if inflation is low.

[14]In Appendix C we report an alternative statistic: the output drop required to bring inflation to 2%.

[15]Ball et al. (1988) use cross-country data to document that the tradeoff between output and inflation decreases with inflation. More recently, Hobijn et al. (2023) show that the sacrifice ratio fell after the onset of the pandemic due to the steepening of the Phillips curve.

3.4 Robustness

In this section, we show that our results are robust to eliminating strategic complementarities in price setting, changing the functional form of the price adjustment cost and adding idiosyncratic shocks to match the dispersion in price changes.

3.4.1 The Role of Strategic Complementarities

In our baseline calibration we assumed a moderate degree of strategic complementarities in pricing by setting $\eta = 2/3$ and $\theta = 6$. Here we gauge the robustness of our results to eliminating strategic complementarities by setting $\eta = 1$. We consider two economies, one in which $\theta = 6$, as in the baseline, and one in which $\theta = 3$. We re-calibrate each of these economies to match the same moments as in the baseline calibration.

The columns labeled $\theta = 6$ and $\theta = 3$ in Panel A of Table 2 show that both economies match the targeted moments exactly. Eliminating strategic complementarities reduces the curvature of the profit function and thus the firms' incentives to adjust prices. Therefore, as Panel B shows, the model requires smaller adjustment costs to match the comovement of the fraction of price changes with inflation.

The top panels of Figure 9 show that eliminating strategic complementarities increases the slope of the Phillips curve considerably, more so when θ is lower. However, our conclusion stands: the slope of the Phillips curve greatly increases in times of high inflation, primarily due to the inflation accelerator.

3.4.2 Alternative Price Adjustment Cost Function

In our baseline model we assumed a quadratic adjustment cost function. Here, we consider a more general specification

$$\frac{\xi}{1+\nu} \left(n_{it} - \bar{n}\right)^{1+\nu},$$

where the parameter $\nu > 0$ governs the convexity of the cost function. This parameter has implications for the convexity of the relationship between inflation and the fraction of price changes: the lower ν is, the more convex this relationship is. We thus calibrate this version of the model by targeting, in addition to the original set of moments in Table 1, the slope coefficient from regressing the fraction of price changes on the square of inflation.

Table 2 shows the results of the parameterization in the column labeled "alt adj cost". As Panel A shows, while the relationship between the fraction of price changes and inflation

Table 2: Calibration: Alternative Parameterizations and Model Specifications

A. Targeted Moments

	Data	$\theta = 6$	$\theta = 3$	alt adj cost	idio
mean inflation	3.517	3.517	3.517	3.517	3.517
s.d. inflation	2.739	2.739	2.739	2.739	2.739
mean frequency	0.297	0.297	0.297	0.297	0.297
slope of n_t on $\lvert\pi_t\rvert$ only	0.016	0.016	0.016	–	0.015
slope of n_t on $\lvert\pi_t\rvert$	0.005	–	–	0.005	–
slope of n_t on $\lvert\pi_t\rvert^2$	0.001	–	–	0.001	–
s.d. price changes	0.129	–	–	–	0.129

B. Calibrated Parameter Values

		$\theta = 6$	$\theta = 3$	alt adj cost	idio
$\log \pi$	inflation target, annual	0.037	0.037	0.039	0.041
σ_R	s.d. monetary shocks	0.011	0.011	0.015	0.019
$\bar{n}$	fraction of free price changes	0.231	0.228	0.261	0.000
ξ	adjustment cost, level	0.323	0.099	0.933	17.716
ν	adjustment cost, elasticity	–	–	0.620	–
σ_z	s.d. idiosyncratic shocks	–	–	–	0.068

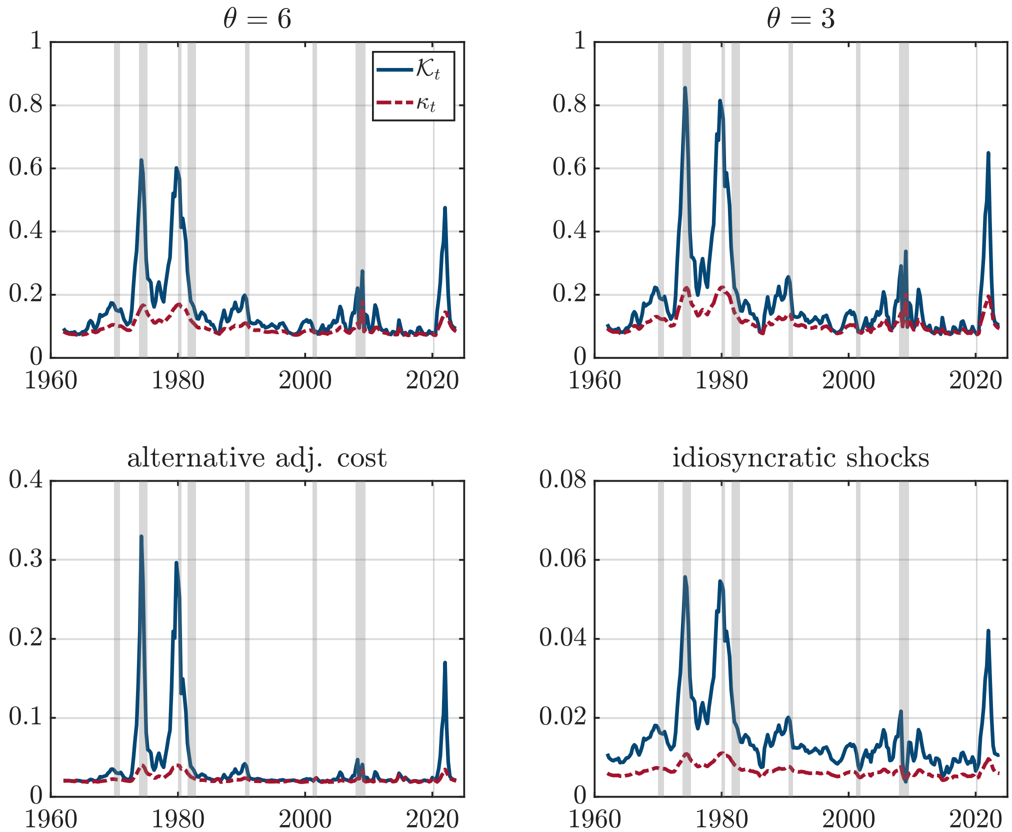
is indeed convex in the data, the coefficient on the quadratic inflation term is small. Thus targeting it only reduces the value of ν from 1 in our baseline to 0.62.

In the bottom-left panel of Figure 9, we illustrate the implications for the slope of the Phillips curve. Because in this version of the model the fraction of price changes increases less with inflation when inflation is low, the slope is stable in these periods. In contrast, high inflation periods are now associated with a larger increase in the slope of the Phillips curve. Thus, our assumption of a quadratic price adjustment cost provides a conservative assessment of how steep the slope of the Phillips curve is, and consequently how strong the inflation accelerator is, in periods of high inflation.

3.4.3 Adding Idiosyncratic Shocks

For clarity of exposition, in our baseline model we abstracted from idiosyncratic shocks. Therefore, that model cannot reproduce the large dispersion in price changes observed in the data. We next extend the model to allow for such shocks and show that our results are

Figure 9: Slope of the Phillips Curve: Alternative Parameterizations and Model Specifications



Notes: The gray bars indicate NBER recessions.

robust. To maintain tractability, we assume that these shocks are at the product level.[16] Since idiosyncratic shocks now generate a motive for price changes even when inflation is zero, we assume that $\bar{n} = 0$, so there are no free price changes.

Environment. The technology used by intermediate good firms is

$$y_{ikt} = z_{ikt} \left(l_{ikt} \right)^{\eta},$$

where z_{ikt} is a product-level shock that evolves according to

$$\log z_{ikt} = \log z_{ikt-1} + \sigma_z \epsilon_{ikt}$$

[16] Assuming, instead, that shocks are to individual firms would require keeping track of the distribution of firm prices, as in menu cost models.

and ϵ_{ikt} is a iid standard normal random variable. Intermediate goods are aggregated into a final good with a CES aggregator

$$y_t = \left(\int_0^1 \int_0^1 \left(\frac{y_{ikt}}{z_{ikt}} \right)^{\frac{\theta-1}{\theta}} \mathrm{d}k\mathrm{d}i \right)^{\frac{\theta}{\theta-1}},$$

so z_{ikt} has the interpretation of quality (Midrigan, 2011, Blanco et al., 2024).

Price Setting and Equilibrium. Adding idiosyncratic shocks requires that we evaluate the price index and misallocation in equations (4) and (5) using the quality-adjusted product prices, $z_{ikt}P_{ikt}$. The firms' price setting problem can then be characterized as in the baseline model. In a symmetric equilibrium, the price index is

$$1 = n_t \left(p_t^*\right)^{1-\theta} + (1-n_t) \exp\left(\frac{\sigma_z^2}{2}(1-\theta)^2 \right) \pi_t^{\theta-1}.$$

The only difference relative to the definition of the price index in equation (9) is the term $\exp\left(\frac{\sigma_z^2}{2}(1-\theta)^2\right)$, which effectively decreases inflation. To see this, notice that if the relative reset price, $p_t^* = 1$, inflation is equal to $\pi_t = \exp\left(-\frac{\sigma_z^2}{2}(\theta-1)\right) < 1$, reflecting that consumers reallocate spending towards cheaper quality-adjusted varieties. A similar adjustment appears in the law of motion of misallocation,

$$x_t^{-\frac{\theta}{\eta}} = n_t \left(p_t^*\right)^{-\frac{\theta}{\eta}} + (1-n_t) \exp\left(\frac{\sigma_z^2}{2} \left(\frac{\theta}{\eta}\right)^2 \right) \left(\frac{x_{t-1}}{\pi_t} \right)^{-\frac{\theta}{\eta}},$$

the optimal fraction of price changes,

$$\xi n_t = b_{1t} \left(\left(p_t^*\right)^{1-\theta} - \exp\left(\frac{\sigma_z^2}{2}(1-\theta)^2 \right) \pi_t^{\theta-1} \right) - \tau b_{2t} \left(\left(p_t^*\right)^{-\frac{\theta}{\eta}} - \exp\left(\frac{\sigma_z^2}{2} \left(\frac{\theta}{\eta}\right)^2 \right) x_{t-1}^{-\frac{\theta}{\eta}} \pi_t^{\frac{\theta}{\eta}} \right),$$

and the expressions for b_{1t} and b_{2t},

$$\begin{aligned} b_{1t} &= 1 + \beta \exp\left(\frac{\sigma_z^2}{2}(1-\theta)^2 \right) \mathbb{E}_t \left((1-n_{t+1}) \pi_{t+1}^{\theta-1} b_{1t+1} \right) \\ b_{2t} &= y_t^{\frac{1}{\eta}} + \beta \exp\left(\frac{\sigma_z^2}{2} \left(\frac{\theta}{\eta}\right)^2 \right) \mathbb{E}_t \left((1-n_{t+1}) \pi_{t+1}^{\frac{\theta}{\eta}} b_{2t+1} \right). \end{aligned}$$

Phillips Curve. The expression for the slope of the Phillips curve is identical to that in the baseline model discussed in Section 3.3.2, except that the terms involving $\pi_t^{\theta-1}$ in the elasticities $\mathcal{A}_t$, $\mathcal{B}_t$, $\mathcal{M}_t$ and $\mathcal{N}_t$ are scaled by $\exp\left(\frac{\sigma_z^2}{2}(1-\theta)^2\right)$ and those involving $\pi_t^{\frac{\theta}{\eta}}$ are scaled by $\exp\left(\frac{\sigma_z^2}{2}\left(\frac{\theta}{\eta}\right)^2\right)$.

Quantification. We assign the same values to the discount factor, returns to scale and demand elasticity as in the baseline calibration. In addition to the moments targeted in the baseline model, we also target the standard deviation of individual price changes, Δp_{ikt}, which is 0.129 in the BLS data (Morales-Jimenez and Stevens, 2024), by appropriately setting the standard deviation of idiosyncratic shocks, σ_z.[17] The last column of Table 2 reports the results of the calibration. The model now requires a larger adjustment cost, ξ, since idiosyncratic shocks increase the marginal benefit of adjusting and also make it more sensitive to inflation.

The bottom-right panel of Figure 9 shows how the slope of the Phillips curve, $\mathcal{K}_t$, evolves over time in this model. As in the baseline, the slope fluctuates considerably, ranging from 0.01 in the 1990s to 0.055 in the 1970s. While idiosyncratic shocks dampen the slope of the Phillips curve because consumers reallocate towards products whose prices have not adjusted, the inflation accelerator is as powerful as in the baseline. For example, absent the inflation accelerator, the slope κ_t peaks at only 0.01 in the 1970s, one-fifth of the overall effect.

4 Implications for Monetary Policy

We next use our model to revisit several themes central to monetary policy analysis. Specifically, we first estimate a richer version of our model to study the drivers of inflation and show that loose monetary policy substantially amplified the inflationary impact of adverse shocks in the late 1970s. We then show that the determinacy region is larger in our model compared to the Calvo model and that the welfare costs of price rigidity are smaller and increase less with trend inflation.

4.1 Historical Analysis

Because our baseline model was purposefully parsimonious so as to transparently illustrate the mechanism, here we study a richer economy. Specifically, we assume that, in addition to shocks to the interest rate rule, firms are also subject to aggregate shocks to productivity and a time-varying tax on labor, which we refer to as a "cost-push" shock. In addition, motivated by the evidence in Coibion and Gorodnichenko (2011), we allow for time-varying Taylor rule coefficients, as well as persistent shocks to the inflation target.

The setup of the model is, to a very large extent, as in the baseline. We highlight next only the new elements. First, we assume a finitely-elastic labor supply with Frisch elasticity

[17] In Appendix D, we derive closed form expressions for the moments of the distribution of price changes.

$1/\gamma$. Second, the technology of intermediate goods producers is

$$y_{ikt} = z_t \left(l_{ikt}\right)^{\eta},$$

where z_t is aggregate productivity and follows an AR(1) process. Third, firms are assumed to face a time-varying labor tax, so their profits from producing a good k are

$$P_{ikt} y_{ikt} - \tau_t W_t l_{ikt},$$

where the tax τ_t also follows an AR(1) process. Lastly, the central bank follows the same Taylor rule as earlier but with time-varying coefficients and inflation target, $\bar{\pi}_t$,

$$\frac{1+i_t}{1+\bar{i}_t} = \left(\frac{1+i_{t-1}}{1+\bar{i}_t}\right)^{\phi_{it}} \left(\left(\frac{\pi_t}{\bar{\pi}_t}\right)^{\phi_{\pi t}} \left(\frac{y_t}{y_{t-1}}\right)^{\phi_{yt}}\right)^{1-\phi_{it}} \varepsilon_t,$$

where

$$1+\bar{i}_t = \frac{\bar{\pi}_t}{\beta}.$$

We estimate the model using quarterly data on inflation, output and the nominal federal funds rate from 1962:Q1 to 2023:Q4.[18] We use the same CPI price index as in the baseline. We use real gross domestic product divided by the civilian labor force for output growth, and the fed funds rate for the nominal interest rate. Since there is no trend growth in output in our model, we de-trend output in the data using a linear trend. For periods when the zero lower bound binds, we substitute the fed funds rate with the shadow interest rate calculated by Wu and Xia (2016). We also use data on the fraction of price changes from Montag and Villar (2023) and Nakamura et al. (2018) that we discussed above.

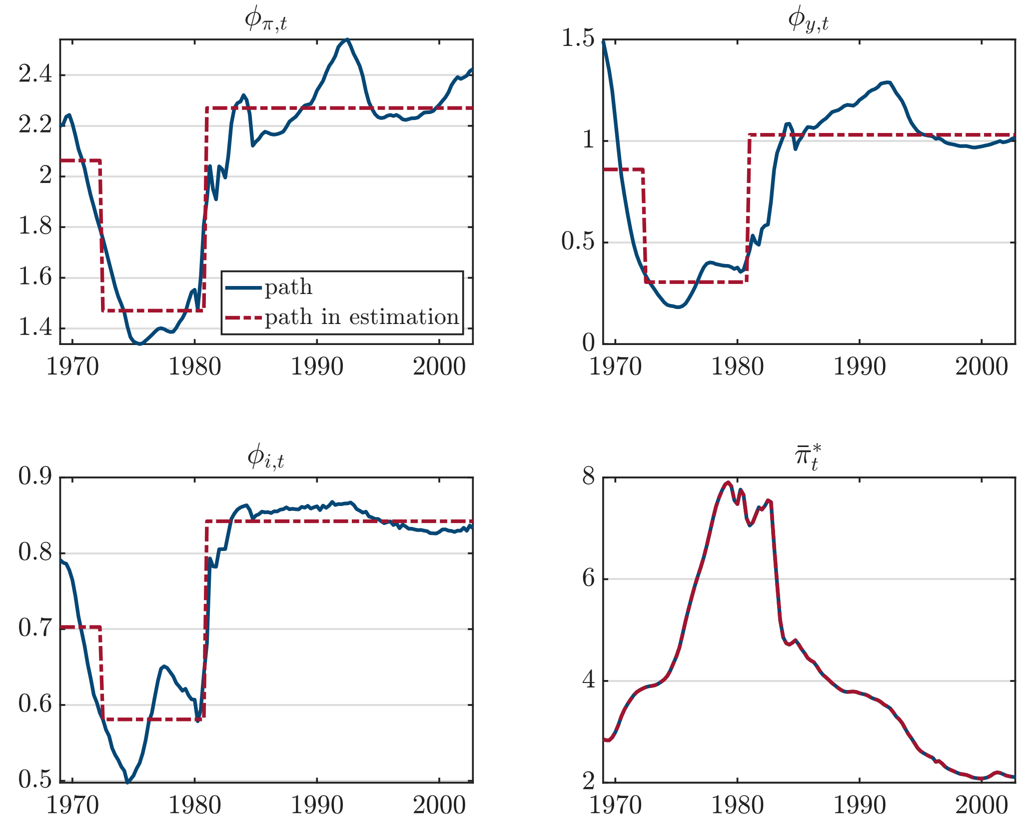
We use the time-varying estimates of the Taylor rule coefficients and the inflation target in Coibion and Gorodnichenko (2011). For computational tractability, we assume that the parameters $\phi_{\pi t}$, ϕ_{yt} and ϕ_{it} were in one of three regimes, illustrated in Figure 10. To identify these regimes, we estimate structural breaks in the time-varying inflation response coefficient, $\phi_{\pi t}$, in Coibion and Gorodnichenko (2011). The break dates are selected to minimize the total residual sum of squares of the path of $\phi_{\pi t}$ relative to the piecewise-constant path. We then apply the same break dates to the other two coefficients, ϕ_{yt} and ϕ_{it}. As the figure shows, the three-regime approximation fits the estimated time-varying coefficients well.

As for the inflation target, we assume that it follows the AR(1) process

$$\bar{\pi}_t = (1-\rho_p)\bar{\pi} + \rho_p \bar{\pi}_{t-1} + \sigma_p \varepsilon_{p,t}$$

[18] We provide more details of the data series that we use in Appendix D.

Figure 10: Time-varying Taylor Rule Coefficients



Notes: The solid lines in each figure are from Coibion and Gorodnichenko (2011). The dash-dotted lines are the paths we use in estimation.

and use the estimated path of the inflation target from Coibion and Gorodnichenko (2011), shown in the bottom-right panel of Figure 10, as an observable in our estimation.[19]

We estimate our model, as well as an otherwise identical Calvo model, using Bayesian likelihood methods. We report the details of the estimation as well as the parameter estimates in Appendix D. We note that the estimates of ξ and $\bar{n}$ are similar to the baseline and that the variance of productivity and cost-push shocks is smaller than in the Calvo model. As we discuss below, this is because our model features important nonlinearities, so it requires less exogenous variation to match the data. We also show in Appendix D that the model's predictions for the dynamics of the slope of the Phillips curve are similar to the baseline.

We first use the estimated model to study the consequences of the elevated slope of the

[19] Relative to Coibion and Gorodnichenko (2011), we also aggregate from the original monthly series to a quarterly one, and use the sum of the two lagged interest rate coefficients as our inertia coefficient. Our sample is also longer than that of Coibion and Gorodnichenko (2011). Thus, we also linearly extrapolate the $\bar{\pi}_t$ series so that our sample starts and ends at an inflation target of 2%. We also use the first and last observations of $\phi_{\pi t}$, ϕ_{yt}, and ϕ_{it} to extend these series.

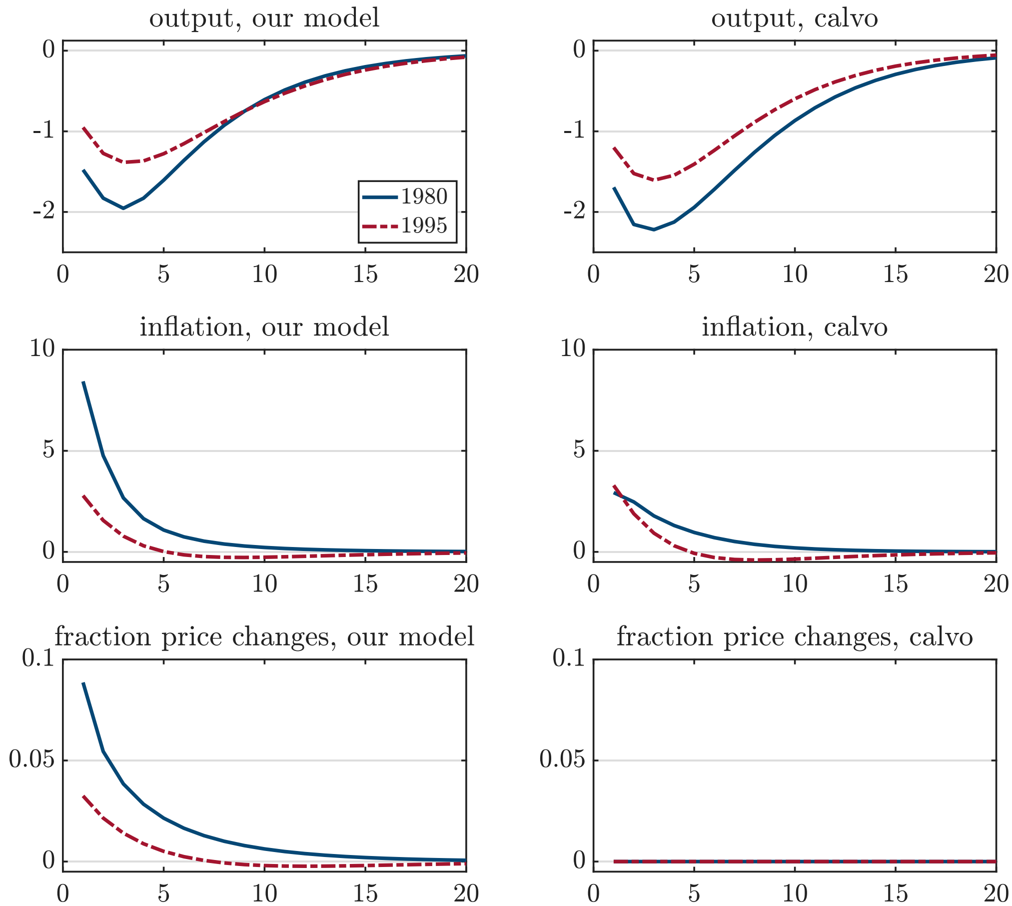
Phillips curve in periods of high inflation for how the economy responds to shocks. We compute the impulse responses of output, inflation and the fraction of price changes to a one standard-deviation cost-push shock in two periods: the first quarter of 1980, when monetary policy was in the accommodative regime, inflation was high (11.7%), as was the slope of the Phillips curve, and the first quarter of 1995, when monetary policy was more aggressive, inflation was much lower (2.8%) and so was the slope of the Phillips curve. To separate the role of the time-varying slope of the Phillips curve from that of the monetary policy stance, we compare the impulse responses to those from the Calvo model.

As Figure 11 shows, in our model the same shock generates a much larger increase in inflation in the 1980s than in the Calvo model, even though the output responses are similar. This is because the slope of the Phillips curve is higher in our model. In contrast, in the low-inflation environment of the 1990s, the responses of inflation and output are similar in the two models because in this period the slope of the Phillips curve is similar. Our model therefore predicts that the economy is more susceptible to negative shocks in periods in which monetary policy is more accommodative, as in the 1970s. In the debate over whether the Great Inflation in the late 1970s reflects primarily bad policy or bad luck, our results point to an interaction: accommodative policy substantially amplifies the inflationary effects of adverse shocks.

We then use the model to study the contribution of the different shocks to inflation in the time series. We compute these decompositions using the model's nonlinear solution. For each period, we first construct a counterfactual inflation path in which all shocks are set to zero. We then generate counterfactual inflation paths by feeding in only one shock at a time. The contribution of each shock is measured as the deviation of the resulting inflation path from the no-shock counterfactual. Because the model is nonlinear, the sum of individual shock contributions does not generally equal the inflation path resulting from the interaction of all four shocks. The residual therefore captures the nonlinear interactions.

Figure 12 plots this decomposition in our model and in the Calvo model. As the left panel shows, our model predicts that cost-push shocks alone were responsible for approximately half of the rise in inflation during the first oil-price shock in the 1970s. In contrast, as the right panel shows, the Calvo model predicts that cost-push shocks account for most of the rise in inflation in this period. As for the second spike in inflation in the late-1970s, shocks to trend inflation were primarily responsible for the rise in both models. Importantly, in both these inflationary episodes, the sum of the contribution of individual shocks in our

Figure 11: State-Dependent Effects of Shocks

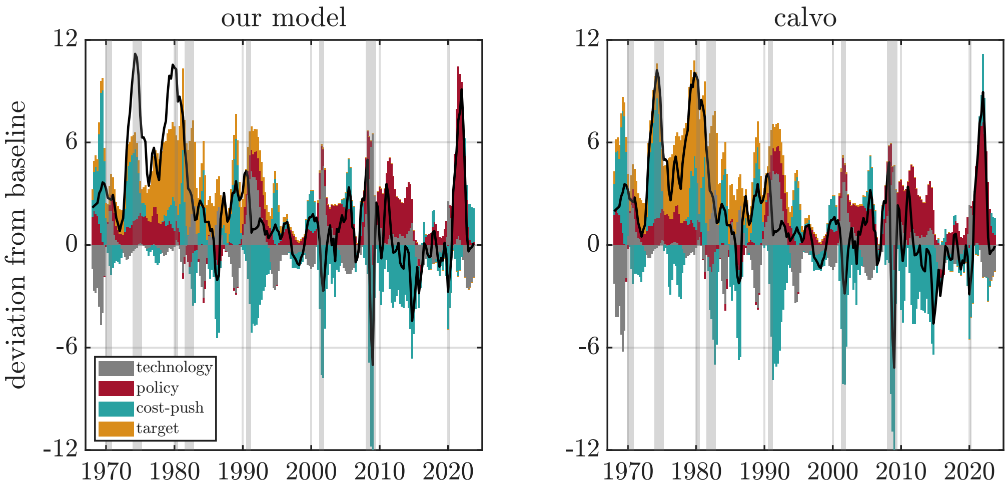


model is smaller than the overall rise in inflation, suggesting an important role for nonlinear interactions. Lastly, both models predict that monetary policy shocks account for almost all of the inflation surge during the pandemic, reflecting the low level of the shadow federal funds rate relative to the rate prescribed by the Taylor rule.

4.2 Determinacy for Different Levels of Trend Inflation

We next investigate the extent to which endogenizing the fraction of price changes alters the determinacy properties of the model for different levels of trend inflation. As is well known (see, e.g., Kiley, 2007 and Coibion and Gorodnichenko, 2011), determinacy requires that the interest rate in equation (1) responds more aggressively to inflation in environments with higher trend inflation. This is because higher trend inflation makes firms' pricing more forward-looking, effectively flattening the Phillips curve, which requires a larger response of real interest rates and therefore the output gap to stabilize inflation expectations.

Figure 12: Drivers of Inflation



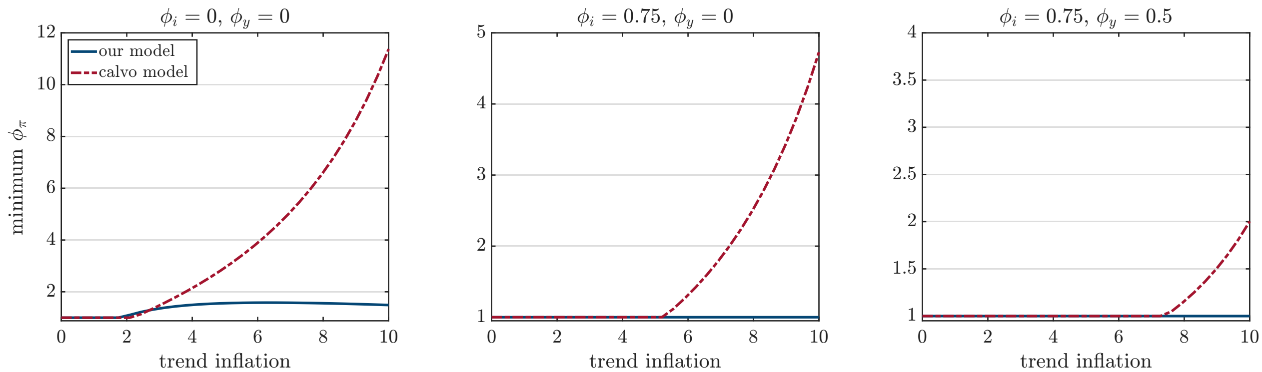
In Figure 13 we plot the minimum value of ϕ_π required for determinacy at different levels of trend inflation π. We contrast the predictions of our baseline model to those of the Calvo model, both parameterized as described in Section 3.1. Because the minimum level of ϕ_π required to achieve determinacy depends on the extent to which the interest rate responds to output and the inertia in the policy rule, we do this for three different scenarios: *(i)* $\phi_i = 0$ and $\phi_y = 0$, *(ii)* $\phi_i = 0.75$ and $\phi_y = 0$, and *(iii)* $\phi_i = 0.75$ and $\phi_y = 0.5$. As the figure shows, the determinacy region is larger in our model with an endogenous fraction of price changes compared to the Calvo model. For example, even when trend inflation is as high as 10% and the Calvo model requires a very aggressive response of interest rates to inflation to achieve determinacy, in our model the response of the interest rate is significantly lower and is close to one. This is because in our model the fraction of price changes increases with trend inflation, steepening the slope of the Phillips curve and requiring a smaller response of real interest rates and therefore the output gap.

4.3 Welfare Analysis

Lastly, we compute the welfare losses associated with inflation and output-gap volatility in our baseline model. In doing this, we follow Woodford (2003) and rely on a second-order approximation of the representative consumer's utility around the flexible-price allocation. We show that in our model the welfare losses are smaller than in the Calvo model.

Let $\mathbb{L}(\pi) = -(1-\beta)\,\mathbb{E}\left(V_t(\pi) - V^F\right)$ denote the welfare losses from price ridigity at trend inflation π, where $V_t(\pi)$ and V^F are the lifetime utility of the representative consumer

Figure 13: Determinacy for Different Values of Trend Inflation



Notes: The figure plots the minimum value of ϕ_π in the Taylor rule in equation (1) required for determinacy. The left panel sets $\phi_i = 0$ and $\phi_y = 0$; the middle panel sets $\phi_i = 0.75$ and $\phi_y = 0$; the right panel sets $\phi_i = 0.75$ and $\phi_y = 0.5$.

in the economy with sticky and flexible prices, respectively, and $\mathbb{E}$ denotes the expectation with respect to the ergodic distribution over the aggregate state variables.[20] Let $V(\pi)$ be the steady state of $V_t(\pi)$. We can decompose the welfare losses into two components

$$\mathbb{L}(\pi) = \underbrace{-(1-\beta)\left(V(\pi) - V^F\right)}_{=\Theta_0(\pi)} + \underbrace{(-1)(1-\beta)\,\mathbb{E}\left(V_t(\pi) - V(\pi)\right)}_{=\Theta_1(\pi)}, \tag{18}$$

where $\Theta_0(\pi)$ captures the losses due to the fact that the steady state is inefficient and $\Theta_1(\pi)$ captures the losses due to fluctuations around the steady state. In what follows, we derive analytical approximations of the welfare losses. For brevity, we do not list terms that are quantitatively unimportant, independent of policy or predetermined at time zero. In Appendix E, we provide full expressions as well as proofs of our results.

To build intuition, we first discuss the welfare losses at zero trend inflation in Proposition 1. To that end, let $\hat{y}_t$ and $\hat{\pi}_t$ denote the log-deviations of output and inflation from their steady-state values and let $\tilde{n}_t = \frac{n_t - \bar{n}}{\bar{n}}$ denote the proportional deviation of the fraction of price changes from $\bar{n}$, the share of free price changes.

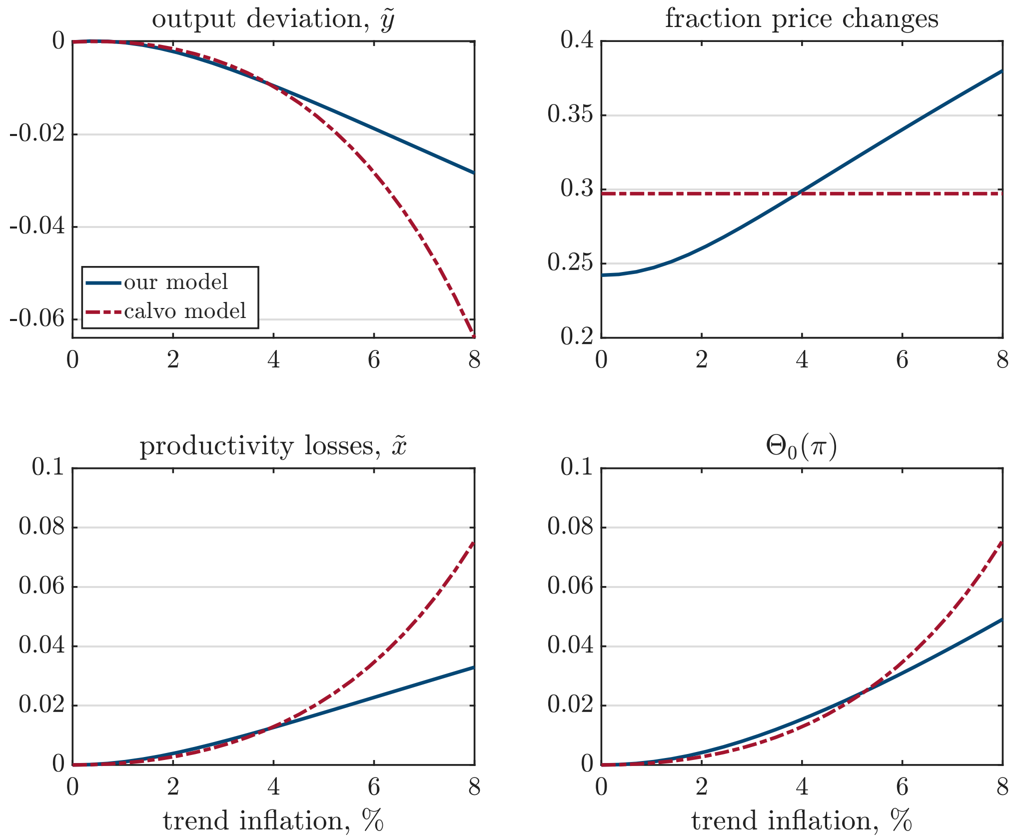
Proposition 1. *At zero trend inflation,* $\Theta_0(1) = 0$ *and*

$$\mathbb{L}(1) = \Theta_1(1) \approx \Gamma_y \mathbb{E}\hat{y}_t^2 + \Gamma_\pi \mathbb{E}\hat{\pi}_t^2 + \Gamma_n \mathbb{E}\tilde{n}_t^2, \tag{19}$$

where $\Gamma_y = \frac{1}{2\eta}$, $\Gamma_n = \frac{\xi \bar{n}^2}{2}$ *and* $\Gamma_\pi = \frac{1}{2}\frac{\theta}{1-\beta(1-\bar{n})}\left(1 + \theta\left(\frac{1}{\eta} - 1\right)\right)\frac{1-\bar{n}}{\bar{n}}$.

[20] Since we consider an economy with monetary shocks only, the flexible price value V_F is time-invariant.

Figure 14: Welfare Costs of Trend Inflation



At zero trend inflation welfare losses arise only due to fluctuations around the steady state. This is because with zero trend inflation the steady-state of our economy is efficient due to the subsidy that corrects the markup distortion. The welfare losses from fluctuations around the steady state arise from three sources, illustrated in each of the terms in equation (19). The first term captures the welfare losses from output gap volatility due to markup fluctuations. The second term reflects the welfare losses from price dispersion induced by fluctuations in inflation. These two terms are identical to those in the Calvo model (see Gali, 2008). The third term captures the costs of adjusting prices.

We next discuss environments with positive trend inflation. We first characterize the steady-state welfare losses, $\Theta_0(\pi)$. We plot these as a function of trend inflation in the bottom-right panel of Figure 14 and contrast them with those predicted by the Calvo model. The welfare losses increase with trend inflation in both models. However, when trend inflation is high, these losses are much larger in the Calvo model. To understand why, in Proposition 2 we derive a quadratic approximation of the welfare losses from steady-state inflation.

Proposition 2. *Let $\tilde{x} = \log x^{-\theta}$ be the productivity losses relative to the flexible price allocation, in which $x = 1$, and let $\tilde{y} = \log y - \eta \log \eta$ be the log-deviation of output from its flexible-price counterpart. Absent aggregate shocks, the welfare losses from price rigidity in a steady state with inflation equal to π are*

$$\Theta_0(\pi) \approx \tilde{x} + \frac{(\tilde{y} + \tilde{x})^2}{2\eta} + \frac{\xi \bar{n}^2}{2} \tilde{n}^2, \tag{20}$$

where

$$\tilde{y} = \eta \left[-\frac{1 + \theta\left(\frac{1}{\eta} - 1\right)}{\theta - 1} \log\left(\frac{1 - (1-n)\pi^{\theta-1}}{n}\right) + \log\left(\frac{1 - \beta(1-n)\pi^{\frac{\theta}{\eta}}}{1 - \beta(1-n)\pi^{\theta-1}}\right)\right]$$

and

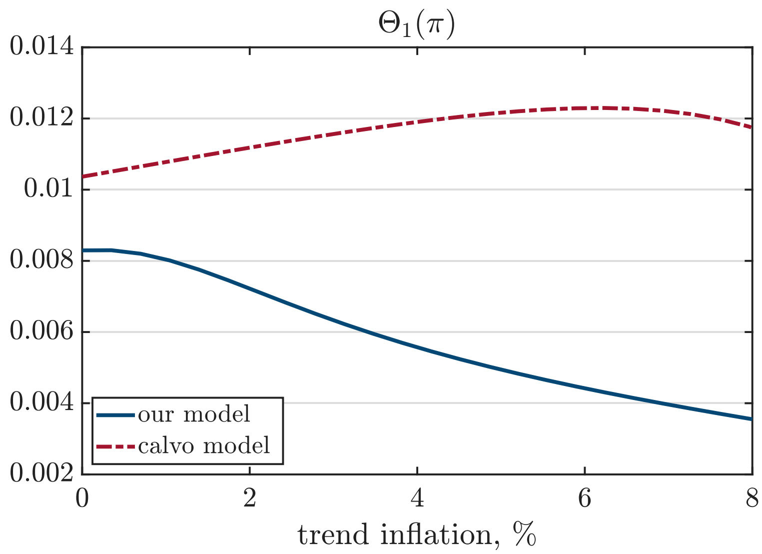
$$\tilde{x} = \frac{\theta}{\theta - 1} \log\left(\frac{1 - (1-n)\pi^{\theta-1}}{n}\right) + \eta \log\left(\frac{n}{1 - (1-n)\pi^{\frac{\theta}{\eta}}}\right).$$

The first term in equation (20) captures the productivity losses from price dispersion, shown in the bottom-left panel of Figure 14. Productivity losses increase with trend inflation, but less so in our model than in the Calvo model, a result reminiscent of Devereux and Yetman (2002), Bakhshi et al. (2007) and the menu cost model of Blanco (2021). This is because, as shown in the top-right panel of the figure, at higher trend inflation a higher share of prices adjust in our model. The second term in equation (20) captures the losses from the deviations of labor from its flexible-price counterpart. The top-left panel of Figure 14 shows that, as trend inflation increases, output falls nearly as much as productivity,[21] implying that labor changes little, so the second term of equation (20) is small. The third term of the equation captures the increased cost of changing prices. Taken together, the bottom panels of Figure 14 show that the bulk of the welfare losses are accounted for by misallocation.

We next characterize the welfare losses due to fluctuations around the steady state, shown in Figure 15 for both our model and the Calvo model. We compute these losses by varying π and leaving all other parameter values unchanged. As the figure shows, the welfare losses from fluctuations are smaller in our model and decline with trend inflation. To understand why that is the case, in Proposition 3 we derive a quadratic approximation of the welfare losses from fluctuations around the steady-state.

[21] As Ascari and Ropele (2009) point out, in the Calvo model the relationship between output and trend inflation is hump-shaped at low rates of inflation. This is also the case in our model: output peaks at a level of approximately 0.02% above its flexible-price (zero trend inflation) level when inflation is equal to 0.5%, but this effect is too small to be visible in the figure.

Figure 15: Welfare Costs of Fluctuations



Proposition 3. *The welfare losses due to fluctuations around the steady-state are*

$$\Theta_1(\pi) \approx \Gamma_y \mathbb{E}\hat{y}_t^2 + \Gamma_\pi \mathbb{E}\hat{\pi}_t^2 + \Gamma_{n\pi}\mathbb{E}\hat{n}_t\hat{\pi}_t + \Gamma_n \mathbb{E}\left(\tilde{n}_t - \tilde{n}\right)^2, \tag{21}$$

where $\Gamma_y = \frac{1}{2\eta}$, $\Gamma_n = \frac{\xi \bar{n}^2}{2}$,

$$\Gamma_\pi = \frac{\eta + \tilde{y} + \tilde{x}}{(1-\beta(1-n)\pi^{\frac{\theta}{\eta}})}\frac{\theta}{2\eta}(1-n)\pi^{\theta-1}\left[\left(1+\theta\left(\frac{1}{\eta}-1\right)\right)\frac{1-(1-n)\pi^{\frac{\theta}{\eta}}}{(1-(1-n)\pi^{\theta-1})^2}(1-n)\pi^{\theta-1}\right.$$
$$\left. -(\theta-1)\frac{1-(1-n)\pi^{\frac{\theta}{\eta}}}{1-(1-n)\pi^{\theta-1}} + \frac{\theta}{\eta}\pi^{1+\theta\left(\frac{1}{\eta}-1\right)}\right] + \frac{\theta^2}{2\eta}(1-n)^2\pi^{2(\theta-1)}\left[-\frac{1-(1-n)\pi^{\frac{\theta}{\eta}}}{1-(1-n)\pi^{\theta-1}} + \pi^{1+\theta\left(\frac{1}{\eta}-1\right)}\right]^2$$

and

$$\Gamma_{n\pi} = -\frac{\eta + \tilde{y} + \tilde{x}}{(1-\beta(1-n)\pi^{\frac{\theta}{\eta}})}\frac{\theta}{\eta}\pi^{\theta-1}n\left[(\pi^{\theta-1}-1)\frac{1+\theta\left(\frac{1}{\eta}-1\right)}{\theta-1}\frac{1-n}{n}\frac{1-(1-n)\pi^{\frac{\theta}{\eta}}}{(1-(1-n)\pi^{\theta-1})^2}\right.$$
$$\left. -\frac{1-(1-n)\pi^{\frac{\theta}{\eta}}}{1-(1-n)\pi^{\theta-1}} + \pi^{1+\theta\left(\frac{1}{\eta}-1\right)}\right] - \theta(1-n)\pi^{\theta-1}\left[-\frac{1-(1-n)\pi^{\frac{\theta}{\eta}}}{1-(1-n)\pi^{\theta-1}} + \pi^{1+\theta\left(\frac{1}{\eta}-1\right)}\right]$$
$$\times\left[\left(1-(1-n)\pi^{\frac{\theta}{\eta}}\right)\left(\frac{\theta}{\eta(\theta-1)}\frac{1-\pi^{\theta-1}}{1-(1-n)\pi^{\theta-1}}-1\right)+n\pi^{\frac{\theta}{\eta}}\right]$$

The first term in equation (21) captures the welfare losses from output-gap volatility. The next two terms capture the impact that inflation fluctuations have on average productivity losses from price dispersion. First, all else equal, inflation volatility increases price dispersion, an effect familiar from the Calvo model (see Woodford, 2003, Coibion et al., 2012). Second, the increase in inflation leads to an increase in the fraction of price changes, reducing misallocation, an effect captured by the covariance between inflation and the fraction of price changes. The last term captures the cost of price changes.

We illustrate the quantitative contribution of each term in Figure 16 for both our model and the Calvo model. We make four points. First, because the slope of the Phillips curve increases with trend inflation in our model, the variance of output-gap fluctuations declines as trend inflation rises. The opposite is true in the Calvo model where, as discussed above, the slope of the Phillips curve decreases with inflation. Thus, and in contrast to the Calvo model, the overall costs from output-gap fluctuations decrease with trend inflation in our model. Second, the welfare costs of inflation alone, $\Gamma_\pi \, \mathbb{E}\hat{\pi}_t^2$, are nearly the same in our model and in the Calvo model. Because our model has a higher steady-state fraction of price changes, the weight Γ_π on inflation volatility decreases with trend inflation, an effect that is offset by the increase in the volatility of inflation. Third, the welfare costs of inflation volatility in our model are offset by the positive comovement between inflation and the fraction of price changes. Intuitively, an increase in inflation in response to shocks is associated with an increase in the fraction of price changes, which reduces misallocation. Fourth, higher trend inflation increases the resource costs of changing prices. Overall, the reduction in misallocation due to a higher fraction of price changes is the dominant force and explains why the welfare costs due to fluctuations decrease with trend inflation in our model.

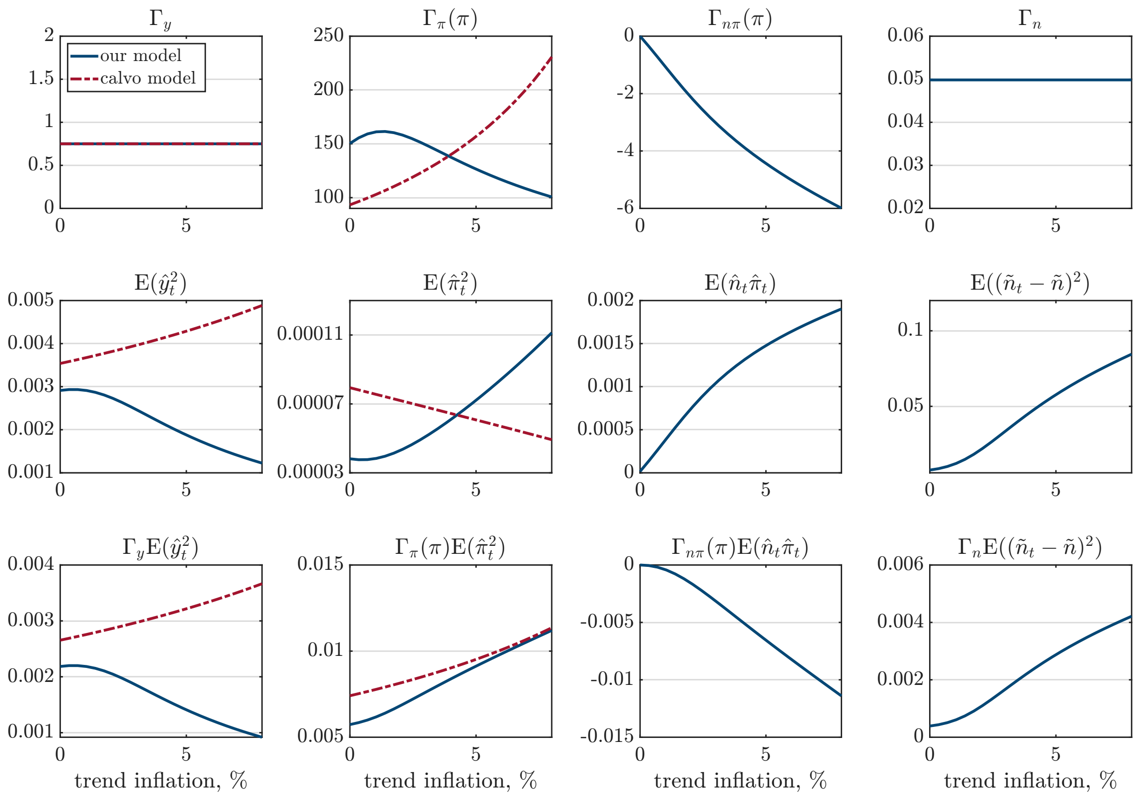
We thus conclude that the welfare losses from inflation are smaller in our model with an endogenous fraction of price changes. That these losses are smaller in our model does not imply that more inflation is desirable from an optimal policy standpoint. This is because in our model the benefits from more inflation are also smaller since more inflation is associated with a steeper Phillips curve. The analysis of optimal monetary policy in environments with an endogenous fraction of price changes is an exciting avenue for future work.[22]

5 Conclusions

A widely documented fact is that the fraction of price changes increases in periods of high inflation. We developed a tractable sticky price model in which the fraction of price changes

[22]For example, Karadi et al. (2024a) argue that optimal monetary policy in a menu cost economy should leverage the lower sacrifice ratio when inflation is high to respond aggressively to a cost-push shock.

Figure 16: Decomposition of Welfare Costs of Fluctuations



varies endogenously over time and increases in times of high inflation. Tractability stems from assuming that firms sell a continuum of products and choose how many, but not which, prices to adjust each period. This eliminates the need to keep track of the price distribution, so our model admits exact aggregation and reduces to a one-equation extension of the Calvo model. The endogenous response of the fraction of price changes to shocks implies a powerful feedback loop between inflation and the fraction of price changes, which we refer to as the *inflation accelerator*. One one hand, an increase in the fraction of price changes increases inflation, more so the higher is inflation to begin with. On the other hand, a increase in inflation increases the benefits to adjusting prices and thus the fraction of price changes.

When applied to the U.S. data, the model predicts that the slope of the Phillips curve fluctuates considerably, with the slope in the 1970s–80s roughly five times larger that in the 1990s. The inflation accelerator is responsible for the bulk of this difference. Our findings imply that the tradeoff between inflation and output stabilization is also time-varying: reducing inflation from 10% to 9% is a lot less costly than reducing it from 3% to 2%. Our model also predicts that loose monetary policy substantially amplifies the inflationary effects of adverse shocks, that the determinacy region is much wider in environments with high trend inflation and that the welfare costs of price rigidity are smaller compared to the Calvo model. Because our model is highly tractable, it can be relatively easily extended to incorporate richer sources of aggregate and sectoral shocks, incorporate additional frictions and be used in quantitative and policy analysis.

References

Alvarez, Fernando and Francesco Lippi, "Price Setting With Menu Cost for Multiproduct Firms," *Econometrica*, 2014, *82* (1), 89–135.

_ **, Hervé Le Bihan, and Francesco Lippi**, "The Real Effects of Monetary Shocks in Sticky Price Models: A Sufficient Statistic Approach," *American Economic Review*, October 2016, *106* (10), 2817–51.

_ **, Martin Beraja, Martín Gonzalez-Rozada, and Pablo Andres Neumeyer**, "From Hyperinflation to Stable Prices: Argentina's Evidence on Menu Cost Models," *The Quarterly Journal of Economics*, 09 2018, *134* (1), 451–505.

Ascari, Guido, Alexandre Carrier, Emanuel Gasteiger, Alex Grimaud, and Gauthier Vermandel, "Monetary Policy in the Euro Area, when Phillips Curves ... are Curves," CEPR Discussion Paper 20489, Centre for Economic Policy Research (CEPR) 2025.

_ **and Tiziano Ropele**, "Trend Inflation, Taylor Principle, and Indeterminacy," *Journal of Money, Credit and Banking*, 2009, *41* (8), 1557–1584.

Auclert, Adrien, Rodolfo Rigato, Matthew Rognlie, and Ludwig Straub, "New Pricing Models, Same Old Phillips Curves?," *The Quarterly Journal of Economics*, 09 2023, *139* (1), 121–186.

Bakhshi, Hasan, Hashmat Khan, Pablo Burriel-Llombart, and Barbara Rudolf, "The New Keynesian Phillips Curve Under Trend Inflation and Strategic Complementarity," *Journal of Macroeconomics*, 2007, *29* (1), 37–59.

Ball, Laurence M and Sandeep Mazumder, "Inflation Dynamics and the Great Recession," *Brookings Papers on Economic Activity*, 2011, pp. 337–81.

Ball, Laurence, N. Gregory Mankiw, and David Romer, "The New Keynesian Economics and the Output-Inflation Trade-Off," *Brookings Papers on Economic Activity*, 1988, *1988* (1).

Blanco, Andres, "Optimal Inflation Target in an Economy with Menu Cost and Zero Lower Bound," *American Economic Journal: Macroeconomics*, 2021, *13* (3), 108–141.

_ **, Corina Boar, Callum Jones, and Virgiliu Midrigan**, "Non-linear Inflation Dynamics in Menu Cost Economies," Technical Report, NBER 2024.

Burstein, Ariel and Christian Hellwig, "Welfare Costs of Inflation in a Menu Cost Model," *American Economic Review*, May 2008, *98* (2), 438–43.

Calvo, Guillermo A., "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, 1983, *12* (3), 383–398.

Caplin, Andrew S. and Daniel F. Spulber, "Menu Costs and the Neutrality of Money," *The Quarterly Journal of Economics*, 1987, *102* (4), 703–726.

Carvalho, Carlos and Oleksiy Kryvtsov, "Price Selection," *Journal of Monetary Economics*, 2021, *122*, 56–75.

Coibion, Olivier and Yuriy Gorodnichenko, "Monetary policy, trend inflation, and the great moderation: An alternative interpretation," *American Economic Review*, 2011, *101* (1), 341–370.

_ , _ , and Johannes Wieland, "The Optimal Inflation Rate in New Keynesian Models: Should Central Banks Raise Their Inflation Targets in Light of the Zero Lower Bound?," *The Review of Economic Studies*, 03 2012, *79* (4), 1371–1406.

Costain, James, Anton Nakov, and Borja Petit, "Flattening of the Phillips Curve with State-Dependent Prices and Wages," *The Economic Journal*, 2022, *132* (642).

Devereux, Michael B. and James Yetman, "Menu Costs and the Long-Run Output–Inflation Trade-off," *Economics Letters*, 2002, *76* (1), 95–100.

Dotsey, Michael, Robert G. King, and Alexander L. Wolman, "State-Dependent Pricing and the General Equilibrium Dynamics of Money and Output," *The Quarterly Journal of Economics*, 1999, *114* (2), 655–690.

Fisher, Timothy CG and Jerzy D Konieczny, "Synchronization of Price Changes by Multiproduct Firms: Evidence from Canadian Newspaper Prices," *Economics Letters*, 2000, *68* (3), 271–277.

Flynn, Joel P., George Nikolakoudis, and Karthik A. Sastry, "A Theory of Supply Function Choice and Aggregate Supply," Technical Report, NBER 2024.

Gagliardone, Luca, Mark Gertler, Simone Lenzu, and Joris Tielens, "Micro and Macro Cost-Price Dynamics in Normal Times and During Inflation Surges," Working Paper 33478, National Bureau of Economic Research February 2025.

Gagnon, Etienne, "Price Setting during Low and High Inflation: Evidence from Mexico," *The Quarterly Journal of Economics*, 08 2009, *124* (3), 1221–1263.

Gali, Jordi, *Monetary Policy, Inflation, and the Business Cycle*, Princeton University Press, 2008.

Gasteiger, Emanuel and Alex Grimaud, "Price Setting Frequency and the Phillips Curve," *European Economic Review*, 2023, *158*.

Gelman, Andrew and Donald B. Rubin, "Inference from iterative simulation using multiple sequences," *Statistical Science*, 1992, *7* (4), 457–472.

Gertler, Mark and John Leahy, "A Phillips Curve with an Ss Foundation," *Journal of Political Economy*, 2008, *116* (3), 533–572.

Golosov, Mikhail and Robert Lucas, "Menu Costs and Phillips Curves," *Journal of Political Economy*, 2007, *115* (2), 171–199.

Greenwald, Daniel, "The Mortgage Credit Channel of Macroeconomic Transmission," Technical Report, NBER 2018.

Hazell, Jonathon, Juan Herreno, Emi Nakamura, and Jon Steinsson, "The Slope of the Phillips Curve: Evidence from U.S. States," *The Quarterly Journal of Economics*, 02 2022, *137* (3), 1299–1344.

Hobijn, Bart, Russell Miles, James Royal, and Jing Zhang, "The Recent Steepening of Phillips Curves," *Chicago Fed Letter*, January 2023.

Justiniano, Alejandro and Giorgio E. Primiceri, "Potential and Natural Output," Technical Report, Federal Reserve Bank of Chicago 2008.

Karadi, Peter and Adam Reiff, "Menu Costs, Aggregate Fluctuations, and Large Shocks," *American Economic Journal: Macroeconomics*, July 2019, *11* (3), 111–46.

_ , Anton Nakov, Galo Nuño Barrau, Ernesto Pasten, and Dominik Thaler, "Strike while the Iron is Hot: Optimal Monetary Policy with a Nonlinear Phillips Curve," Technical Report August 2024.

_ , Raphael Schoenle, and Jesse Wursten, "Price Selection in the Microdata," *Journal of Political Economy Macroeconomics*, 2024, *2* (2), 228–271.

Kehoe, Patrick and Virgiliu Midrigan, "Prices Are Sticky After All," *Journal of Monetary Economics*, 2015, *75*, 35–53.

Kiley, Michael T., "Endogenous Price Stickiness and Business Cycle Persistence," *Journal of Money, Credit and Banking*, 2000, *32* (1), 28–53.

Kiley, Michael T, "Is Moderate-to-High Inflation Inherently Unstable?," *International Journal of Central Banking*, 2007.

Klenow, Peter J. and Oleksiy Kryvtsov, "State-Dependent or Time-Dependent Pricing: Does it Matter for Recent U.S. Inflation?," *The Quarterly Journal of Economics*, 08 2008, *123* (3), 863–904.

Levy, Daniel, Mark Bergen, Shantanu Dutta, and Robert Venable, "The Magnitude of Menu Costs: Direct Evidence From Large U. S. Supermarket Chains," *The Quarterly Journal of Economics*, 1997, *112* (3), 791–825.

Luo, Shaowen and Daniel Villar, "The Price Adjustment Hazard Function: Evidence from High Inflation Periods," *Journal of Economic Dynamics and Control*, 2021, *130.*

Midrigan, Virgiliu, "Menu Costs, Multi-product Firms, and Aggregate Fluctuations," *Econometrica*, 2011, *79* (4), 1139–1180.

Montag, Hugh and Daniel Villar, "Price-Setting During the Covid Era," Technical Report, Board of Governors of the Federal Reserve System August 2023.

Morales-Jimenez, Camilo and Luminita Stevens, "Price Rigidities in U.S. Business Cycles," *Working paper*, 2024.

Nakamura, Emi and Jon Steinsson, "Five Facts about Prices: A Reevaluation of Menu Cost Models," *Quarterly Journal of Economics*, 2008, *123* (4), 1415–1464.

_ , Jón Steinsson, Patrick Sun, and Daniel Villar, "The Elusive Costs of Inflation: Price Dispersion during the U.S. Great Inflation," *The Quarterly Journal of Economics*, 08 2018, *133* (4), 1933–1980.

Peltzman, Sam, "Prices Rise Faster than They Fall," *Journal of Political Economy*, 2000, *108* (3).

Reiter, Michael and Adrian Wende, "Generalized Rotemberg Price-Setting," CESifo Working Paper Series 11297 2024.

Romer, David, "Staggered Price Setting with Endogenous Frequency of Adjustment," *Economics Letters*, 1990, *32* (3), 205–210.

Rotemberg, Julio J., "Sticky Prices in the United States," *Journal of Political Economy*, 1982, *90* (6), 1187–1211.

Sara-Zaror, Francisca, "Inflation, Price Dispersion, and Welfare: The Role of Consumer Search," Technical Report 4127502, SSRN October 2025.

Taylor, John B., "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy*, 1980, *88* (1), 1–23.

Woodford, Michael, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, 2003.

Wu, Jing Cynthia and Fan Dora Xia, "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound," *Journal of Money, Credit, and Banking*, 2016, *48*, 253–291.

Appendix

For Online Publication

A Evidence on Price Setting in Multi-Product Firms

We assume in our model that variation in the aggregate fraction of price changes arises because of variation in the fraction of prices that each individual multi-product firm adjusts, and not because of variation in the share of firms that adjust all of their prices. In this section, we use the NielsenIQ microdata to provide evidence for this assumption.

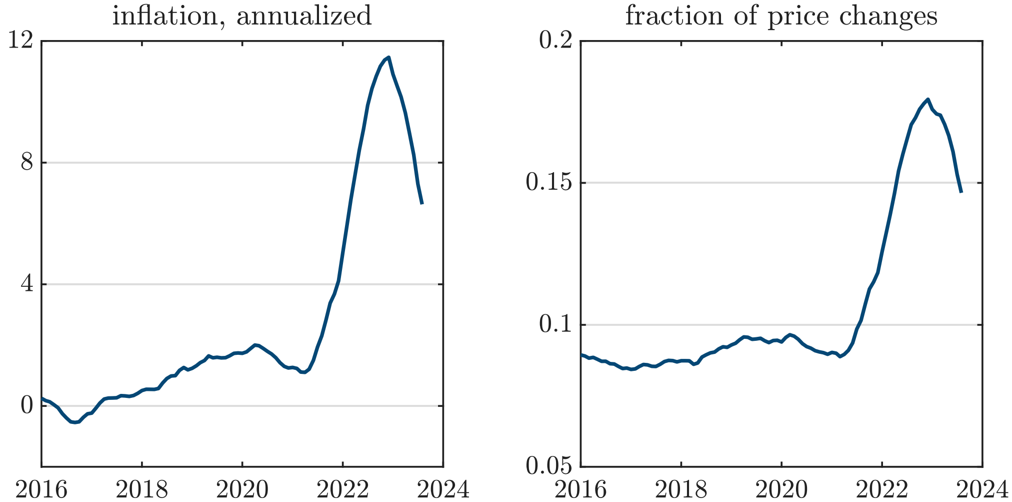
Dataset Description. The NielsonIQ Retail Scanner Dataset is comprised of weekly pricing, volume, and store-level data from post-of-sales (POS) systems for grocery stores, drug stores, and mass merchandisers. The dataset represents over 90 participating retail chains, comprised of about 65,000 stores across all U.S. markets. Weekly product data include UPC codes, units, and price, among other fields. The weekly data equate to about 4.5 million UPCs combined for food, nonfood grocery items, health, and beauty care aids, and selected general merchandise. These are aggregated into 1,100 product categories, which are futher categorized into 125 product groups and 10 departments.

Data Cleaning Steps. To make the dataset more manageable, we restrict the sample to the post-2014 period, and use observations from the last week of every month. We process the data store-by-store and apply the following filters:

- We first drop UPC observations that are missing product information or belong to certain categories for which UPC information may represent multiple products, including magnet data, packaged meats, bread and baked goods, fresh produce, non-food products (such as cookware, electronics, and diapers). We also drop 'remaining' product categories (such as 'CANNED FRUIT - REMAINING') and certain perishables.
- The price is rounded to the nearest cent.
- Next, we construct contiguous price spells at the UPC-store level. A new spell is assumed to occur when the UPC appears in the dataset for the first time, or when there is a gap of more than two months.[23] We then treat each UPC-spell-store as the unit of observation.
- We filter any UPCs with less than six months of data and store-periods for which there are fewer than 10 products sold by that store.
- As in Sara-Zaror (2025), we then filter the UPC posted prices for sales to obtain regular prices by first filtering out for up to 3 month price spikes, as in Nakamura and Steinsson (2008) and Blanco et al. (2024), second, applying the Kehoe and Midrigan (2015) sales

[23] To prevent spuriously short spells, we fill one month gaps with its observation in the previous month while setting that observation's sales in that month to zero, so that the filled price does not affect the aggregated statistics.

Figure A.1: Inflation and the Fraction of Price Changes in NielsenIQ



Notes: We use a 12-month moving average to smooth out variation due to seasonality.

filter. Finally, we require price decreases to persist for at least 2 months before updating the regular price.

- We finally drop the first three and last three regular price observations.

Using the regular prices, we compute log price changes and store-level inflation as the sales-weighted sum of log price changes. We then drop price changes of less than .1% and remove outliers (price changes in the top and bottom 2% of observations). Using these price changes, we compute the sales-weighted fraction of price changes at the product-store-month level. At this level of aggregation, we drop thinly-populated products (those with less than 20 UPCs), and products with price adjustment frequencies greater than 0.75 each month. We then aggregate across products at the store level using sales weights to get a store-month frequency of adjustment.

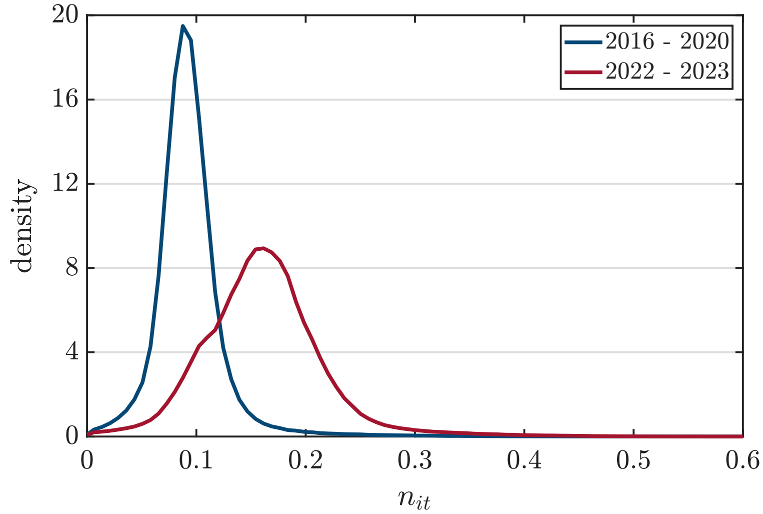
Following these procedures, the dataset on which we conduct our synchronization analysis consists of store-month inflation rate, the store-month price frequency of adjustment, and the store-month sales, for approximately 65,000 stores.

Results. In Figure A.1 we plot the time-series of average inflation and fraction of price changes, weighing each store by its sales in a given period. As the figure shows, the increase in inflation in the 2022-2023 period was associated with a nearly doubling of the average fraction of price changes.

We next split the sample into two sub-periods: one with relatively low inflation (2016-2020) and one with relatively high inflation (2022-2023). In Figure A.2 we plot the distribution of the fraction of price changes across stores and months, for each of these sub-periods.[24]

[24]Because of seasonality and permanent differences in the fraction of price changes across stores, we residualize the data by removing store and calendar month fixed effects and add back the average fraction of price changes.

Figure A.2: Distribution of Fraction of Price Changes in NielsenIQ



As the figure shows, both distributions are unimodal, suggesting that stores do not generally change all their prices at once. More importantly, in the high-inflation period, the distribution of the fraction of price changes shifts to the right, suggesting that all stores change a larger fraction of their prices, consistent with our model.

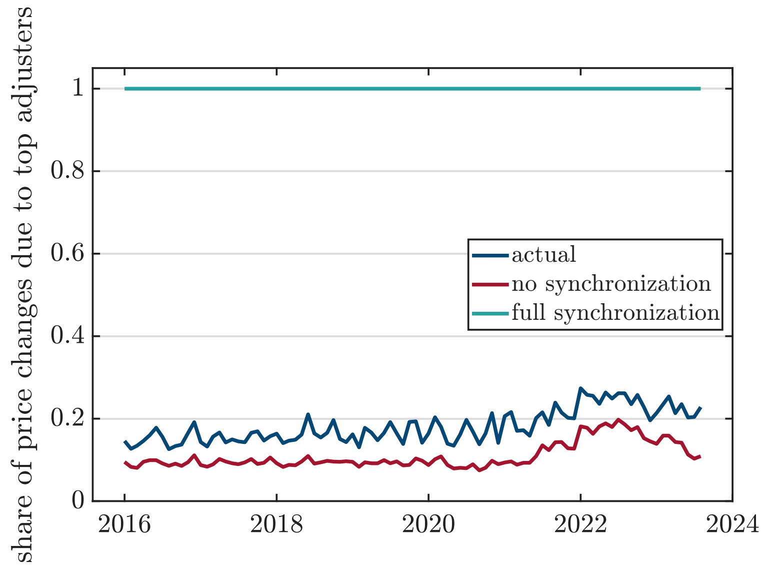
We finally provide systematic evidence in favor of our assumption by computing two measures of synchronization in price changes. The first is the statistic proposed by Fisher and Konieczny (2000), namely the standard deviation of the fraction of price changes across time for a given store, relative to the standard deviation implied by full synchronization. Letting n_{it} denote the fraction of price changes for store i in month t and n_i the corresponding time-series average, this statistic is equal to

$$\sqrt{\frac{\frac{1}{T}\sum_{t=1}^{T}\left(n_{it}-n_i\right)^2}{n_i\left(1-n_i\right)}}.$$

Intuitively, the denominator $n_i\left(1-n_i\right)$ is the variance of a binomial random variable with mean n_i, so if the store i would have adjusted all of its prices in a fraction n_i of the periods, then the numerator, which is the variance of n_{it}, would be equal to the denominator and the statistic would be equal to 1. If, in contrast, price changes were perfectly staggered, so that $n_{it}=n_i$, the numerator and therefore the statistic would be equal to 0. In other words, a value of this statistic close to zero indicates lack of synchronization of price changes within a store, as in our model, while a value close to one indicated strong synchronization. Since this statistic assumes a stationary environment (i.e. a constant aggregate fraction of price changes), we first remove time fixed effects from n_{it} and add back the average fraction of price changes. We find that in the NielsenIQ sample, this statistic ranges from a 10^{th} percentile of 0.05 to a 90^{th} percentile of 0.21, with a median of 0.09, suggesting very little synchronization of price changes within a store.

We next propose a second measure of synchronization that more directly speaks to the question of what accounts for the increase in the average fraction of price changes across

Figure A.3: Share of Price Changes Due to Top Adjusters



stores. For each month t, we compute the (sales-weighted) fraction of prices that change, n_t. We then rank stores within that month in descending order by their fraction of price changes and identify the smallest set of ”top adjusters” whose cumulative sales share is equal to n_t. For this set of stores, we compute the share of all price changes that they account for in that month. If price changes in a given store were fully synchronized, so that $n_{it} \in \{0, 1\}$, then these top adjusters would account for all of the price changes observed in the data. If, in contrast, the fraction of price changes were the same in all stores, as in our model, the top adjusters would only account for a share n_t of price changes. Our second statistic therefore ranges from n_t to 1, with the lower bound indicating no synchronization and the upper bound indicating full synchronization. Figure A.3 plots the evolution of this statistic over time. The share of price changes accounted for by top adjusters is very close to the lower bound of no synchronization, even during the post-Covid high-inflation period.

B Detailed Derivations

Here we provide detailed derivations of the main results discussed in text.

B.1 Steady State

We start by characterizing how the key equilibrium variables depend on the steady-state inflation rate, denoted by π. In steady state, the equilibrium conditions are

$$(p^*)^{1-\theta} = \frac{1 - (1-n)\,\pi^{\theta-1}}{n}, \tag{22}$$

$$x^{-\frac{\theta}{\eta}} = \frac{n\,(p^*)^{-\frac{\theta}{\eta}}}{1 - (1-n)\,\pi^{\frac{\theta}{\eta}}}, \tag{23}$$

$$(p^*)^{1+\theta\left(\frac{1}{\eta}-1\right)} = \frac{1}{\eta}\frac{b_2}{b_1}, \tag{24}$$

$$b_1 = \frac{1}{1-\beta(1-n)\pi^{\theta-1}}, \tag{25}$$

$$b_2 = \frac{y^{\frac{1}{\eta}}}{1-\beta(1-n)\pi^{\frac{\theta}{\eta}}}, \tag{26}$$

$$\xi(n-\bar{n}) = b_1\left((p^*)^{1-\theta} - \pi^{\theta-1} - \tau\frac{b_2}{b_1}\left((p^*)^{-\frac{\theta}{\eta}} - x^{-\frac{\theta}{\eta}}\pi^{\frac{\theta}{\eta}}\right)\right). \tag{27}$$

We first derive an expression for p and x as a function of n and π. Combining equations (24), (25) and (26) implies that

$$(p^*)^{1+\theta\left(\frac{1}{\eta}-1\right)} = \frac{1}{\eta}\frac{1-\beta(1-n)\pi^{\theta-1}}{1-\beta(1-n)\pi^{\frac{\theta}{\eta}}}y^{\frac{1}{\eta}}. \tag{28}$$

Using equation (22), we have that output satisfies

$$y^{\frac{1}{\eta}} = \eta\frac{1-\beta(1-n)\pi^{\frac{\theta}{\eta}}}{1-\beta(1-n)\pi^{\theta-1}}\left(\frac{n}{1-(1-n)\pi^{\theta-1}}\right)^{\frac{1+\theta\left(\frac{1}{\eta}-1\right)}{\theta-1}}. \tag{29}$$

To find the losses from misallocation, combine equations (22) and (23) and write

$$x^{-\frac{\theta}{\eta}} = \frac{n}{1-(1-n)\pi^{\frac{\theta}{\eta}}}\left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right)^{\frac{1}{\eta}\frac{\theta}{\theta-1}},$$

which implies

$$x^{\theta} = \left(\frac{1-(1-n)\pi^{\frac{\theta}{\eta}}}{n}\right)^{\eta}\left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right)^{-\frac{\theta}{\theta-1}}.$$

To find n we use equation (28), which can be rearranged as

$$\xi(n-\bar{n}) = \frac{1}{1-\beta(1-n)\pi^{\theta-1}}\left((1-\tau\eta)(p^*)^{1-\theta} - \pi^{\theta-1} + \tau\eta(p^*)^{1+\theta\left(\frac{1}{\eta}-1\right)}x^{-\frac{\theta}{\eta}}\pi^{\frac{\theta}{\eta}}\right),$$

or, using equations (22) and (23),

$$\xi(n-\bar{n}) = \frac{1}{1-\beta(1-n)\pi^{\theta-1}}\left((1-\tau\eta)\frac{1-(1-n)\pi^{\theta-1}}{n} - \pi^{\theta-1} + \tau\eta\frac{1-(1-n)\pi^{\theta-1}}{n}\frac{n\pi^{\frac{\theta}{\eta}}}{1-(1-n)\pi^{\frac{\theta}{\eta}}}\right).$$

Since

$$\frac{1-(1-n)\pi^{\theta-1}}{n} - \pi^{\theta-1} = \frac{1-\pi^{\theta-1}}{n},$$

this expression simplifies to

$$\xi(n-\bar{n}) = \frac{1}{1-\beta(1-n)\pi^{\theta-1}}\frac{1}{n}\left(1-\pi^{\theta-1} - \tau\eta\frac{1-(1-n)\pi^{\theta-1}}{1-(1-n)\pi^{\frac{\theta}{\eta}}}\left(1-\pi^{\frac{\theta}{\eta}}\right)\right).$$

B.2 Proof for Steady State Frequency of Price Changes

Lemma 1. *Assume that $\beta \to 1$. Then the steady-state frequency of price changes n satisfies*

$$\xi(n-\bar{n}) = \underbrace{\frac{1}{n}\left[\frac{\eta+\theta(1-\eta)}{\theta} + (\theta-1)\Big(H_{\theta/\eta}(\pi,n) - H_{\theta-1}(\pi,n)\Big)\right]}_{=:F(\pi,n)}, \tag{30}$$

$$H_a(\pi,n) := \frac{1}{a}\frac{n\pi^a}{1-(1-n)\pi^a}. \tag{31}$$

Define

$$\Gamma := \frac{\theta-1}{2} \times \left(1+\theta\left(\frac{1}{\eta}-1\right)\right) \times \frac{1}{\xi}. \tag{32}$$

Under a second-order approximation of $F(\pi,n)$ around zero inflation ($\pi = 1$), we obtain

$$n(\pi) = \frac{\bar{n} + \sqrt{\bar{n}^2 + 4\Gamma\frac{2-\bar{n}}{\bar{n}^2}(\pi-1)^2}}{2}. \tag{33}$$

Under a third-order approximation of $F(\pi,n)$ around zero inflation ($\pi = 1$), we obtain

$$\begin{aligned} n(\pi) = \bar{n} &- \frac{\bar{n} + \frac{4-\bar{n}}{\bar{n}^3}\Gamma(\pi-1)^2}{2} \\ &+ \frac{1}{2}\sqrt{\left(\bar{n}+\frac{4-\bar{n}}{\bar{n}^3}\Gamma(\pi-1)^2\right)^2 + 4\Gamma\left(\frac{2-\bar{n}}{\bar{n}^2}(\pi-1)^2 + \frac{\theta\frac{1+\eta}{\eta}\big(\bar{n}^2+6(1-\bar{n})\big)+2(\bar{n}^2-3)}{3\bar{n}^3}(\pi-1)^3\right)}. \end{aligned} \tag{34}$$

We verify numerically the accuracy of the approximations across different levels of trend inflation. For trend inflation rates up to 25%, the approximation in equation (34) is very accurate, and for rates below 5% the approximation in equation (33) is also highly accurate.

Proof. We proceed in three steps. Step 1 simplifies the equilibrium condition that characterizes the steady-state frequency of price adjustment as a function of trend inflation. Step 2 approximates the marginal benefit of increasing the frequency of price changes using a third-order Taylor expansion around zero inflation. Step 3 derives the closed-form expressions in equations (33) and (34) by solving the resulting quadratic equation.

Step 1. *Assume $\beta \to 1$. The steady-state frequency of price changes satisfies*

$$\xi(n-\bar{n}) = \frac{1}{n}\left[\frac{\eta+\theta(1-\eta)}{\theta} + (\theta-1)\left(\frac{1}{\theta/\eta}\frac{n\pi^{\theta/\eta}}{1-(1-n)\pi^{\theta/\eta}} - \frac{1}{\theta-1}\frac{n\pi^{\theta-1}}{1-(1-n)\pi^{\theta-1}}\right)\right]. \tag{35}$$

Proof of Step 1. The steady-state equilibrium conditions are

$$(p^*)^{1+\theta\left(\frac{1}{\eta}-1\right)} = \frac{1}{\eta}\frac{b_2}{b_1}, \tag{36}$$

$$b_2 = y^{1/\eta} + \beta(1-n)\pi^{\theta/\eta} b_2, \tag{37}$$

$$b_1 = 1 + \beta(1-n)\pi^{\theta-1} b_1, \tag{38}$$

$$1 = n(p^*)^{1-\theta} + (1-n)\pi^{\theta-1}, \tag{39}$$

$$x^{-\theta/\eta} = n(p^*)^{-\theta/\eta} + (1-n)\pi^{\theta/\eta} x^{-\theta/\eta}, \tag{40}$$

$$\xi(n-\bar{n}) = b_1\left[(p^*)^{1-\theta} - \pi^{\theta-1}\right] - b_2\tau\left[(p^*)^{-\theta/\eta} - \pi^{\theta/\eta}x^{-\theta/\eta}\right]. \tag{41}$$

From (39),

$$p^* = \left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right)^{\frac{1}{1-\theta}}. \tag{42}$$

Plugging (42) into (40) yields

$$x^{-\theta/\eta} = n\left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right)^{\frac{-\theta}{\eta(1-\theta)}} + (1-n)\pi^{\theta/\eta}x^{-\theta/\eta}, \tag{43}$$

so

$$x^{-\theta/\eta} = \frac{n}{1-(1-n)\pi^{\theta/\eta}}\left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right)^{\frac{-\theta}{\eta(1-\theta)}}. \tag{44}$$

From (38),

$$b_1 = \frac{1}{1-\beta(1-n)\pi^{\theta-1}}. \tag{45}$$

Using (42) in (41), we obtain

$$\begin{aligned}\xi(n-\bar{n}) = b_1\Bigg[&\frac{1-(1-n)\pi^{\theta-1}}{n} - \pi^{\theta-1} \\ &- \frac{b_2}{b_1}\tau\left(\left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right)^{\frac{-\theta}{\eta(1-\theta)}} - \pi^{\theta/\eta}x^{-\theta/\eta}\right)\Bigg].\end{aligned} \tag{46}$$

From (36), $\frac{b_2}{b_1} = \eta(p^*)^{1+\theta\left(\frac{1}{\eta}-1\right)}$, and using (42) we have

$$(p^*)^{1+\theta\left(\frac{1}{\eta}-1\right)} = \left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right)^{\frac{1+\theta\left(\frac{1}{\eta}-1\right)}{1-\theta}}.$$

Substituting into (46) and using

$$\frac{-\theta}{\eta(1-\theta)} + \frac{1+\theta\left(\frac{1}{\eta}-1\right)}{1-\theta} = 1,$$

we get

$$\xi(n-\bar{n}) = b_1\Bigg[\frac{1-(1-n)\pi^{\theta-1}}{n} - \pi^{\theta-1} + \eta\tau\left(\left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right)^{\frac{1+\theta\left(\frac{1}{\eta}-1\right)}{1-\theta}}\pi^{\theta/\eta}x^{-\theta/\eta} - \frac{1-(1-n)\pi^{\theta-1}}{n}\right)\Bigg]. \tag{47}$$

Using $\tau = \frac{\theta-1}{\theta}$ implies $1-\eta\tau = \frac{\eta+\theta(1-\eta)}{\theta}$ and

$$\frac{1+\theta\left(\frac{1}{\eta}-1\right)}{1-\theta} = 1 + \frac{\theta}{\eta(1-\theta)}.$$

Hence (47) becomes

$$\xi(n-\bar{n}) = b_1\left[\frac{1-(1-n)\pi^{\theta-1}}{n}\frac{\eta+\theta(1-\eta)}{\theta} + \eta\frac{\theta-1}{\theta}\left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right)^{1+\frac{\theta}{\eta(1-\theta)}}\pi^{\theta/\eta}x^{-\theta/\eta} - \pi^{\theta-1}\right]. \tag{48}$$

Using (45) and taking the limit $\beta \to 1$,

$$\xi(n-\bar{n}) = \frac{1-(1-n)\pi^{\theta-1}}{1-\beta(1-n)\pi^{\theta-1}}\frac{1}{n} \tag{49}$$

$$\times\left[\frac{\eta+\theta(1-\eta)}{\theta} + \eta\frac{\theta-1}{\theta}\left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right)^{\frac{\theta}{\eta(1-\theta)}}\pi^{\theta/\eta}x^{-\theta/\eta} - \frac{n\pi^{\theta-1}}{1-(1-n)\pi^{\theta-1}}\right]$$

$$\to \frac{1}{n}\left[\frac{\eta+\theta(1-\eta)}{\theta} + \eta\frac{\theta-1}{\theta}\left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right)^{\frac{\theta}{\eta(1-\theta)}}\pi^{\theta/\eta}x^{-\theta/\eta} - \frac{n\pi^{\theta-1}}{1-(1-n)\pi^{\theta-1}}\right]. \tag{50}$$

Finally, substituting (44) into (50) yields

$$\begin{aligned}\xi(n-\bar{n}) &= \frac{1}{n}\left[\frac{\eta+\theta(1-\eta)}{\theta} + \eta\frac{\theta-1}{\theta}\frac{n\pi^{\theta/\eta}}{1-(1-n)\pi^{\theta/\eta}} - \frac{n\pi^{\theta-1}}{1-(1-n)\pi^{\theta-1}}\right]\\ &= \frac{1}{n}\left[\frac{\eta+\theta(1-\eta)}{\theta} + (\theta-1)\left(\frac{1}{\theta/\eta}\frac{n\pi^{\theta/\eta}}{1-(1-n)\pi^{\theta/\eta}} - \frac{1}{\theta-1}\frac{n\pi^{\theta-1}}{1-(1-n)\pi^{\theta-1}}\right)\right],\end{aligned} \tag{51}$$

which is exactly (35).

Step 2. *Define*

$$F(\pi,n) := \frac{\eta+\theta(1-\eta)}{\theta} + (\theta-1)\left(\frac{1}{\theta/\eta}\frac{n\pi^{\theta/\eta}}{1-(1-n)\pi^{\theta/\eta}} - \frac{1}{\theta-1}\frac{n\pi^{\theta-1}}{1-(1-n)\pi^{\theta-1}}\right). \tag{52}$$

Then

$$F(\pi,n)=\xi\Gamma\left(\frac{2-\bar{n}}{\bar{n}^2}(\pi-1)^2+\frac{\theta\frac{1+\eta}{\eta}\big(\bar{n}^2+6(1-\bar{n})\big)+2\bar{n}^2-6}{3\bar{n}^3}(\pi-1)^3\right)$$
$$-\xi\Gamma\,\frac{4-\bar{n}}{\bar{n}^3}(\pi-1)^2(n-\bar{n})+O\big(\|(\pi-1,n-\bar{n})\|^4\big)\,. \tag{53}$$

Proof of Step 2. This step is a straightforward application of Taylor expansions. Define

$$H_a(\pi,n):=\frac{1}{a}\,\frac{n\pi^a}{1-(1-n)\pi^a}. \tag{54}$$

Then

$$F(\pi,n)=\frac{\eta+\theta(1-\eta)}{\theta}+(\theta-1)\Big(H_{\theta/\eta}(\pi,n)-H_{\theta-1}(\pi,n)\Big). \tag{55}$$

At $\pi=1$ we have $H_a(1,n)=\frac{1}{a}$ for any n, and hence

$$F(1,n)=\frac{\eta+\theta(1-\eta)}{\theta}+(\theta-1)\left(\frac{\eta}{\theta}-\frac{1}{\theta-1}\right)=0,$$

so the steady-state condition $\xi(n-\bar{n})=\frac{1}{n}F(\pi,n)$ implies $n=\bar{n}$ at $\pi=1$.

The first-order derivatives of H_a are

$$H^a_n(\pi,n)=\frac{1}{a}\frac{\pi^a-\pi^{2a}}{\big(1-(1-n)\pi^a\big)^2},\qquad H^a_n(1,\bar{n})=0, \tag{56}$$

$$H^a_\pi(\pi,n)=\frac{n\pi^{a-1}}{\big(1-(1-n)\pi^a\big)^2},\qquad H^a_\pi(1,\bar{n})=\frac{1}{\bar{n}}. \tag{57}$$

The second-order derivatives are

$$H^a_{n^2}(\pi,n)=-\frac{2}{a}\frac{\pi^{2a}-\pi^{3a}}{\big(1-(1-n)\pi^a\big)^3},\qquad H^a_{n^2}(1,\bar{n})=0, \tag{58}$$

$$H^a_{\pi^2}(\pi,n)=n\pi^{a-2}\frac{(a-1)+(a+1)(1-n)\pi^a}{\big(1-(1-n)\pi^a\big)^3},\qquad H^a_{\pi^2}(1,\bar{n})=\frac{1}{\bar{n}^2}\big(2a-\bar{n}(a+1)\big), \tag{59}$$

$$H^a_{\pi n}(\pi,n)=\pi^{a-1}\frac{1-(n+1)\pi^a}{\big(1-(1-n)\pi^a\big)^3},\qquad H^a_{\pi n}(1,\bar{n})=-\frac{1}{\bar{n}^2}. \tag{60}$$

The third-order derivatives are

$$H^a_{n^3}(\pi,n) = \frac{6}{a}\frac{\pi^{3a}-\pi^{4a}}{\left(1-(1-n)\pi^a\right)^4}, \qquad H^a_{n^3}(1,\bar{n}) = 0, \tag{61}$$

$$H^a_{n^2\pi}(\pi,n) = 2\pi^{2a-1}\frac{-2+(2+n)\pi^a}{\left(1-(1-n)\pi^a\right)^4}, \qquad H^a_{n^2\pi}(1,\bar{n}) = \frac{2}{\bar{n}^3}, \tag{62}$$

$$H^a_{n\pi^2}(\pi,n) = a\pi^{2(a-1)}\frac{-(n+1)(1-(1-n)\pi^a)+(1-(n+1)\pi^a)3(1-n)}{(1-(1-n)\pi^a)^4} \tag{63}$$

$$+(a-1)\pi^{a-2}\frac{1-(n+1)\pi^a}{(1-(1-n)\pi^a)^3} \tag{64}$$

$$H^a_{n\pi^2}(1,\bar{n}) = \frac{a\bar{n}-4a+\bar{n}}{\bar{n}^3}, \tag{65}$$

$$H^a_{\pi^3}(\pi,n) = n\pi^{a-3}\frac{6(1-n)^2a^2\pi^{2a}+6a(a-1)(1-n)\pi^a\left(1-(1-n)\pi^a\right)+(a-1)(a-2)\left(1-(1-n)\pi^a\right)^2}{\left(1-(1-n)\pi^a\right)^4}, \tag{66}$$

$$H^a_{\pi^3}(1,\bar{n}) = \frac{6(1-\bar{n})^2a^2+6(1-\bar{n})a(a-1)\bar{n}+(a-1)(a-2)\bar{n}^2}{\bar{n}^3}. \tag{67}$$

Applying these derivatives to the Taylor expansion of (55) around $(\pi,n)=(1,\bar{n})$ and collecting terms by order yields

$$\begin{aligned} F(\pi,n) = {} & \underbrace{0}_{\text{zero order}} + \underbrace{0}_{\text{first order}} + \underbrace{\xi\Gamma\frac{2-\bar{n}}{\bar{n}^2}(\pi-1)^2}_{\text{second order}} \\ & + \underbrace{\xi\Gamma\left(-\frac{4-\bar{n}}{\bar{n}^3}(\pi-1)^2(n-\bar{n}) + \frac{\theta^{\frac{1+\eta}{\eta}}\left(\bar{n}^2+6(1-\bar{n})\right)+2\bar{n}^2-6}{3\bar{n}^3}(\pi-1)^3\right)}_{\text{third order}} \\ & + O\left(\|(\pi-1,n-\bar{n})\|^4\right), \end{aligned} \tag{68}$$

which proves (53).

Step 3. *The steady-state frequency can be approximated up to second order as*

$$n(\pi) = \frac{\bar{n}+\sqrt{\bar{n}^2+4\Gamma\frac{2-\bar{n}}{\bar{n}^2}(\pi-1)^2}}{2}. \tag{69}$$

Under a third-order approximation,

$$n(\pi) = \bar{n} - \frac{\bar{n}+\frac{4-\bar{n}}{\bar{n}^3}\Gamma(\pi-1)^2}{2} \tag{70}$$

$$+\frac{1}{2}\sqrt{\left(\bar{n}+\frac{4-\bar{n}}{\bar{n}^3}\Gamma(\pi-1)^2\right)^2+4\Gamma\left(\frac{2-\bar{n}}{\bar{n}^2}(\pi-1)^2+\frac{\theta^{\frac{1+\eta}{\eta}}\left(\bar{n}^2+6(1-\bar{n})\right)+2(\bar{n}^2-3)}{3\bar{n}^3}(\pi-1)^3\right)}. \tag{71}$$

Proof of Step 3. We prove the third-order case; the second-order formula follows by dropping the cubic term. Let $\hat{\pi} := \pi - 1$. Using the Step 1 condition $\xi(n-\bar{n}) = \frac{1}{n}F(\pi, n)$ together with the Step 2 expansion (53), we obtain

$$(n-\bar{n})\,n = \Gamma\left(\frac{2-\bar{n}}{\bar{n}^2}\hat{\pi}^2 + \frac{\theta\frac{1+\eta}{\eta}\left(\bar{n}^2+6(1-\bar{n})\right)+2\bar{n}^2-6}{3\bar{n}^3}\hat{\pi}^3 - \frac{4-\bar{n}}{\bar{n}^3}(n-\bar{n})\hat{\pi}^2\right). \tag{72}$$

Define $z := n - \bar{n}$. Then $n = z + \bar{n}$ and (72) becomes

$$z(z+\bar{n}) = \Gamma\left(\frac{2-\bar{n}}{\bar{n}^2}\hat{\pi}^2 + \frac{\theta\frac{1+\eta}{\eta}\left(\bar{n}^2+6(1-\bar{n})\right)+2\bar{n}^2-6}{3\bar{n}^3}\hat{\pi}^3 - \frac{4-\bar{n}}{\bar{n}^3}z\hat{\pi}^2\right), \tag{73}$$

or, equivalently,

$$0 = z^2 + \left(\bar{n} + \Gamma\frac{4-\bar{n}}{\bar{n}^3}\hat{\pi}^2\right)z - \Gamma\left(\frac{2-\bar{n}}{\bar{n}^2}\hat{\pi}^2 + \frac{\theta\frac{1+\eta}{\eta}\left(\bar{n}^2+6(1-\bar{n})\right)+2\bar{n}^2-6}{3\bar{n}^3}\hat{\pi}^3\right). \tag{74}$$

Equation (74) is quadratic in z. Imposing $n > \bar{n}$ (i.e., $z > 0$), the relevant root is

$$n(\hat{\pi}) = \bar{n} + \frac{-\left(\bar{n}+\frac{4-\bar{n}}{\bar{n}^3}\Gamma\hat{\pi}^2\right) + \sqrt{\left(\bar{n}+\frac{4-\bar{n}}{\bar{n}^3}\Gamma\hat{\pi}^2\right)^2 + 4\Gamma\left(\frac{2-\bar{n}}{\bar{n}^2}\hat{\pi}^2 + \frac{\theta\frac{1+\eta}{\eta}\left(\bar{n}^2+6(1-\bar{n})\right)+2(\bar{n}^2-3)}{3\bar{n}^3}\hat{\pi}^3\right)}}{2}, \tag{75}$$

which is (70) after substituting $\hat{\pi} = \pi - 1$. ☐

☐

B.3 Log-Linear Approximation Around the Steady State

Recall that the system is

$$1 = n_t\left(p_t^*\right)^{1-\theta} + (1-n_t)\,\pi_t^{\theta-1} \tag{76}$$

$$x_t^{-\frac{\theta}{\eta}} = n_t\left(p_t^*\right)^{-\frac{\theta}{\eta}} + (1-n_t)\left(x_{t-1}\right)^{-\frac{\theta}{\eta}}\pi_t^{\frac{\theta}{\eta}} \tag{77}$$

$$\xi\left(n_t-\bar{n}\right) = b_{1t}\left((1-\tau\eta)\left(p_t^*\right)^{1-\theta} - \left(\pi_t\right)^{\theta-1} + \tau\eta\left(p_t^*\right)^{1+\theta\left(\frac{1}{\eta}-1\right)}\left(x_{t-1}\right)^{-\frac{\theta}{\eta}}\pi_t^{\frac{\theta}{\eta}}\right) \tag{78}$$

$$b_{1t} = 1 + \beta\mathbb{E}_t\left(1-n_{t+1}\right)\left(\pi_{t+1}\right)^{\theta-1}b_{1t+1} \tag{79}$$

$$b_{2t} = y_t^{\frac{1}{\eta}} + \beta\mathbb{E}_t\left(1-n_{t+1}\right)\left(\pi_{t+1}\right)^{\frac{\theta}{\eta}}b_{2t+1} \tag{80}$$

$$\left(p_t^*\right)^{1+\theta\left(\frac{1}{\eta}-1\right)} = \frac{1}{\eta}\frac{b_{2t}}{b_{1t}} \tag{81}$$

$$\frac{1+i_t}{1+i} = \left(\frac{1+i_{t-1}}{1+i}\right)^{\phi_i}\left(\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}\left(\frac{y_t}{y_{t-1}}\right)^{\phi_y}\right)^{1-\phi_i}\varepsilon_t \tag{82}$$

$$1 = \beta(1+i_t)\mathbb{E}_t\frac{y_t}{y_{t+1}}\frac{1}{\pi_{t+1}} \tag{83}$$

We next log-linearize the equations that describe price setting. We use hats to denote the log-deviation of variables from their non-stochastic steady state levels.

B.3.1 Price Index

Log-linearizing equation (76) gives

$$\hat{\pi}_t = \underbrace{\frac{1}{(1-n)\,\pi^{\theta-1}}\frac{\pi^{\theta-1}-1}{\theta-1}}_{\mathcal{M}}\hat{n}_t + \underbrace{\frac{1-(1-n)\,\pi^{\theta-1}}{(1-n)\,\pi^{\theta-1}}}_{\mathcal{N}}\hat{p}_t^* \tag{84}$$

B.3.2 Fraction of Price Changes

Log-linearizing equation (78) we have

$$\xi n \hat{n}_t = \xi\,(n-\bar{n})\,\hat{b}_{1t} + \frac{1}{1-\beta\,(1-n)\,\pi^{\theta-1}}\left((1-\tau\eta)\,\frac{1-(1-n)\,\pi^{\theta-1}}{n}\,(1-\theta)\,\hat{p}_t^* - \pi^{\theta-1}\,(\theta-1)\,\hat{\pi}_t\right)$$
$$+\frac{1}{1-\beta\,(1-n)\,\pi^{\theta-1}}\tau\eta\frac{1-(1-n)\,\pi^{\theta-1}}{1-(1-n)\,\pi^{\frac{\theta}{\eta}}}\pi^{\frac{\theta}{\eta}}\left(\left(1+\theta\left(\frac{1}{\eta}-1\right)\right)\hat{p}_t^* - \frac{\theta}{\eta}\hat{x}_{t-1} + \frac{\theta}{\eta}\hat{\pi}_t\right)$$

or, equivalently,

$$\begin{aligned}
\hat{n}_t \;\; = \;\; & \frac{n-\bar{n}}{n}\hat{b}_{1t} + \underbrace{\frac{1}{\xi n}\frac{1}{1-\beta\,(1-n)\,\pi^{\theta-1}}\left(\tau\theta\frac{1-(1-n)\,\pi^{\theta-1}}{1-(1-n)\,\pi^{\frac{\theta}{\eta}}}\pi^{\frac{\theta}{\eta}} + \pi^{\theta-1}\,(1-\theta)\right)}_{\mathcal{A}}\hat{\pi}_t \\
+ \;\; & \underbrace{\frac{1}{\xi n}\frac{1-(1-n)\,\pi^{\theta-1}}{1-\beta\,(1-n)\,\pi^{\theta-1}}\left((1-\tau\eta)\,\frac{1}{n}\,(1-\theta) + \tau\eta\frac{\pi^{\frac{\theta}{\eta}}}{1-(1-n)\,\pi^{\frac{\theta}{\eta}}}\left(1+\theta\left(\frac{1}{\eta}-1\right)\right)\right)}_{\mathcal{B}}\hat{p}_t^* + \\
- \;\; & \underbrace{\frac{1}{\xi n}\frac{1}{1-\beta\,(1-n)\,\pi^{\theta-1}}\tau\theta\frac{1-(1-n)\,\pi^{\theta-1}}{1-(1-n)\,\pi^{\frac{\theta}{\eta}}}\pi^{\frac{\theta}{\eta}}}_{\mathcal{C}}\hat{x}_{t-1}
\end{aligned} \tag{85}$$

Since $\tau\theta = \theta - 1$, we can simply the expression for $\mathcal{A}$ to

$$\mathcal{A} = \frac{\theta-1}{\xi n}\frac{1}{1-\beta\,(1-n)\,\pi^{\theta-1}}\frac{\pi^{\frac{\theta}{\eta}} - \pi^{\theta-1}}{1-(1-n)\,\pi^{\frac{\theta}{\eta}}},$$

which is zero when $\pi = 1$ and is increasing in π. Thus, not only is inflation more responsive to changes in the fraction of price changes in economies with higher trend inflation, but the fraction of price changes is itself more sensitive to inflation when trend inflation is higher.

Similarly, we can simplify the expression for $\mathcal{B}$ to

$$\mathcal{B} = (1-\tau\eta)\,\frac{\theta-1}{\xi n}\frac{1-(1-n)\,\pi^{\theta-1}}{1-\beta\,(1-n)\,\pi^{\theta-1}}\frac{1}{n}\frac{\pi^{\frac{\theta}{\eta}}-1}{1-(1-n)\,\pi^{\frac{\theta}{\eta}}},$$

which is also zero when $\pi = 1$ and is increasing with π. Finally,

$$\mathcal{C} = \frac{\theta-1}{\xi n}\frac{1-(1-n)\,\pi^{\theta-1}}{1-\beta\,(1-n)\,\pi^{\theta-1}}\frac{\pi^{\frac{\theta}{\eta}}}{1-(1-n)\,\pi^{\frac{\theta}{\eta}}}.$$

B.3.3 Optimal Reset Price

Log-linearizing equations (79)-(81) and rearranging implies

$$\begin{aligned}\hat{p}_t^* &= \tfrac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\tfrac{1}{\eta}\left(1-\beta\left(1-n\right)\pi^{\frac{\theta}{\eta}}\right)\hat{y}_t+\tfrac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\beta\left(1-n\right)\left(\tfrac{\theta}{\eta}\pi^{\frac{\theta}{\eta}}-\left(\theta-1\right)\pi^{\theta-1}\right)\mathbb{E}_t\hat{\pi}_{t+1}\\ &\quad+\beta\left(1-n\right)\pi^{\frac{\theta}{\eta}}\mathbb{E}_t\hat{p}_{t+1}^*+\tfrac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\beta\left(\pi^{\frac{\theta}{\eta}}-\pi^{\theta-1}\right)\mathbb{E}_t\left(\left(1-n\right)\hat{b}_{1t+1}-n\hat{n}_{t+1}\right) \qquad (86)\end{aligned}$$

B.3.4 Losses from Misallocation

Log-linearizing equation (77) we have

$$\hat{x}_t=\left(1-\left(1-n\right)\pi^{\frac{\theta}{\eta}}\right)\hat{p}_t^*-\frac{\eta}{\theta}\left(1-\pi^{\frac{\theta}{\eta}}\right)\hat{n}_t+\left(1-n\right)\pi^{\frac{\theta}{\eta}}\left(\hat{x}_{t-1}-\hat{\pi}_t\right)$$

B.3.5 Equation for b_{1t}

Log-linearizing equation (79) we have

$$\hat{b}_{1t}=\beta\left(1-n\right)\pi^{\theta-1}\left(\theta-1\right)\mathbb{E}_t\hat{\pi}_{t+1}+\beta\left(1-n\right)\pi^{\theta-1}\mathbb{E}_t\hat{b}_{1t+1}-\beta n\pi^{\theta-1}\mathbb{E}_t\hat{n}_{t+1}$$

B.3.6 Slope of Phillips Curve

Combining equations (84) and (85) implies

$$\hat{\pi}_t=\frac{\mathcal{MB}+\mathcal{N}}{1-\mathcal{MA}}\hat{p}_t^*-\frac{\mathcal{MC}}{1-\mathcal{MA}}\hat{x}_{t-1}+\frac{\mathcal{M}}{1-\mathcal{MA}}\frac{n-\bar{n}}{n}\hat{b}_{1t}$$

To derive an expression for inflation, we multiply both sides of equation (86) by $\frac{\mathcal{MB}+\mathcal{N}}{1-\mathcal{MA}}$ and add $-\frac{\mathcal{MC}}{1-\mathcal{MA}}x_{t-1}+\frac{\mathcal{M}}{1-\mathcal{MA}}\frac{n-\bar{n}}{n}\hat{b}_{1t}$. Then, the LHS of equation (86) is equal to $\hat{\pi}_t$. Adding and subtracting $\beta\left(1-n\right)\pi^{\frac{\theta}{\eta}}\left(-\frac{\mathcal{MC}}{1-\mathcal{MA}}\hat{x}_t+\frac{\mathcal{M}}{1-\mathcal{MA}}\frac{n-\bar{n}}{n}\mathbb{E}_t\hat{b}_{1t+1}\right)$ to the RHS of (86) to express $\mathbb{E}_t\hat{p}_{t+1}^*$ as a function of expected inflation and rearranging, implies that

$$\begin{aligned}\hat{\pi}_t &= \mathcal{K}\widehat{mc}_t+\beta\left(1-n\right)\left(\frac{\frac{\theta}{\eta}\pi^{\frac{\theta}{\eta}}-\left(\theta-1\right)\pi^{\theta-1}}{1+\theta\left(\frac{1}{\eta}-1\right)}\frac{\mathcal{MB}+\mathcal{N}}{1-\mathcal{MA}}+\pi^{\frac{\theta}{\eta}}\right)\mathbb{E}_t\hat{\pi}_{t+1}+\\ &\quad+\beta\left(1-n\right)\left(\frac{\pi^{\frac{\theta}{\eta}}-\pi^{\theta-1}}{1+\theta\left(\frac{1}{\eta}-1\right)}\frac{\mathcal{MB}+\mathcal{N}}{1-\mathcal{MA}}-\pi^{\frac{\theta}{\eta}}\frac{\mathcal{M}}{1-\mathcal{MA}}\frac{n-\bar{n}}{n}\right)\mathbb{E}_t\hat{b}_{1t+1}\\ &\quad-\beta n\frac{\pi^{\frac{\theta}{\eta}}-\pi^{\theta-1}}{1+\theta\left(\frac{1}{\eta}-1\right)}\frac{\mathcal{MB}+\mathcal{N}}{1-\mathcal{MA}}\mathbb{E}_t\hat{n}_{t+1}\\ &\quad+\beta\left(1-n\right)\pi^{\frac{\theta}{\eta}}\frac{\mathcal{MC}}{1-\mathcal{MA}}\hat{x}_t-\frac{\mathcal{MC}}{1-\mathcal{MA}}\hat{x}_{t-1}+\frac{\mathcal{M}}{1-\mathcal{MA}}\frac{n-\bar{n}}{n}\hat{b}_{1t},\end{aligned}$$

where we used that $\widehat{mc}_t=\frac{1}{\eta}\hat{y}_t$.

The slope of the Phillips curve is

$$\mathcal{K} = \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\left(1-\beta\left(1-n\right)\pi^{\frac{\theta}{\eta}}\right)\frac{\mathcal{M}\mathcal{B}+\mathcal{N}}{1-\mathcal{M}\mathcal{A}}.$$

B.4 Log-Linear Approximation Around Each Point in Time

We next log-linearized the model around each point in time. To do this we consider the impact of an additional monetary shock that occurs in period t. We let tildes denote the value of the equilibrium variables following this additional shock and hats denote the log-difference between the tilde equilbrium variable and the original one, e.g. $\hat{\pi}_t = \log\tilde{\pi}_t - \log\pi_t$. The equilibrium of the model can then be described by the system of equations (76)-(83), where each equilibrium variable is replaced by its tilde counterpart. In what follows, we log-linearize the pricing block of this new system, but refer to the original equations for brevity.

B.4.1 Price Index

Log-linearizing equation (76) and using

$$n_t\left(p_t^*\right)^{1-\theta} = 1-\left(1-n_t\right)\pi_t^{\theta-1}$$

implies

$$\hat{\pi}_t = \underbrace{\frac{1}{\left(1-n_t\right)\pi_t^{\theta-1}}\frac{\pi_t^{\theta-1}-1}{\theta-1}}_{\mathcal{M}_t}\hat{n}_t + \underbrace{\frac{1-\left(1-n_t\right)\pi_t^{\theta-1}}{\left(1-n_t\right)\pi_t^{\theta-1}}}_{\mathcal{N}_t}\hat{p}_t^*.$$

B.4.2 Fraction of Price Changes

Log-linearizing (78) implies

$$\begin{aligned}\xi n_t\hat{n}_t = {} & \xi\left(n_t-\bar{n}\right)\hat{b}_{1t} + b_{1t}\left(1-\tau\eta\right)\left(p_t^*\right)^{1-\theta}\left(1-\theta\right)\hat{p}_t^* - b_{1t}\pi_t^{\theta-1}\left(\theta-1\right)\hat{\pi}_t + \\ & + b_{1t}\tau\eta\left(p_t^*\right)^{1+\theta\left(\frac{1}{\eta}-1\right)}\pi_t^{\frac{\theta}{\eta}}\left(x_{t-1}\right)^{-\frac{\theta}{\eta}}\left(\left(1+\theta\left(\frac{1}{\eta}-1\right)\right)\hat{p}_t^* + \frac{\theta}{\eta}\hat{\pi}_t - \frac{\theta}{\eta}\hat{x}_{t-1}\right),\end{aligned}$$

which can be rearranged as

$$\begin{aligned}\hat{n}_t = {} & \frac{n_t-\bar{n}}{n_t}\hat{b}_{1t} + \underbrace{\frac{\theta-1}{\xi n_t}b_{1t}\left(\left(p_t^*\right)^{1+\theta\left(\frac{1}{\eta}-1\right)}\pi_t^{\frac{\theta}{\eta}}\left(x_{t-1}\right)^{-\frac{\theta}{\eta}} - \pi_t^{\theta-1}\right)}_{\mathcal{A}_t}\hat{\pi}_t + \\ & + \underbrace{\frac{b_{1t}}{\xi n_t}\left(\left(1-\tau\eta\right)\left(p_t^*\right)^{1-\theta}\left(1-\theta\right) + \tau\eta\left(1+\theta\left(\frac{1}{\eta}-1\right)\right)\left(p_t^*\right)^{1+\theta\left(\frac{1}{\eta}-1\right)}\pi_t^{\frac{\theta}{\eta}}\left(x_{t-1}\right)^{-\frac{\theta}{\eta}}\right)}_{\mathcal{B}_t}\hat{p}_t^* \\ & \underbrace{-\left(\theta-1\right)\frac{b_{1t}}{\xi n_t}\left(p_t^*\right)^{1+\theta\left(\frac{1}{\eta}-1\right)}\pi_t^{\frac{\theta}{\eta}}\left(x_{t-1}\right)^{-\frac{\theta}{\eta}}}_{\mathcal{C}_t}\hat{x}_{t-1}.\end{aligned}$$

Using

$$\tau\eta\left(1+\theta\left(\frac{1}{\eta}-1\right)\right)=(\theta-1)(1-\tau\eta)$$

allows us to write

$$\mathcal{B}_t=(1-\tau\eta)(\theta-1)\frac{b_{1t}}{\xi n_t}\left((p_t^*)^{1+\theta\left(\frac{1}{\eta}-1\right)}\pi_t^{\frac{\theta}{\eta}}(x_{t-1})^{-\frac{\theta}{\eta}}-(p_t^*)^{1-\theta}\right).$$

B.4.3 Optimal Reset Price

The log-linearized versions of equations (79)-(81) are

$$\begin{aligned}
b_{1t}\hat{b}_{1t} &= \beta\mathbb{E}_t(1-n_{t+1})(\pi_{t+1})^{\theta-1}b_{1t+1}\left((\theta-1)\hat{\pi}_{t+1}+\hat{b}_{1t+1}\right)-\beta\mathbb{E}_t n_{t+1}(\pi_{t+1})^{\theta-1}b_{1t+1}\hat{n}_{t+1}\\
b_{2t}\hat{b}_{2t} &= (y_t)^{\frac{1}{\eta}}\frac{1}{\eta}\hat{y}_t+\beta\mathbb{E}_t(1-n_{t+1})(\pi_{t+1})^{\frac{\theta}{\eta}}b_{2t+1}\left(\frac{\theta}{\eta}\hat{\pi}_{t+1}+\hat{b}_{2t+1}\right)-\beta\mathbb{E}_t n_{t+1}(\pi_{t+1})^{\frac{\theta}{\eta}}b_{2t+1}\hat{n}_{t+1}
\end{aligned}$$

and

$$\hat{p}_t^*=\frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\left(\hat{b}_{2t}-\hat{b}_{1t}\right).$$

Because we consider a one-time unanticipated shock, the resulting transition, that is, the evolution of variables involving hats is deterministic, so we can write

$$\begin{aligned}
\hat{b}_{1t} &= \beta\left(\mathbb{E}_t(1-n_{t+1})(\pi_{t+1})^{\theta-1}\frac{b_{1t+1}}{b_{1t}}\right)\left((\theta-1)\hat{\pi}_{t+1}+\hat{b}_{1t+1}\right)-\beta\left(\mathbb{E}_t n_{t+1}(\pi_{t+1})^{\theta-1}\frac{b_{1t+1}}{b_{1t}}\right)\hat{n}_{t+1}\\
\hat{b}_{2t} &= \frac{(y_t)^{\frac{1}{\eta}}}{b_{2t}}\left(\frac{1}{\eta}\hat{y}_t\right)+\beta\left(\mathbb{E}_t(1-n_{t+1})(\pi_{t+1})^{\frac{\theta}{\eta}}\frac{b_{2t+1}}{b_{2t}}\right)\left(\frac{\theta}{\eta}\hat{\pi}_{t+1}+\hat{b}_{2t+1}\right)\\
&- \beta\left(\mathbb{E}_t n_{t+1}(\pi_{t+1})^{\frac{\theta}{\eta}}\frac{b_{2t+1}}{b_{2t}}\right)\hat{n}_{t+1}.
\end{aligned}$$

Subtracting the first expression from the second and multiplying by $\frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}$ gives

$$\begin{aligned}
\hat{p}_t^* &= \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\frac{(y_t)^{\frac{1}{\eta}}}{b_{2t}}\widehat{mc}_t+\\
&+ \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\beta\underbrace{\mathbb{E}_t(1-n_{t+1})\left(\frac{\theta}{\eta}(\pi_{t+1})^{\frac{\theta}{\eta}}\frac{b_{2t+1}}{b_{2t}}-(\theta-1)(\pi_{t+1})^{\theta-1}\frac{b_{1t+1}}{b_{1t}}\right)}_{\mathcal{F}_t}\hat{\pi}_{t+1}+\\
&+ \beta\underbrace{\mathbb{E}_t(1-n_{t+1})(\pi_{t+1})^{\frac{\theta}{\eta}}\frac{b_{2t+1}}{b_{2t}}}_{\mathcal{G}_t}\hat{p}_{t+1}^*\\
&+ \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\beta\underbrace{\mathbb{E}_t(1-n_{t+1})\left((\pi_{t+1})^{\frac{\theta}{\eta}}\frac{b_{2t+1}}{b_{2t}}-(\pi_{t+1})^{\theta-1}\frac{b_{1t+1}}{b_{1t}}\right)}_{\mathcal{H}_t}\hat{b}_{1t+1}\\
&- \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\beta\underbrace{\mathbb{E}_t n_{t+1}\left((\pi_{t+1})^{\frac{\theta}{\eta}}\frac{b_{2t+1}}{b_{2t}}-(\pi_{t+1})^{\theta-1}\frac{b_{1t+1}}{b_{1t}}\right)}_{\mathcal{I}_t}\hat{n}_{t+1}
\end{aligned}$$

B.4.4 Losses from Misallocation

Log-linearizing equation (77) implies

$$-\frac{\theta}{\eta}x_t^{-\frac{\theta}{\eta}}\hat{x}_t = n_t\left(p_t^*\right)^{-\frac{\theta}{\eta}}\left(\hat{n}_t-\frac{\theta}{\eta}\hat{p}_t^*\right)+\left(1-n_t\right)\left(x_{t-1}\right)^{-\frac{\theta}{\eta}}\pi_t^{\frac{\theta}{\eta}}\left(-\frac{\theta}{\eta}\hat{x}_{t-1}+\frac{\theta}{\eta}\hat{\pi}_t\right)-n_t\left(x_{t-1}\right)^{-\frac{\theta}{\eta}}\pi_t^{\frac{\theta}{\eta}}\hat{n}_t,$$

which can be rearranged as

$$\hat{x}_t = n_t x_t^{\frac{\theta}{\eta}}\left(p_t^*\right)^{-\frac{\theta}{\eta}}\hat{p}_t^*+\frac{\eta}{\theta}n_t\left(\left(\frac{x_t}{x_{t-1}}\right)^{\frac{\theta}{\eta}}\pi_t^{\frac{\theta}{\eta}}-x_t^{\frac{\theta}{\eta}}\left(p_t^*\right)^{-\frac{\theta}{\eta}}\right)\hat{n}_t+\left(1-n_t\right)\left(\frac{x_t}{x_{t-1}}\right)^{\frac{\theta}{\eta}}\pi_t^{\frac{\theta}{\eta}}\left(\hat{x}_{t-1}-\hat{\pi}_t\right).$$

B.4.5 Equation for b_{1t}

Lastly, log-linearizing equation (79) gives

$$\hat{b}_{1t} = \underbrace{\beta\,\mathbb{E}_t\left(1-n_{t+1}\right)\left(\pi_{t+1}\right)^{\theta-1}\frac{b_{1t+1}}{b_{1t}}}_{\mathcal{D}_t}\left(\left(\theta-1\right)\hat{\pi}_{t+1}+\hat{b}_{1t+1}\right)-\underbrace{\beta\,\mathbb{E}_t n_{t+1}\left(\pi_{t+1}\right)^{\theta-1}\frac{b_{1t+1}}{b_{1t}}}_{\mathcal{E}_t}\hat{n}_{t+1}.$$

B.4.6 Phillips Curve

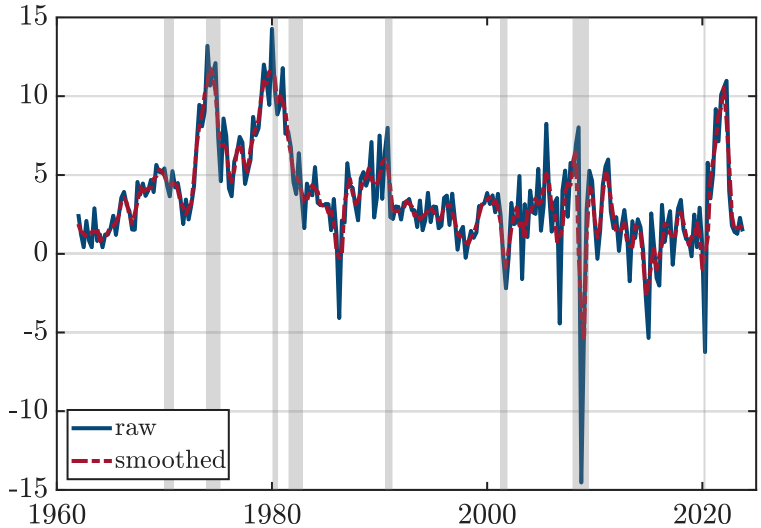
Following the same steps as in Section B.3.6 allows us to write the Phillips curve

$$\begin{aligned}
\hat{\pi}_t \;=\;& \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\frac{\left(y_t\right)^{\frac{1}{\eta}}}{b_{2t}}\frac{\mathcal{M}_t\mathcal{B}_t+\mathcal{N}_t}{1-\mathcal{M}_t\mathcal{A}_t}\widehat{mc}_t+\frac{\frac{\mathcal{M}_t\mathcal{B}_t+\mathcal{N}_t}{1-\mathcal{M}_t\mathcal{A}_t}}{\frac{\mathcal{M}_{t+1}\mathcal{B}_{t+1}+\mathcal{N}_{t+1}}{1-\mathcal{M}_{t+1}\mathcal{A}_{t+1}}}\beta\mathbb{E}_t\left(1-n_{t+1}\right)\left(\pi_{t+1}\right)^{\frac{\theta}{\eta}}\frac{b_{2t+1}}{b_{2t}}\hat{\pi}_{t+1}\\
+\;& \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\frac{\mathcal{M}_t\mathcal{B}_t+\mathcal{N}_t}{1-\mathcal{M}_t\mathcal{A}_t}\beta\mathbb{E}_t\left(1-n_{t+1}\right)\left(\frac{\theta}{\eta}\left(\pi_{t+1}\right)^{\frac{\theta}{\eta}}\frac{b_{2t+1}}{b_{2t}}-\left(\theta-1\right)\left(\pi_{t+1}\right)^{\theta-1}\frac{b_{1t+1}}{b_{1t}}\right)\hat{\pi}_{t+1}+\\
+\;& \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\frac{\mathcal{M}_t\mathcal{B}_t+\mathcal{N}_t}{1-\mathcal{M}_t\mathcal{A}_t}\beta\mathbb{E}_t\left(1-n_{t+1}\right)\left(\left(\pi_{t+1}\right)^{\frac{\theta}{\eta}}\frac{b_{2t+1}}{b_{2t}}-\left(\pi_{t+1}\right)^{\theta-1}\frac{b_{1t+1}}{b_{1t}}\right)\hat{b}_{1t+1}\\
-\;& \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\frac{\mathcal{M}_t\mathcal{B}_t+\mathcal{N}_t}{1-\mathcal{M}_t\mathcal{A}_t}\beta\mathbb{E}_t n_{t+1}\left(\left(\pi_{t+1}\right)^{\frac{\theta}{\eta}}\frac{b_{2t+1}}{b_{2t}}-\left(\pi_{t+1}\right)^{\theta-1}\frac{b_{1t+1}}{b_{1t}}\right)\hat{n}_{t+1}\\
-\;& \frac{\mathcal{M}_t\mathcal{C}_t}{1-\mathcal{M}_t\mathcal{A}_t}\hat{x}_{t-1}+\frac{\mathcal{M}_t}{1-\mathcal{M}_t\mathcal{A}_t}\frac{n_t-\bar{n}}{n_t}\hat{b}_{1t}\\
-\;& \frac{\frac{\mathcal{M}_t\mathcal{B}_t+\mathcal{N}_t}{1-\mathcal{M}_t\mathcal{A}_t}}{\frac{\mathcal{M}_{t+1}\mathcal{B}_{t+1}+\mathcal{N}_{t+1}}{1-\mathcal{M}_{t+1}\mathcal{A}_{t+1}}}\beta\mathbb{E}_t\left(1-n_{t+1}\right)\left(\pi_{t+1}\right)^{\frac{\theta}{\eta}}\frac{b_{2t+1}}{b_{2t}}\left(-\frac{\mathcal{M}_{t+1}\mathcal{C}_{t+1}}{1-\mathcal{M}_{t+1}\mathcal{A}_{t+1}}\hat{x}_t+\frac{\mathcal{M}_{t+1}}{1-\mathcal{M}_{t+1}\mathcal{A}_{t+1}}\frac{n_{t+1}-\bar{n}}{n_{t+1}}\hat{b}_{1t+1}\right),
\end{aligned}$$

so the slope of the Phillips curve is

$$\mathcal{K}_t = \frac{1}{1+\theta\left(\frac{1}{\eta}-1\right)}\frac{\left(y_t\right)^{\frac{1}{\eta}}}{b_{2t}}\frac{\mathcal{M}_t\mathcal{B}_t+\mathcal{N}_t}{1-\mathcal{M}_t\mathcal{A}_t}.$$

Figure C.4: Inflation Data



Notes: The gray bars indicate NBER recessions.

C Additional Figures

C.1 Raw Inflation Data

Figure C.4 contrasts the raw quarterly annualized inflation series with its counterpart smoothed using a centered 3-quarter moving average. We target the smoothed series in Section 3.3. Figure C.5 reports the slope of the Phillips curve we obtain when targeting the raw inflation data. Notice that the slope spikes to a level above 0.2 for one quarter in 1980, but is otherwise comparable to the magnitudes we report in the main text.

C.2 Extensive Margin Decomposition of Inflation

To illustrate the role of the extensive margin of price changes in explaining inflation fluctuations, we follow Klenow and Kryvtsov (2008) and decompose inflation

$$\pi_t = \Delta_t n_t$$

into two components: Δ_t, the average price change conditional on adjustment, and n_t, the fraction of price changes. We isolate the role of the intensive margin by computing a counterfactual inflation series

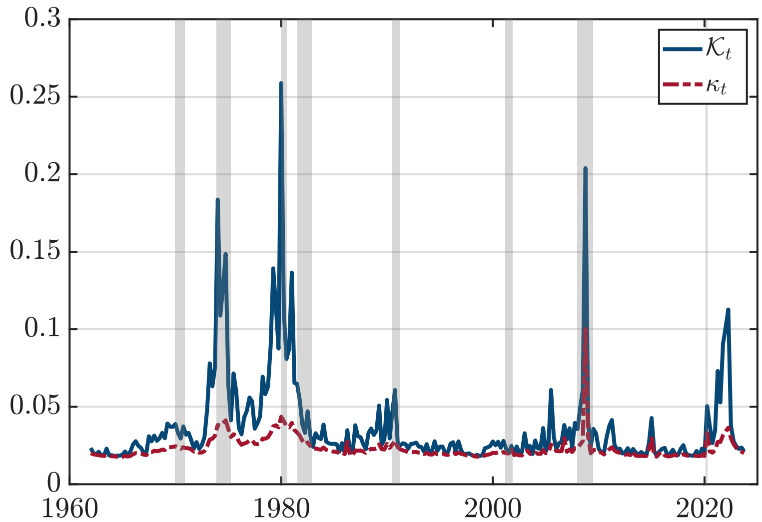
$$\pi_t^i = \Delta_t \bar{n},$$

where we set the fraction of price changes equal to its time series average $\bar{n}$. We isolate the role of the extensive margin by computing a counterfactual inflation series

$$\pi_t^e = \bar{\Delta} n_t,$$

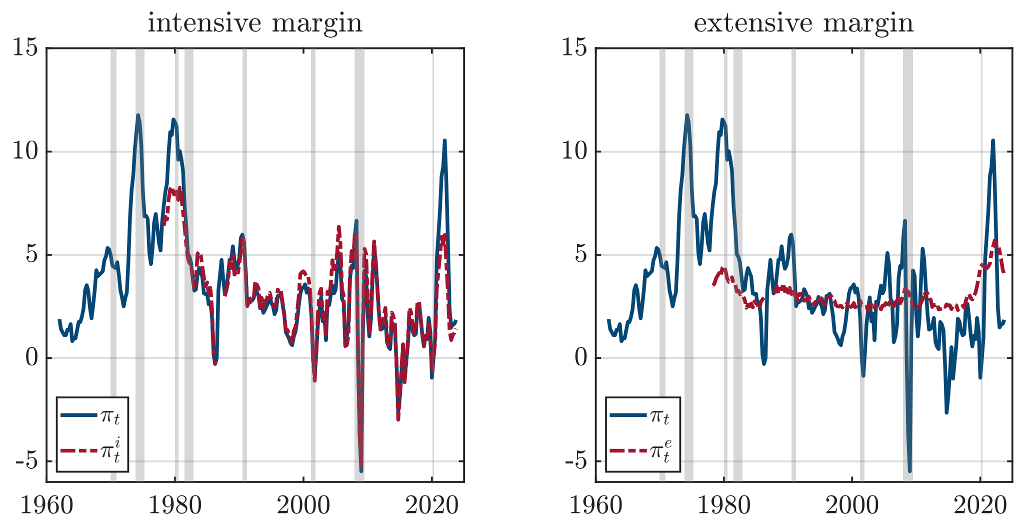
where we set the average size of price changes equal to its time series average $\bar{\Delta}$. Figure C.6 plots this decomposition. As Klenow and Kryvtsov (2008) point out, the intensive margin of

Figure C.5: Slope of the Phillips Curve, Target Raw Inflation Data



Notes: The gray bars indicate NBER recessions.

Figure C.6: Inflation Decomposition



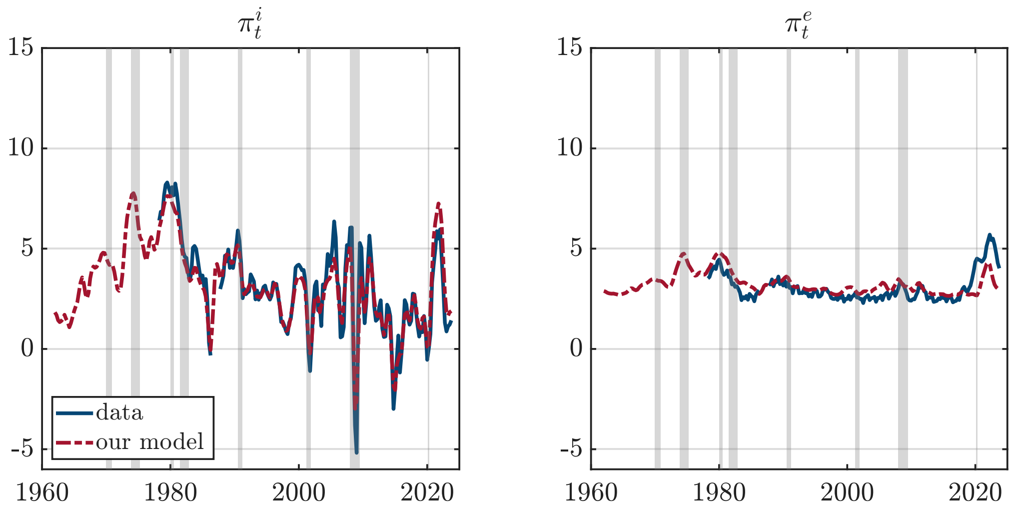
Notes: The gray bars indicate NBER recessions.

price changes accounts for the bulk of the fluctuations in inflation, except for periods of high inflation, when the extensive margin plays a bigger role.

In Figure C.7, we repeat this decomposition with data simulated from our model, which recall matches the inflation time series by construction. As the figure shows, our model reproduces both the intensive and extensive margins well.

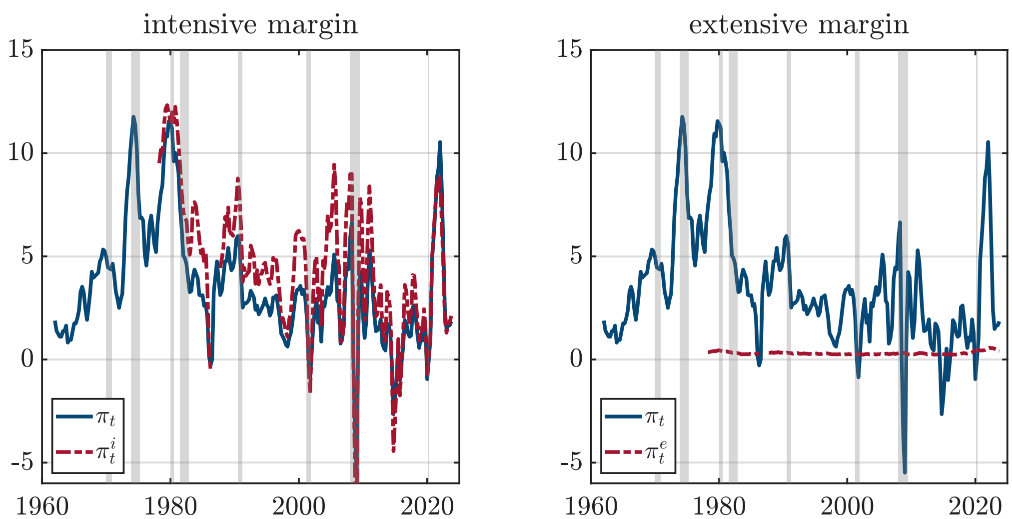
Our results may appear to contradict those of Montag and Villar (2023), who argued that the extensive margin of price changes played a minor role in the post-Covid increase in inflation. We show that their conclusion follows from the choice of the value at which to fix the fraction of price changes and the average size of price changes in calculating counterfactual

Figure C.7: Inflation Decomposition, Data vs. Model



Notes: The gray bars indicate NBER recessions.

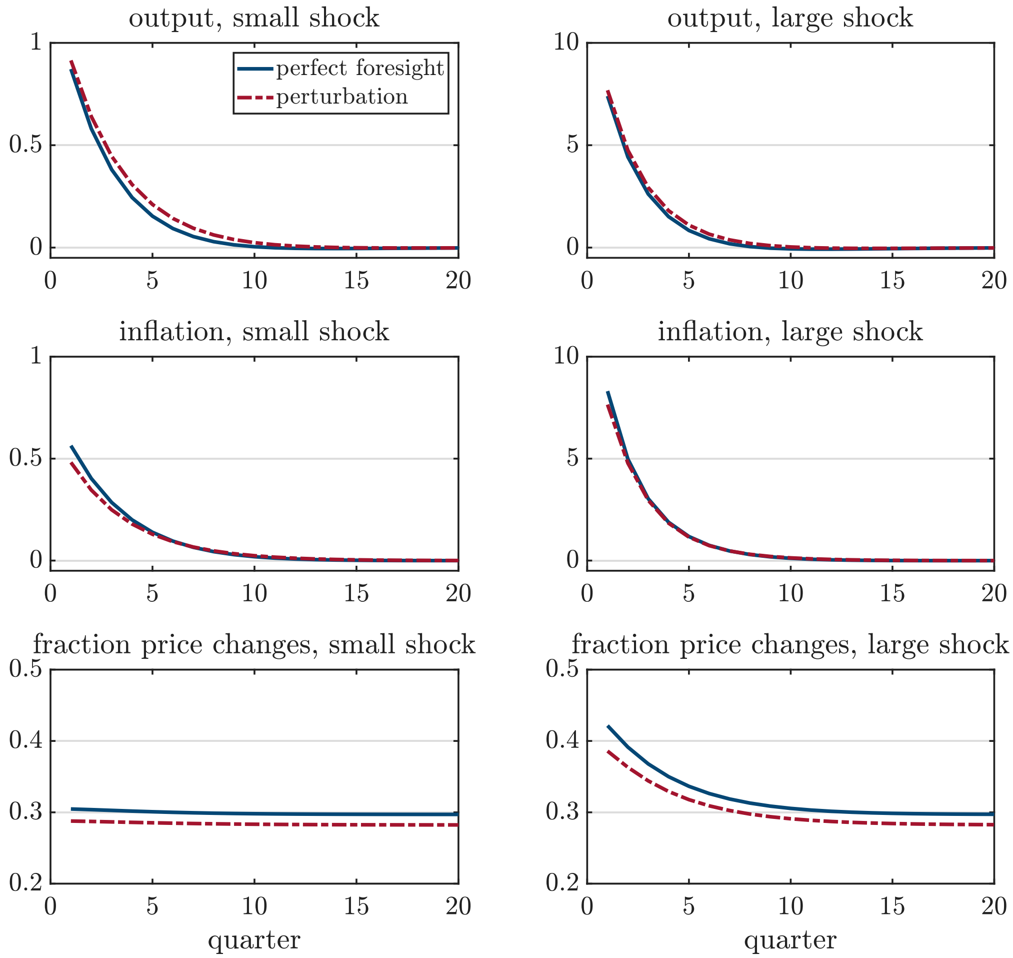
Figure C.8: Inflation Decomposition, Montag and Villar



Notes: The gray bars indicate NBER recessions.

inflation series π_t^i and π_t^e. Specifically, they set $\bar{n}$ and $\bar{\Delta}$ equal to the value of the fraction of price changes and the average size of price changes in January 2020. Because of seasonality, January is a month with an unusually large fraction of price change and an unusually low average size of price changes, biasing the decomposition towards finding no role for the extensive margin. We repeat the decomposition above using their approach and confirm their results in Figure C.8.

Figure C.9: Impulse Response to Monetary Shocks: Perfect Foresight vs. Perturbation

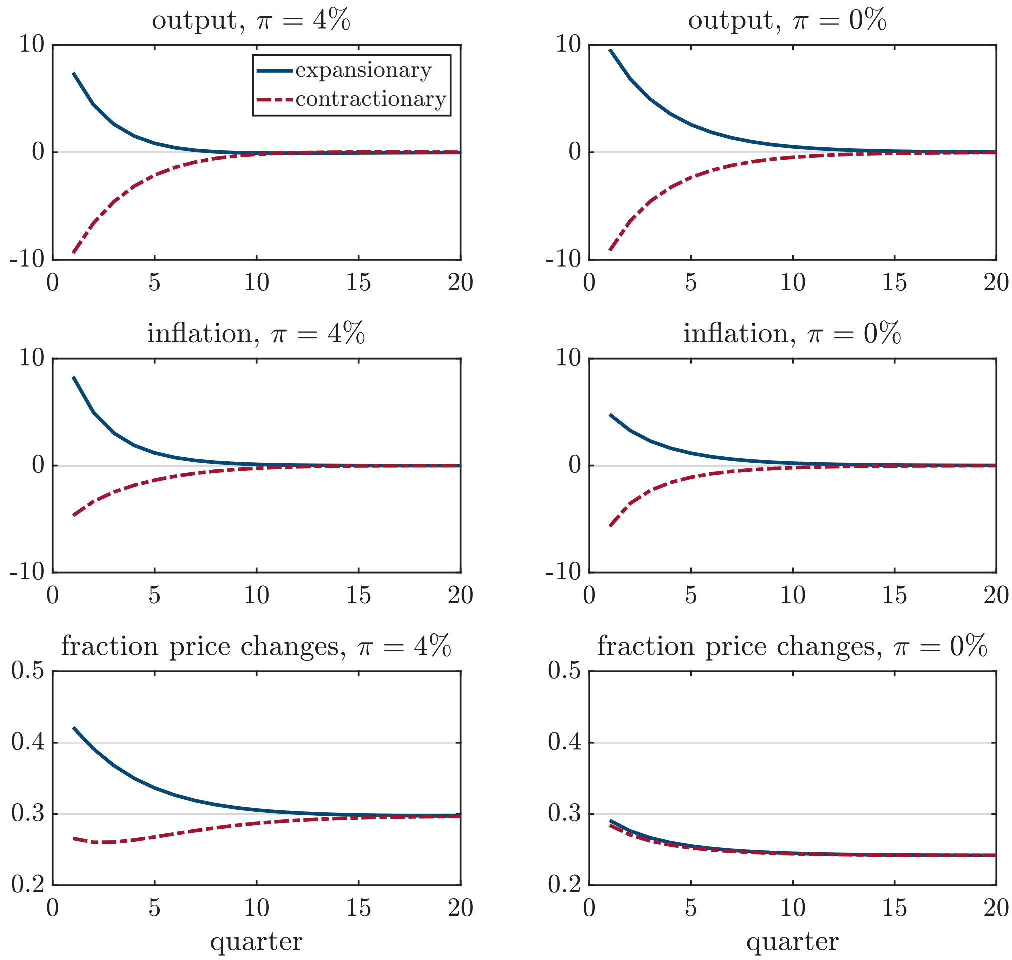


C.3 Impulse Response to Monetary Shocks

Figure C.9 plots impulse responses to small (0.25 standard deviations) and large (2.5 standard deviations) shocks to the Taylor rule. We compute these under perfect foresight, as in Section 3.2.2, and with a third order perturbation. We compute the perfect foresight responses starting from the non-stochastic steady state of the model and those using the third-order perturbation starting from the stochastic steady state. Notice that the impulse responses are very similar, despite differences in the steady state values of output, inflation and the fraction of price changes.

Figure C.10 plots impulse respones to large (2.5 standard deviations) expansionary and contractionary monetary policy shocks. To illustrate the asymmetry implied by our model, we consider two scenarios: our benchmark economy where steady-state inflation is 4% and an alternarive economy with zero trend inflation. Notice that the fraction of price changes decreases after a contractionary shock in the economy with 4% trend inflation because the contractionary shock reduces inflation to nearly zero and decreases incentives to adjust prices. For this reason, the response of inflation to the contractionary shock is smaller in absolute

Figure C.10: Impulse Response to Expansionary and Contractionary Monetary Shocks

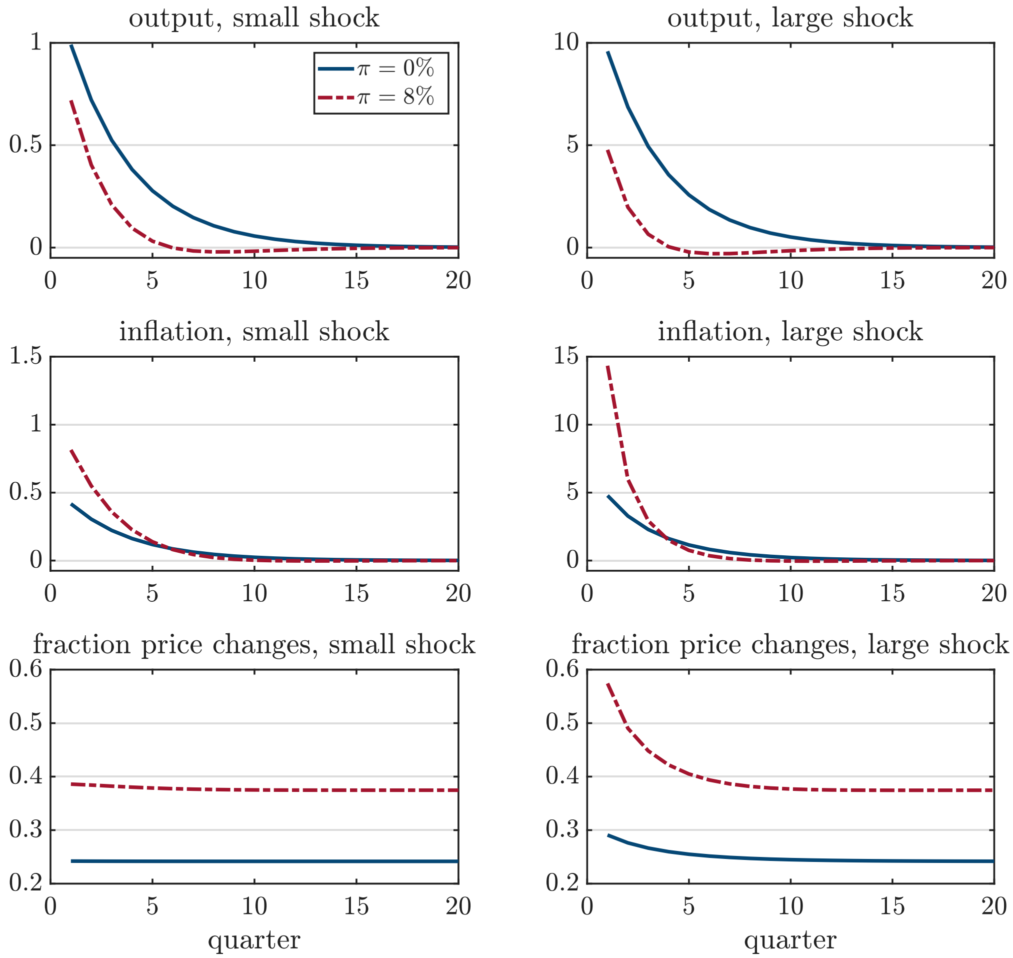


value than to an expansionary shock and the response of output is lager. In contrast, when steady-state inflation is zero, the two shocks have nearly identical effects. Thus, in environments with positive trend inflation, our model is consistent with the evidence that prices rise faster than they fall (Peltzman, 2000).

Figure C.11 plots impulse responses to small (0.25 standard deviations) and large (2.5 standard deviations) expansionary monetary shocks in two economies: one with zero trend inflation and one with 8% trend inflation. We note that the same shock generates a larger response of inflation and the fraction of price changes in the economy with higher trend inflation. This is because firms desire larger price changes when trend inflation is high.

Figure C.12 computes a measure of the total sacrifice ratio: the output drop that would be required to bring inflation to 2% in our model and in the Calvo model. The figure shows that the output drop required to achieve 2% would be larger in the Calvo model in periods of high inflation.

Figure C.11: Impulse Response as a Function of Trend Inflation



D Robustness

D.1 A Model with Idiosyncratic Shocks

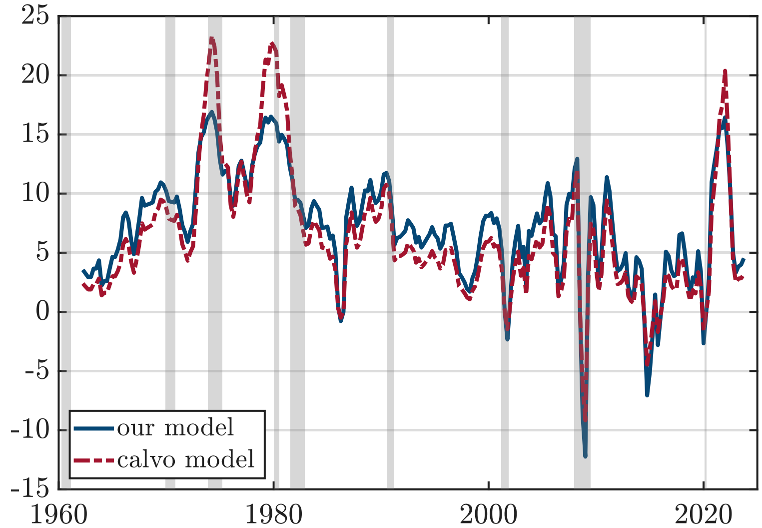
In this section, we derive expressions that characterize moments of the distribution of price changes. Let $\hat{p}_{ikt} := -\log(z_{ikt}p_{ikt})$ be the inverse of the log-price gap and $M_t(q) = \int_0^1 \int_0^1 \hat{p}^q_{ikt} \mathrm{d}k\mathrm{d}i$ be the q-th moment of the distribution in period t. We can then recursively express $M_t(1)$ and $M_t(2)$ as

$$\begin{aligned} M_t(1) &= -n_t \log(p_t^*) + (1-n_t)[\log(\pi_t) + M_{t-1}(1)], \\ M_t(2) &= n_t \log(p_t^*)^2 + (1-n_t)\left[\log(\pi_t)^2 + \sigma_z^2 + 2\log(\pi_t)M_{t-1}(1) + M_{t-1}(2)\right]. \end{aligned}$$

Let $\mathbb{E}_t[\Delta p^m]$ denote the m-th moment of the log of price changes in period t. By definition, and under the assumption of random adjustment, it satisfies

$$\mathbb{E}_t[\Delta p^m] = \int_0^1 \int_0^1 \left[\log\left(\frac{p^*_{ikt}}{p_{ikt-1}}\pi_t\right)\right]^m \mathrm{d}k\mathrm{d}i,$$

Figure C.12: Total Sacrifice Ratio



where p^*_{ikt} is the optimal relative reset price and p_{ikt-1} is the relative price in the previous period. It is straightforward to show that

$$\begin{aligned}
\mathbb{E}_t[\Delta p] &= \log(p^*_t) + \log(\pi_t) + M_{t-1}(1), \\
\mathbb{E}_t[\Delta p^2] &= \log(p^*_t \pi_t)^2 + \sigma_z^2 + 2\log(p^*_t \pi_t) M_{t-1}(1) + M_{t-1}(2), \\
\mathbb{S}td_t[\Delta p] &= \sqrt{\mathbb{E}_t[\Delta p^2] - \mathbb{E}_t[\Delta p]^2}.
\end{aligned}$$

To provide intuition for the micro-price statistics, notice that in the steady state, $M(1) = -\log(p^*) + \frac{1-n}{n}\log(\pi)$. Since $\mathbb{E}[\Delta p] = \log(p^*) + \log(\pi) + M(1)$, we obtain

$$\begin{aligned}
\mathbb{E}[\Delta p] &= \log(p^*) + \log(\pi) + M(1) \\
&= \log(p^*) + \log(\pi) - \log(p^*) + \frac{1-n}{n}\log(\pi) \\
&= \frac{\log(\pi)}{n}.
\end{aligned}$$

Thus, inflation is equal to the fraction of price changes times the size of price changes. Similarly, assuming zero inflation in the steady state ($\log(\pi) = 0$), we find that $M(2) = -\log(p^*)^2 + \frac{1-n}{n}\sigma_z^2$. Since $\mathbb{E}[\Delta p^2] = -\log(p^*)^2 + \sigma_z^2 + M_{t-1}(2)$, we obtain

$$\begin{aligned}
\mathbb{E}[\Delta p^2] &= \log(p^*)^2 + \sigma_z^2 - 2\log(p^*)^2 - \log(p^*)^2 + \frac{1-n}{n}\sigma_z^2 \\
&= \frac{\sigma_z^2}{n} - 2\log(p^*)^2 \\
&\approx \frac{\sigma_z^2}{n}.
\end{aligned}$$

Thus, the variance of idiosyncratic shocks is equal to the fraction of price changes times the variance of price changes, plus a small correction term related to consumer substitution effects.

D.2 Historical Analysis

In this section, we describe how we estimate the model with multiple aggregate shocks that we use for the historical analysis in Section 4.1. We assume that the Taylor rule shock ε_t is an i.i.d. process with standard deviation σ_ε, and that the other three aggregate shocks, z_t, τ_t, and $\bar{\pi}_t^*$, follow an AR(1) process with persistence ρ_z, ρ_τ and ρ_p, and standard deviation of innovations σ_z, σ_τ and σ_p

$$\begin{aligned}
\log z_t &= (1-\rho_z)\log z + \rho_z \log z_{t-1} + \sigma_z \epsilon_{zt} \\
\log \tau_t &= (1-\rho_\tau)\log \tau + \rho_\tau \log \tau_{t-1} + \sigma_\tau \epsilon_{\tau t} \\
\log \bar{\pi}_t^* &= (1-\rho_p)\log \bar{\pi}^* + \rho_p \log \bar{\pi}_{t-1}^* + \sigma_p \epsilon_{pt}
\end{aligned}$$

where $z = 1$, and $\tau = 1 - 1/\theta$.

To parameterize this model, we first set a number of parameters to values typical in the literature. We then conduct a Bayesian likelihood estimation to estimate the persistence and standard deviations of the shock processes, as well as the parameter ξ determining the cost of changing prices and the fraction of free price changes $\bar{n}$.

D.2.1 Data

We use quarterly data on inflation, output and the nominal federal funds rate from 1962:Q1 to 2023:Q4. The estimation observables are constructed from quarterly U.S. data pulled from FRED and then transformed into annualized rates or demeaned growth deviations in the following way:

- The nominal interest rate observable is based on the effective federal funds rate (FRED: FEDFUNDS), which is first converted from monthly to quarterly (simple quarterly average), then expressed as a quarterly net rate, shifted by the model steady-state nominal rate, and finally annualized back to percent units. For periods when the zero lower bound binds, we substitute the fed funds rate with the shadow interest rate calculated by Wu and Xia (2016).
- Inflation is computed from the CPI less shelter (FRED: CUSR0000SA0L2) by first converting monthly data to quarterly averages, then taking quarter-to-quarter log differences, converting to gross inflation, and annualizing.
- Output growth is constructed using real GDP (FRED: GDPC1) scaled by the civilian labor force index (FRED: CLF16OV, monthly averaged to quarterly and indexed to 1992Q3), forming per–labor-force output. Quarterly log growth $\Delta \log y_t$ is then demeaned over the sample and converted to percent units.

We also use available data on the fraction of price changes from Montag and Villar (2023) and Nakamura et al. (2018), as well as the series of the inflation target from Coibion and Gorodnichenko (2011), both of which we discuss in the main text.

D.2.2 Solution and Estimation

State-Space Representation. Let s_t denote the vector of model state variables, y_t the vector of observables, and ϑ the vector of structural parameters. The model can be written in state-space form as

$$s_t = \Phi(s_{t-1}, \varepsilon_t; \vartheta), \tag{87}$$

$$y_t = H_t s_t + \nu_t, \tag{88}$$

where $\varepsilon_t \sim \mathcal{N}(0, \Omega)$ are structural shocks and $\nu_t \sim \mathcal{N}(0, R)$ are measurement errors. The measurement matrix H_t is allowed to vary over time to accommodate changes in data availability across the sample.

The transition function $\Phi(\cdot)$ is nonlinear. We approximate it using a third-order perturbation solution. The measurement equation is linear in the states.

Likelihood and Posterior. Let $y^T = \{y_1, \ldots, y_T\}$. Given a prior density $p(\vartheta)$, the posterior distribution satisfies

$$p(\vartheta \mid y^T) \propto p(y^T \mid \vartheta)\, p(\vartheta).$$

The likelihood admits the prediction-error decomposition

$$p(y^T \mid \vartheta) = \prod_{t=1}^{T} p(y_t \mid y^{t-1}, \vartheta), \tag{89}$$

where $p(y_t \mid y^{t-1}, \vartheta)$ is the one-step-ahead predictive density.

Extended Kalman Filter. The nonlinearities in our model are smooth, so the Extended Kalman Filter (EKF) can be an effective estimator for our application. The EKF combines nonlinear state prediction with local linearization for uncertainty propagation.

At each time t, the filter proceeds as follows.

1. Prediction. Given the filtered state $\hat{s}_{t|t}$, the predicted state is obtained by directly evaluating the nonlinear policy function,

   $$\hat{s}_{t+1|t} = \Phi(\hat{s}_{t|t}, 0; \vartheta),$$

   using the third-order perturbation. Structural shocks are shut down during prediction so that uncertainty enters exclusively through the covariance recursion.

2. Local Linearization To propagate uncertainty, the model is locally linearized each period around the filtered state. Specifically, we evaluate the Jacobian of the model's dynamic equations at the point implied by the lagged, current, and predicted state. This yields time-varying linear transition matrices

   $$s_{t+1} = C_t + P_t s_t + D_t \varepsilon_{t+1},$$

   which are used to update the covariance matrix.

3. Update. Given observed data y_t, the update step is

$$\tilde{y}_t = y_t - H_t \hat{s}_{t|t-1}, \tag{90}$$

$$S_t = H_t P_{t|t-1} H_t' + R, \tag{91}$$

$$K_t = P_{t|t-1} H_t' S_t^{-1}, \tag{92}$$

$$\hat{s}_{t|t} = \hat{s}_{t|t-1} + K_t \tilde{y}_t. \tag{93}$$

The log-likelihood contribution is

$$\log p(y_t \mid y^{t-1}, \vartheta) = -\tfrac{1}{2} \left(n_t \log(2\pi) + \log |S_t| + \tilde{y}_t' S_t^{-1} \tilde{y}_t \right),$$

where n_t is the number of observed variables at time t.

Measurement error variances in R are specified as a fraction (10%) of the sample variance of each observable, with zero measurement error for the policy rate.

Regime Changes. Owing to the time-varying policy rule coefficients, the model's structural parameters have regime shifts. For each regime, the model's third-order perturbation solution is recomputed. The filter switches between the corresponding policy functions and local linearizations at the appropriate regime break dates.

Posterior Simulation, Metropolis Hastings Algorithm. The final step is to summarize the procedure that we use to generate the Markov Chain that yields the posterior distribution of ϑ. It has a single block, corresponding to parameters ϑ. The algorithm is as follows.

Let N be the length of the chain. For $j = 1, \ldots, N$,

1. Draw a new proposal of ϑ, denoted by $\hat{\vartheta}_j$, from a thick-tailed proposal density centered at $\hat{\vartheta}_{j-1}$ to ensure sufficient coverage of the parameter space. In practice, we draw random values from the Student t-distribution, and scale the draws by the inverse Hessian at the posterior mode of the linearized model, with parameter-specific scaling to ensure efficient exploration.

2. The proposal $\hat{\vartheta}_j$ is accepted with probability $\frac{p(\hat{\vartheta}_j | y^T)}{p(\hat{\vartheta}_{j-1} | y^T)}$. If $\hat{\vartheta}_j$ is accepted, then set $\hat{\vartheta}_{j-1} = \hat{\vartheta}_j$. In practice, it is important to compute both $p(\hat{\vartheta}_j \mid y^T)$ and $p(\hat{\vartheta}_{j-1} \mid y^T)$ within in each step j, due to the variance of the estimate of the likelihood using the particle filter.

We use two chains of length 200,000 and discard the first 10% of the chain as a burn-in. We choose a scaling parameter for the proposal density for the Metropolis Hastings algorithm so as to obtain an acceptance probability of around 25%.

Posterior Chain Convergence. Figure D.13 reports the $\widehat{R}$ diagnostic for each estimated parameter, computed following Gelman and Rubin (1992). The statistic compares the dispersion of draws across independent Markov chains to the dispersion within each chain, and provides a measure of whether the chains have converged to a common stationary distribution.

Specifically, let $\theta_j^{(r,t)}$ denote draw $t = 1, \ldots, T$ of parameter j in chain $r = 1, \ldots, R$. For each parameter, we compute the within-chain variance

$$W_j = \frac{1}{R} \sum_{r=1}^{R} \mathrm{Var}_t \left(\theta_j^{(r,t)} \right), \tag{94}$$

and the between-chain variance

$$B_j = \frac{T}{R-1} \sum_{r=1}^{R} \left(\bar{\theta}_j^{(r)} - \bar{\theta}_j \right)^2, \tag{95}$$

where $\bar{\theta}_j^{(r)}$ is the mean of chain r and $\bar{\theta}_j$ is the grand mean across chains. An estimate of the marginal posterior variance is then

$$\widehat{\mathrm{Var}}(\theta_j) = \frac{T-1}{T} W_j + \frac{1}{T} B_j, \tag{96}$$

and the reported diagnostic is

$$\widehat{R}_j = \sqrt{\frac{\widehat{\mathrm{Var}}(\theta_j)}{W_j}}. \tag{97}$$

Values of $\widehat{R}_j$ close to one indicate that the between-chain and within-chain variances are similar, suggesting that the chains have mixed well and converged. Values substantially above one indicate lack of convergence, reflecting persistent differences across chains. In our application, the $\widehat{R}$ statistics converge to below 1.1 for both models, a typical threshold value suggesting that the posterior distributions across chains have converged.

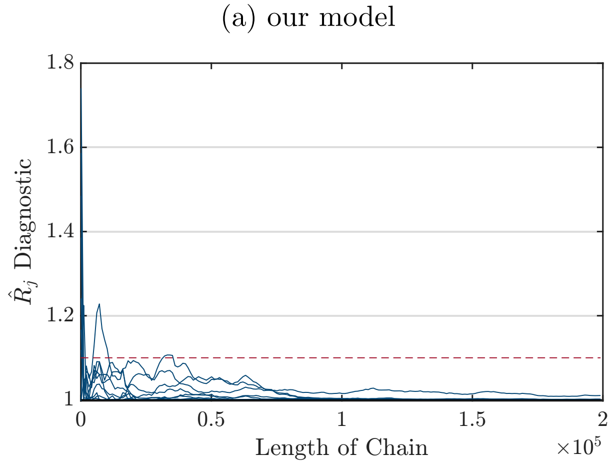
Particle Filter. We additionally compute the likelihood using a bootstrap particle filter to compare relative likelihood evaluations across filtering methods. The agreement in likelihood values and posterior implications across filters suggests that the Extended Kalman Filter provides a reliable approximation in our application.

Under the particle filter, particles are propagated using the fully nonlinear transition equation implied by the third-order solution, weighted using the Gaussian measurement density, and resampled using systematic resampling when the effective sample size falls below a threshold.

The algorithm we use recursively produces discrete approximations to the distribution of states s_t conditional on time $t-1$ information (forecasting distribution) as well as time t information (updated distribution). The algorithm takes existing particles s_{t-1}^j, simulating forward using the state equation to obtain particles s_t^j, and then reweights those particles based on the new data. We can compute the likelihood as a byproduct of computing those weights.

The algorithm proceeds as follows.

Figure D.13: MCMC Chain Diagnostics



1. Initialization ($t = 0$). Initialize N particles $\{s_0^i\}_{i=1}^N$ at the deterministic steady state of the model. These particles are subsequently dispersed through the nonlinear state transition equation.

2. For each period $t = 1, \ldots, T$:

   (a) Forecast step (state propagation). Given particles $\{s_{t-1}^i\}_{i=1}^N$ approximating $p(s_{t-1} \mid y^{t-1}, \vartheta)$, propagate each particle forward using the nonlinear state equation:

   $$s_t^i = \Phi(s_{t-1}^i, \epsilon_t^i, \vartheta), \qquad \epsilon_t^i \sim \mathcal{N}(0, \Omega),$$

   yielding a sample from the predictive distribution $p(s_t \mid y^{t-1}, \vartheta)$.

   (b) Measurement density evaluation. For each particle s_t^i, evaluate the likelihood of the observed data using the measurement equation

   $$y_t = H_t s_t + \nu_t, \qquad \nu_t \sim \mathcal{N}(0, R).$$

   The measurement density is therefore

   $$p(y_t \mid s_t^i, \vartheta) = \mathcal{N}\left(y_t; H_t s_t^i, R\right).$$

   (c) Weighting and likelihood evaluation. Compute unnormalized importance weights

   $$\tilde{w}_t^i = p(y_t \mid s_t^i, \vartheta),$$

   and approximate the one-step-ahead predictive density by Monte Carlo integration:

   $$p(y_t \mid y^{t-1}, \vartheta) \approx \frac{1}{N} \sum_{i=1}^N \tilde{w}_t^i.$$

   Normalize the weights so that $w_t^i = \tilde{w}_t^i / \sum_j \tilde{w}_t^j$.

(d) Resampling. Compute the effective sample size

$$\text{ESS}_t = \frac{1}{\sum_{i=1}^{N}(w_t^i)^2}.$$

If ESS_t/N falls below a pre-specified threshold, or during periods with unusually large shocks, resample the particles using systematic resampling. After resampling, reset all weights to $1/N$. This produces an equally weighted sample approximating $p(s_t \mid y^t, \vartheta)$.

3. Log-likelihood construction. The log-likelihood of the sample is obtained from the prediction error decomposition:

$$\log p(y^T \mid \vartheta) = \sum_{t=1}^{T} \log p(y_t \mid y^{t-1}, \vartheta).$$

We parallelize the algorithm across particles.

We use 100,000 particles, a measurement error that is 25% of the variance of the data (and a somewhat higher measurement error for output growth), and drop observations from 2008Q4, 2009Q1, 2020Q2, and 2020Q3, for which there are exceptionally large movements in the data.

Figure D.14 plots the value of the negative log-likelihoods at different parameter values for the EKF and the particle filter, holding fixed all other parameter values. The figure shows that the two filters give similar relative likelihood rankings across parameter values near to the modal posterior values computed under the EKF.

D.3 Parameter Estimates.

We assign the same values as in the baseline to the discount factor, demand elasticity, returns to scale. We set γ, the inverse of the labor supply elasticity, equal to 2.

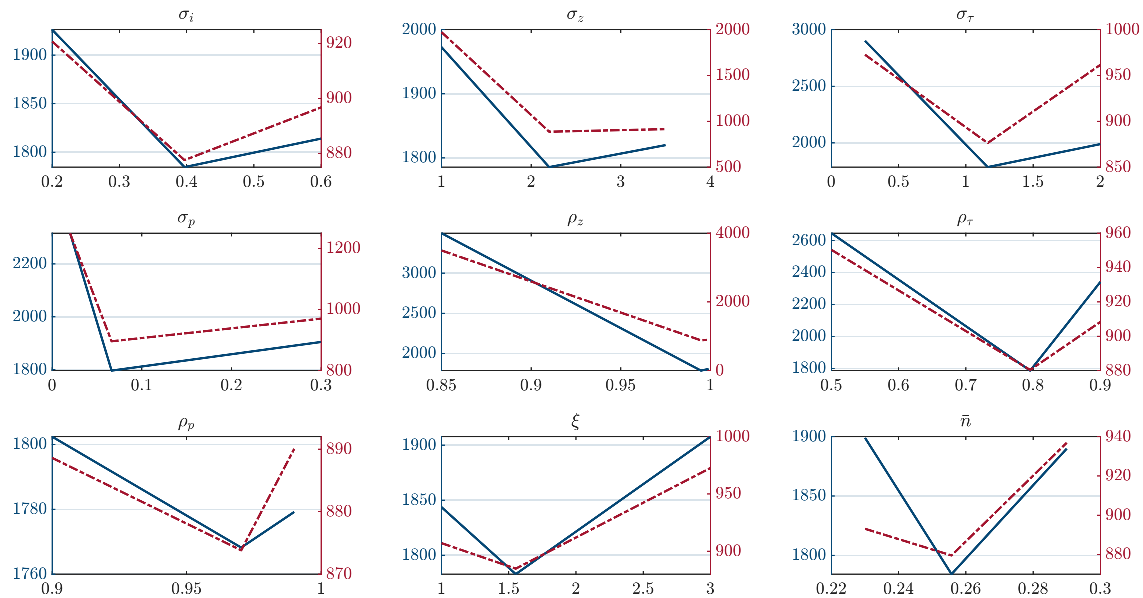
Table D.1 reports the parameters estimates from the Bayesian estimation. We choose standard priors for the model's parameters. We find that, at the median of the posterior distribution, the value of ξ and $\bar{n}$ are similar to those estimated in the baseline model.

D.4 Model Fit and Slope of the Phillips Curve

As in the baseline, to gauge how the Phillips curve evolves over time, we initialize the economy in the stochastic steady state in 1962 and use the non-linear solution of the model to back out the sequences of productivity, cost-push and monetary shocks that reproduce the observed time series for inflation, output growth, the nominal interest rate, and the inflation target series. Figure D.15 plots the shocks. Figure D.16 shows the time series except for the inflation target, which we match by construction. Importantly, as the bottom-right panel of the figure shows, the model reproduces relatively well the time series for the fraction of price changes.

Figure D.17 plots the slope of the model in the left panel, as well as the slope implied by each of the shocks individually in the right panel. As the left panel shows, the slope is similar to that implied by our baseline framework; the right panel shows the importance of

Figure D.14: Comparing EKF and Particle Filter



Notes: For each parameter, we choose 3 values than span a range around the modal posterior estimate under the EKF. The LHS axis shows the value of the negative log-likelihood computed under the EKF. The RHS axis shows the value of the negative log-likelihood computed under the particle filter.

nonlinearities in driving the slope during the high inflation episodes of the 1970s and early 1980s.

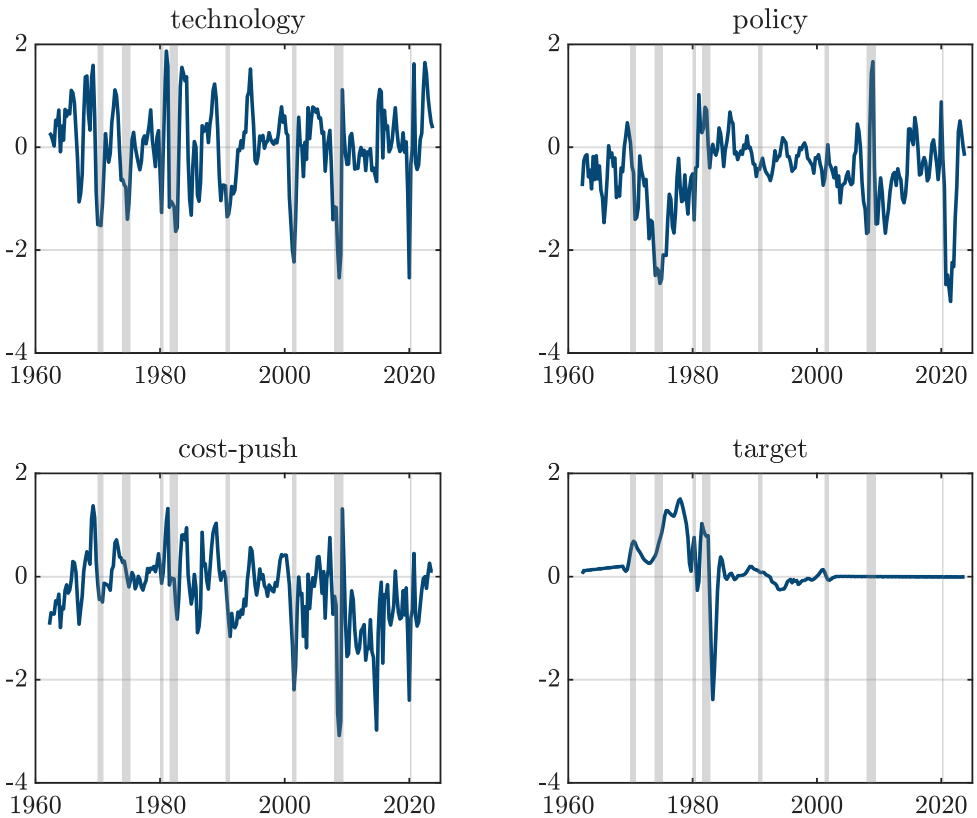
E Proof for the Welfare Function

The proof is divided into a sequence of lemmas. Lemma 2 computes the steady-state TFP losses and output gap as functions of inflation and the fraction of price changes. Lemma 3 derives the second-order approximation of the period utility. Lemma 4 approximates the law of motion for price dispersion. Lemma 5 combines these results to express the welfare function in terms of output, inflation, and the frequency of price changes. Using Lemma 5, we derive Propositions 1, 2, and 3 in the main text.

We remind the reader that for any variable z_t, we use the following notation:

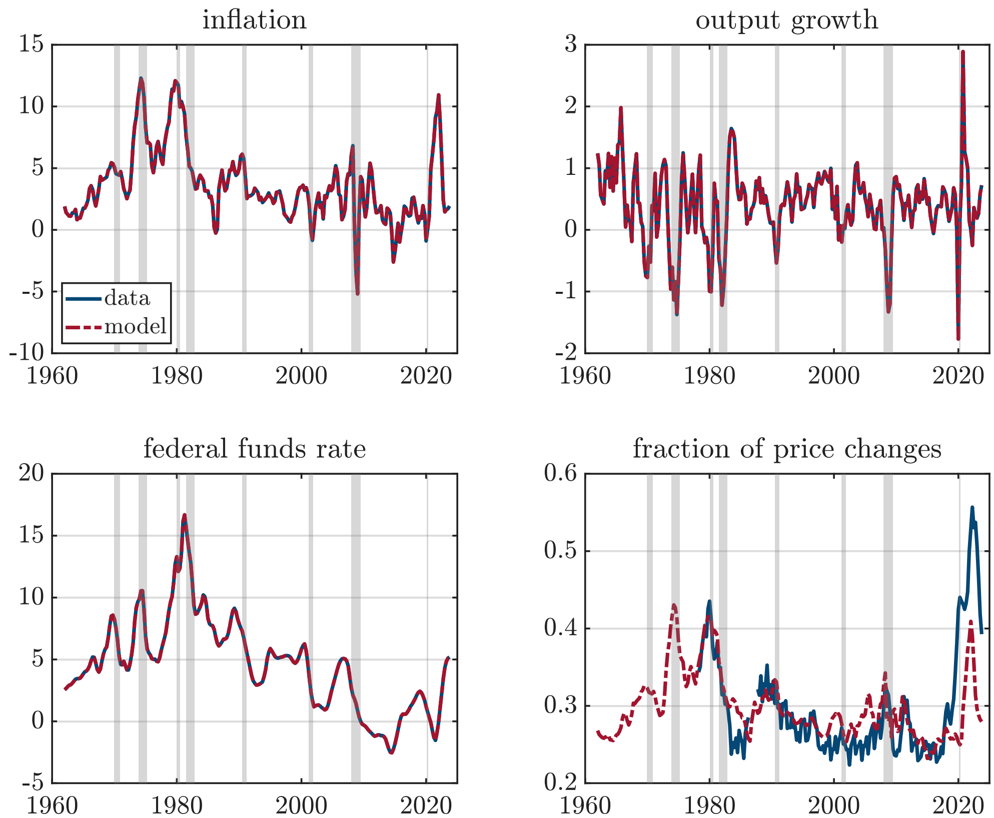
- its steady-state value is denoted by z,
- its flexible-price counterpart is denoted by z_t^F,
- its log-deviation from steady state is $\hat{z}_t = \log(z_t/z)$,
- its log-deviation from the flexible-price counterpart is $\tilde{z}_t = \log(z_t/z_t^F)$, except for the fraction of price changes, where we define $\tilde{n}_t = \frac{n_t - \bar{n}}{\bar{n}}$.

Figure D.15: Shocks



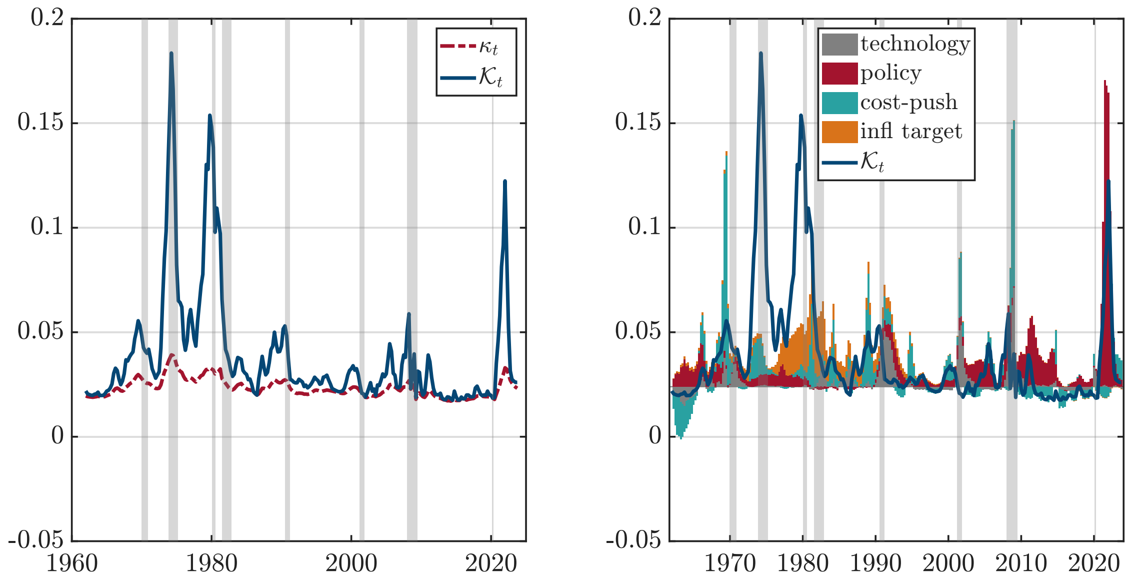
Notes: The gray bars indicate NBER recessions.

Figure D.16: Inflation, Output Growth, Interest Rate and the Fraction of Price Changes



Notes: The gray bars indicate NBER recessions.

Figure D.17: Slope of the Phillips Curve



Notes: The gray bars indicate NBER recessions.

Table D.1: Parameter Estimates, Model with Multiple Aggregate Shocks

	Prior			Posterior			
	Type	Mean	SD	Mode	Median	5%	95%
				A. our model			
$100 \times \sigma_\varepsilon$	IG	1.0	2.0	0.397	0.393	0.361	0.429
$100 \times \sigma_z$	IG	1.0	2.0	2.201	2.215	2.037	2.399
$10 \times \sigma_\tau$	IG	1.0	2.0	1.162	1.161	1.017	1.275
$100 \times \sigma_p$	IG	1.0	2.0	0.066	0.066	0.057	0.077
ρ_z	B	0.5	0.1	0.995	0.995	0.993	0.998
ρ_τ	B	0.5	0.1	0.796	0.797	0.781	0.814
ρ_p	B	0.5	0.1	0.970	0.969	0.959	0.977
ξ	N	1.75	0.1	1.552	1.547	1.410	1.698
$\bar{n}$	N	0.25	0.03	0.256	0.256	0.252	0.260
				B. calvo			
$100 \times \sigma_\varepsilon$	IG	1.0	2.0	0.393	0.395	0.363	0.430
$100 \times \sigma_z$	IG	1.0	2.0	2.602	2.604	2.340	2.868
$10 \times \sigma_\tau$	IG	1.0	2.0	1.771	1.803	1.666	1.980
$100 \times \sigma_p$	IG	1.0	2.0	0.056	0.057	0.049	0.066
ρ_z	B	0.5	0.1	0.999	0.997	0.994	0.999
ρ_τ	B	0.5	0.1	0.769	0.767	0.735	0.780
ρ_p	B	0.5	0.1	0.979	0.977	0.968	0.985
$\bar{n}$	N	0.3	0.05	0.313	0.314	0.308	0.323

Notes: IG = Inverse Gamma, B = Beta, N = Normal distribution.

Lemma 2. *The TFP losses due to price dispersion in the steady state, $\tilde{x} = \log\left(x^{-\theta}/1\right)$, and the output gap, $\tilde{y} = \log(y/y^F)$, are given by:*

$$\tilde{y} = \eta\left[-\frac{\eta+\theta(1-\eta)}{\eta(\theta-1)}\log\left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right)+\log\left(\frac{1-\beta(1-n)\pi^{\frac{\theta}{\eta}}}{1-\beta(1-n)\pi^{\theta-1}}\right)\right], \tag{98}$$

$$\tilde{x} = \frac{\theta}{\theta-1}\log\left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right)+\eta\log\left(\frac{n}{1-(1-n)\pi^{\frac{\theta}{\eta}}}\right). \tag{99}$$

Proof. If prices are flexible, the relevant equilibrium conditions are:

$$y^F = (h^F)^\eta, \tag{100}$$

$$y^F = \eta\frac{y^F}{h^F}, \tag{101}$$

where the first equation represents feasibility, and the second follows from the optimal labor choice: the marginal rate of substitution between consumption and labor equals the marginal product of labor. Solving, we find $h^F = \eta$ and $y^F = \eta^\eta$.

Under sticky prices, the steady-state equilibrium conditions are:

$$(p^*)^{1+\theta\left(\frac{1}{\eta}-1\right)} = \frac{1}{\eta}\frac{b_2}{b_1}, \tag{102}$$

$$b_2 = y^{1/\eta} + \beta(1-n)\pi^{\theta/\eta} b_2, \tag{103}$$

$$b_1 = 1 + \beta(1-n)\pi^{\theta-1} b_1, \tag{104}$$

$$1 = n(p^*)^{1-\theta} + (1-n)\pi^{\theta-1}, \tag{105}$$

$$x^{-\theta/\eta} = n(p^*)^{-\theta/\eta} + (1-n)\pi^{\theta/\eta} x^{-\theta/\eta}, \tag{106}$$

$$y = x^\theta h^\eta. \tag{107}$$

Importantly, equation (107) describes the labor allocated to production, not to price-setting.

From equations (102)–(105), we obtain:

$$\left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right)^{\frac{1+\theta\left(\frac{1}{\eta}-1\right)}{1-\theta}} = \frac{y^{1/\eta}}{\eta}\frac{1-\beta(1-n)\pi^{\theta-1}}{1-\beta(1-n)\pi^{\theta/\eta}}. \tag{108}$$

Since $y^F = \eta^\eta$, we can rewrite the expression as:

$$\left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right)^{\frac{1+\theta\left(\frac{1}{\eta}-1\right)}{1-\theta}} = \left(\frac{y}{y^F}\right)^{1/\eta}\frac{1-\beta(1-n)\pi^{\theta-1}}{1-\beta(1-n)\pi^{\theta/\eta}}. \tag{109}$$

Taking logs, the steady-state output gap is:

$$\tilde{y} = \eta\left[-\frac{\eta+\theta(1-\eta)}{\eta(\theta-1)}\log\left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right) + \log\left(\frac{1-\beta(1-n)\pi^{\theta/\eta}}{1-\beta(1-n)\pi^{\theta-1}}\right)\right]. \tag{110}$$

From equation (106), steady-state price dispersion is given by:

$$x^{-\theta/\eta} = \frac{n}{1-(1-n)\pi^{\theta/\eta}}\left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right)^{\frac{-\theta}{\eta(1-\theta)}}, \tag{111}$$

which implies:

$$x^{-\theta} = \left[\frac{n}{1-(1-n)\pi^{\theta/\eta}}\right]^\eta\left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right)^{\frac{-\theta}{1-\theta}}. \tag{112}$$

Taking logs:

$$\tilde{x} = \frac{\theta}{\theta-1}\log\left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right) + \eta\log\left(\frac{n}{1-(1-n)\pi^{\theta/\eta}}\right). \tag{113}$$

□

Lemma 3. *Define $\Delta_t := x_t^{-\theta/\eta}$. Then, the flow utility function satisfies*

$$u(y_t, h_t, n_t) = -\frac{1}{2}\left[\frac{1}{\eta}\tilde{y}_t^2 + 2\eta\tilde{\Delta}_t + 2\tilde{\Delta}_t\tilde{y}_t + \eta\tilde{\Delta}_t^2\right] - \frac{\xi\bar{n}^2}{2}\tilde{n}_t^2 + \eta\log(\eta) - \eta + O\left(\|(\tilde{y}_t, \tilde{\Delta}_t)\|^3\right).$$

Proof. We approximate the utility function around the flexible-price allocation. Three points are important. First, since the model includes only monetary shocks, we omit time subscripts for variables under flexible prices: $(Y^F, h^F) = (\eta^\eta, \eta)$. Second, we approximate the frequency of price changes around $\bar{n}$ rather than n or 1, i.e., around the point where labor supply for price adjustment is zero. Finally, as mentioned before, $\tilde{n}_t = (n_t - \bar{n})/\bar{n}$.

We expand the utility function up to second order:

$$\begin{aligned}
u(y_t, h_t, n_t) &= \log(y_t) - h_t - \frac{\xi}{2}(n_t - \bar{n})^2 \\
&=^{(1)} \underbrace{\log(y^F) - h^F}_{=\eta\log(\eta)-\eta} + \underbrace{\frac{1}{y^F}(y_t - y^F) - (h_t - h^F)}_{\text{first order}} \underbrace{-\frac{1}{2}\frac{(y_t - y^F)^2}{(y^F)^2} - \frac{\xi}{2}(n_t - \bar{n})^2}_{\text{second order}} + O\left(\|y_t - y^F\|^3\right) \\
&=^{(2)} \frac{y_t - y^F}{y^F} - \frac{1}{2}\left(\frac{y_t - y^F}{y^F}\right)^2 - \underbrace{h^F}_{=\eta}\frac{h_t - h^F}{h^F} \\
&\quad - \frac{\xi\bar{n}^2}{2}\left(\frac{n_t - \bar{n}}{\bar{n}}\right)^2 + \eta\log(\eta) - \eta + O\left(\|y_t - y^F\|^3\right) \\
&=^{(3)} \tilde{y}_t + \frac{1}{2}\tilde{y}_t^2 - \frac{1}{2}\left(\tilde{y}_t^2 + \tilde{y}_t^3 + \frac{1}{4}\tilde{y}_t^4\right) - \eta\left(\tilde{h}_t + \frac{1}{2}\tilde{h}_t^2\right) \\
&\quad - \frac{\xi\bar{n}^2}{2}\tilde{n}_t^2 + \eta\log(\eta) - \eta + O\left(\|(\tilde{y}_t, \tilde{h}_t)\|^3\right) \\
&=^{(4)} \tilde{y}_t - \eta\left(\tilde{h}_t + \frac{1}{2}\tilde{h}_t^2\right) - \frac{\xi\bar{n}^2}{2}\tilde{n}_t^2 + \eta\log(\eta) - \eta + O\left(\|(\tilde{y}_t, \tilde{h}_t)\|^3\right)
\end{aligned} \tag{114}$$

In step (1), we apply a second-order Taylor expansion around the flexible-price allocation. In step (2), we regroup terms and use the flexible-price equilibrium conditions. In step (3), we use the second-order approximation:

$$\frac{z_t - z^F}{z^F} = \frac{z^F e^{\tilde{z}_t} - z^F}{z^F} \approx \tilde{z}_t + \frac{1}{2}\tilde{z}_t^2 + O\left(\|\tilde{z}_t\|^3\right).$$

In step (4), we simplify and collect higher-order terms into the residual.

Now, observe that from the production function:

$$y_t = \frac{h_t^\eta}{\Delta_t^\eta}, \quad \text{and} \quad y^F = (h^F)^\eta,$$

so that

$$\tilde{y}_t = -\eta\tilde{\Delta}_t + \eta\tilde{h}_t, \quad \text{and} \quad \tilde{h}_t = \frac{1}{\eta}\tilde{y}_t + \tilde{\Delta}_t.$$

Substituting into the expression above, we obtain:

$$
\begin{aligned}
u(y_t, h_t, n_t) &= \tilde{y}_t - \eta \left(\frac{1}{\eta}\tilde{y}_t + \tilde{\Delta}_t + \frac{1}{2}\left(\frac{1}{\eta}\tilde{y}_t + \tilde{\Delta}_t\right)^2 \right) - \frac{\xi \bar{n}^2}{2}\tilde{n}_t^2 + \eta \log(\eta) - \eta + O\left(\|(\tilde{y}_t, \tilde{h}_t)\|^3 \right) \\
&= -\eta \tilde{\Delta}_t - \frac{\eta}{2}\left(\frac{1}{\eta^2}\tilde{y}_t^2 + 2 \cdot \frac{1}{\eta}\tilde{y}_t \tilde{\Delta}_t + \tilde{\Delta}_t^2 \right) - \frac{\xi \bar{n}^2}{2}\tilde{n}_t^2 + \eta \log(\eta) - \eta + O\left(\|(\tilde{y}_t, \tilde{\Delta}_t)\|^3 \right) \\
&= -\frac{1}{2}\left(\frac{1}{\eta}\tilde{y}_t^2 + 2\eta \tilde{\Delta}_t + 2\tilde{y}_t \tilde{\Delta}_t + \eta \tilde{\Delta}_t^2 \right) - \frac{\xi \bar{n}^2}{2}\tilde{n}_t^2 + \eta \log(\eta) - \eta + O\left(\|(\tilde{y}_t, \tilde{\Delta}_t)\|^3 \right)
\end{aligned} \tag{115}
$$

which proves the result. □

Lemma 4. *If we ignore the second-order terms of the form $\hat{\Delta}_{t-1} z_t$, where $z_t \in \{\hat{\pi}_t, \hat{n}_t, \hat{\Delta}_{t-1}\}$ for all t, then the law of motion for price dispersion, up to second order, is given by:*

$$
\hat{\Delta}_t = \Lambda_\pi \hat{\pi}_t - \Lambda_n \hat{n}_t + \rho_\Delta \hat{\Delta}_{t-1} + \frac{1}{2}\left[\Lambda_{\pi^2}\hat{\pi}_t^2 - 2\Lambda_{n\pi}\hat{\pi}_t \hat{n}_t + \Lambda_{n^2}\hat{n}_t^2 \right] + O(||(\hat{\Delta}_t, \hat{\pi}_t, \hat{n}_t, \hat{\Delta}_{t-1})||^3) \tag{116}
$$

where

$$
\Lambda_\pi = \frac{\theta}{\eta}(1-n)\pi^{\theta-1}\left[-\frac{1-(1-n)\pi^{\theta/\eta}}{1-(1-n)\pi^{\theta-1}} + \pi^{1+\theta\left(\frac{1}{\eta}-1\right)} \right] \tag{117}
$$

$$
\Lambda_n = \left(1-(1-n)\pi^{\frac{\theta}{\eta}} \right)\left[\frac{\theta}{\eta(\theta-1)} \frac{1-\pi^{\theta-1}}{1-(1-n)\pi^{\theta-1}} - 1 \right] + n\pi^{\frac{\theta}{\eta}} \tag{118}
$$

$$
\Lambda_{\pi^2} = \frac{\theta}{\eta}(1-n)\pi^{\theta-1}\left[\frac{\eta+\theta(1-\eta)}{\eta} \frac{1-(1-n)\pi^{\frac{\theta}{\eta}}}{(1-(1-n)\pi^{\theta-1})^2}(1-n)\pi^{\theta-1} - (\theta-1)\frac{1-(1-n)\pi^{\frac{\theta}{\eta}}}{1-(1-n)\pi^{\theta-1}} + \frac{\theta}{\eta}\pi^{\frac{\eta+\theta(1-\eta)}{\eta}} \right] \tag{119}
$$

$$
\Lambda_{n\pi} = \frac{\theta}{\eta}\pi^{\theta-1} n \left[-\frac{1-(1-n)\pi^{\frac{\theta}{\eta}}}{1-(1-n)\pi^{\theta-1}} + \pi^{\frac{\eta+\theta(1-\eta)}{\eta}} + (\pi^{\theta-1}-1)\frac{\eta+\theta(1-\eta)}{\eta(\theta-1)} \frac{1-n}{n} \frac{1-(1-n)\pi^{\frac{\theta}{\eta}}}{(1-(1-n)\pi^{\theta-1})^2} \right] \tag{120}
$$

$$
\Lambda_{n^2} = \frac{\eta+\theta(1-\eta)}{\eta(\theta-1)} \frac{1-(1-n)\pi^{\theta/\eta}}{(1-(1-n)\pi^{\theta-1})^2}(\pi^{\theta-1}-1)\left[\frac{\eta+\theta(1-\eta)}{\eta(\theta-1)}\left(\pi^{\theta-1}-1\right) + n\pi^{\theta-1} \right] \tag{121}
$$

$$
+ \frac{1-(1-n)\pi^{\theta/\eta}}{1-(1-n)\pi^{\theta-1}} n\pi^{\theta-1} - n\pi^{\frac{\theta}{\eta}} \tag{122}
$$

$$
\rho_\Delta = (1-n)\pi^{\frac{\theta}{\eta}} \tag{123}
$$

with the following properties:

1. *At zero trend inflation:*

$$
\Lambda_\pi(\bar{n}, 1) = \Lambda_n(\bar{n}, 1) = \Lambda_{n^2}(\bar{n}, 1) = \Lambda_{\pi n}(\bar{n}, 1) = 0 \tag{124}
$$

and

$$
\Lambda_{\pi^2}(\bar{n}, 1) = \frac{\theta}{\eta} \frac{\eta+\theta(1-\eta)}{\eta} \frac{1-\bar{n}}{\bar{n}}. \tag{125}
$$

2. Λ_π, Λ_n, Λ_{π^2}, *and* $\Lambda_{\pi n}$ *are positive and increasing in* π *for* π *close to 1.*

Proof. We proceed to do a second order Taylor approximation over

$$\Delta_t = n_t \left(\frac{1-(1-n_t)\pi_t^{\theta-1}}{n_t} \right)^{\frac{-\theta}{\eta(1-\theta)}} + (1-n_t)\pi_t^{\frac{\theta}{\eta}} \Delta_{t-1}. \tag{126}$$

The steady-state price dispersion satisfies:

$$\Delta = \frac{n \left(\frac{1-(1-n)\pi^{\theta-1}}{n} \right)^{\frac{-\theta}{\eta(1-\theta)}}}{1-(1-n)\pi^{\frac{\theta}{\eta}}}. \tag{127}$$

Replacing each variable by its log deviation $z_t = ze^{\hat{z}_t}$.

$$H(\hat{\Delta}_t, \hat{\Delta}_{t-1}, \hat{n}_t, \hat{\pi}_t) := -\Delta e^{\hat{\Delta}_t} + ne^{\hat{n}_t} \left(\frac{1-(1-ne^{\hat{n}_t})\pi^{\theta-1} e^{\hat{\pi}_t(\theta-1)}}{ne^{\hat{n}_t}} \right)^{\frac{-\theta}{\eta(1-\theta)}} + (1-ne^{\hat{n}_t})\pi^{\frac{\theta}{\eta}} e^{\hat{\pi}_t \frac{\theta}{\eta}} \Delta e^{\hat{\Delta}_{t-1}}. \tag{128}$$

The second order approximation around $(\hat{\Delta}_t, \hat{\Delta}_{t-1}, \hat{n}_t, \hat{\pi}_t) = (0,0,0,0)$—re-scaled by Δ^{-1}—is given by

$$= 0 \tag{129}$$

$$
\begin{aligned}
&= \underbrace{-1+1-(1-n)\pi^{\frac{\theta}{\eta}}+(1-n)\pi^{\frac{\theta}{\eta}}}_{H(0,0,0,0)/\Delta=0}\\
&\quad + -\hat{\Delta}_t + \underbrace{\frac{\theta}{\eta}(1-n)\pi^{\theta-1}\left[-\frac{1-(1-n)\pi^{\theta/\eta}}{1-(1-n)\pi^{\theta-1}}+\pi^{\frac{\eta+\theta(1-\eta)}{\eta}}\right]}_{=\partial_{\hat{\pi}}H(0,0,0,0)/\Delta}\hat{\pi}_t + \underbrace{(1-n)\pi^{\frac{\theta}{\eta}}}_{=\partial_{\hat{\Delta}_{t-1}}H(0,0,0,0)/\Delta}\hat{\Delta}_{t-1}\\
&\quad + \underbrace{\left[\left(1-(1-n)\pi^{\frac{\theta}{\eta}}\right)\left[1+\frac{\theta}{\eta(\theta-1)}\frac{\pi^{\theta-1}-1}{1-(1-n)\pi^{\theta-1}}\right]-n\pi^{\frac{\theta}{\eta}}\right]}_{=\partial_{\hat{n}_t}H(0,0,0,0)/\Delta}\hat{n}_t\\
&\quad + \frac{1}{2}\Bigg[\underbrace{\frac{\theta}{\eta}(1-n)\pi^{\theta-1}\Bigg[\left(1+\theta\left(\frac{1}{\eta}-1\right)\right)\frac{1-(1-n)\pi^{\frac{\theta}{\eta}}}{(1-(1-n)\pi^{\theta-1})^2}(1-n)\pi^{\theta-1}}\\
&\quad \underbrace{-(\theta-1)\frac{1-(1-n)\pi^{\frac{\theta}{\eta}}}{1-(1-n)\pi^{\theta-1}}+\frac{\theta}{\eta}\pi^{1+\theta\left(\frac{1}{\eta}-1\right)}\Bigg]}_{=\partial^2_{\hat{\pi}_t^2}H(0,0,0,0)/\Delta}\hat{\pi}_t^2\\
&\quad + 2\underbrace{(-1)\frac{\theta}{\eta}\pi^{\theta-1}n\Bigg[-\frac{1-(1-n)\pi^{\frac{\theta}{\eta}}}{1-(1-n)\pi^{\theta-1}}+\pi^{1+\theta\left(\frac{1}{\eta}-1\right)}}\\
&\quad \underbrace{+(\pi^{\theta-1}-1)\frac{\eta+\theta(1-\eta)}{\eta(\theta-1)}\frac{1-n}{n}\frac{1-(1-n)\pi^{\frac{\theta}{\eta}}}{(1-(1-n)\pi^{\theta-1})^2}\Bigg]}_{=\partial^2_{\hat{\pi}_t\hat{n}_t}H(0,0,0,0)/\Delta}\hat{\pi}_t\hat{n}_t\\
&\quad + \underbrace{\Bigg[\frac{\eta+\theta(1-\eta)}{\eta(\theta-1)}\frac{1-(1-n)\pi^{\theta/\eta}}{(1-(1-n)\pi^{\theta-1})^2}(\pi^{\theta-1}-1)\left[\frac{\eta+\theta(1-\eta)}{\eta(\theta-1)}\left(\pi^{\theta-1}-1\right)+n\pi^{\theta-1}\right]}\\
&\quad \underbrace{+\frac{1-(1-n)\pi^{\theta/\eta}}{1-(1-n)\pi^{\theta-1}}n\pi^{\theta-1}-n\pi^{\frac{\theta}{\eta}}\Bigg]}_{=\partial^2_{\hat{n}_t}H(0,0,0,0)/\Delta}\hat{n}_t^2+\frac{\partial^2_{\hat{\Delta}_t^2}H(0,0,0,0)\hat{\Delta}_t^2+2\partial^2_{\hat{\Delta}_{t-1}\hat{\pi}_t}H(0,0,0,0)\hat{\Delta}_{t-1}\hat{\pi}_t}{\Delta}\\
&\quad + \frac{2\partial^2_{\hat{\Delta}_{t-1}\hat{n}_t}H(0,0,0,0)\hat{\Delta}_{t-1}\hat{n}_t+\partial^2_{\hat{\Delta}_{t-1}^2}H(0,0,0,0)\hat{\Delta}_{t-1}^2}{\Delta}\Bigg]\\
&\quad + O(||(\hat{\Delta}_t,\hat{\pi}_t,\hat{n}_t,\hat{\Delta}_{t-1})||^3)
\end{aligned}
\tag{130}
$$

Let us simplify a little the notation with the following definitions:

$$
\begin{aligned}
\Lambda_\pi &= \frac{\theta}{\eta}(1-n)\pi^{\theta-1}\left[-\frac{1-(1-n)\pi^{\theta/\eta}}{1-(1-n)\pi^{\theta-1}} + \pi^{1+\theta\left(\frac{1}{\eta}-1\right)}\right] \\
\Lambda_n &= \left(1-(1-n)\pi^{\frac{\theta}{\eta}}\right)\left[\frac{\theta}{\eta(\theta-1)}\frac{1-\pi^{\theta-1}}{1-(1-n)\pi^{\theta-1}} - 1\right] + n\pi^{\frac{\theta}{\eta}} \\
\Lambda_{\pi^2} &= \frac{\theta}{\eta}(1-n)\pi^{\theta-1}\Bigg[\left(1+\theta\left(\frac{1}{\eta}-1\right)\right)\frac{1-(1-n)\pi^{\frac{\theta}{\eta}}}{(1-(1-n)\pi^{\theta-1})^2}(1-n)\pi^{\theta-1} \\
&\qquad -(\theta-1)\frac{1-(1-n)\pi^{\frac{\theta}{\eta}}}{1-(1-n)\pi^{\theta-1}} + \frac{\theta}{\eta}\pi^{1+\theta\left(\frac{1}{\eta}-1\right)}\Bigg] \\
\Lambda_{n\pi} &= \frac{\theta}{\eta}\pi^{\theta-1}n\Bigg[-\frac{1-(1-n)\pi^{\frac{\theta}{\eta}}}{1-(1-n)\pi^{\theta-1}} + \pi^{\frac{\eta+\theta(1-\eta)}{\eta}} \\
&\qquad + (\pi^{\theta-1}-1)\frac{\eta+\theta(1-\eta)}{\eta(\theta-1)}\frac{1-n}{n}\frac{1-(1-n)\pi^{\frac{\theta}{\eta}}}{(1-(1-n)\pi^{\theta-1})^2}\Bigg] \\
\Lambda_{n^2} &= \frac{\eta+\theta(1-\eta)}{\eta(\theta-1)}\frac{1-(1-n)\pi^{\theta/\eta}}{(1-(1-n)\pi^{\theta-1})^2}(\pi^{\theta-1}-1)\left[\frac{\eta+\theta(1-\eta)}{\eta(\theta-1)}\left(\pi^{\theta-1}-1\right) + n\pi^{\theta-1}\right] \\
&\qquad + \frac{1-(1-n)\pi^{\theta/\eta}}{1-(1-n)\pi^{\theta-1}}n\pi^{\theta-1} - n\pi^{\frac{\theta}{\eta}} \\
\rho_\Delta &= (1-n)\pi^{\frac{\theta}{\eta}}
\end{aligned}
$$

Using these definition, we can write

$$
\hat{\Delta}_t = \Lambda_\pi\hat{\pi}_t - \Lambda_n\hat{n}_t + \rho_\Delta\hat{\Delta}_{t-1} + \frac{1}{2}[\Lambda_{\pi^2}\hat{\pi}_t^2 - 2\Lambda_{n\pi}\hat{\pi}_t\hat{n}_t + \Lambda_{n^2}\hat{n}_t^2] + O(||(\hat{\Delta}_t,\hat{\pi}_t,\hat{n}_t,\hat{\Delta}_{t-1})||^3). \tag{131}
$$

It is easy to verify that at zero trend inflation: $\Lambda_n = \Lambda_\pi = \Lambda_{n\pi} = \Lambda_{n^2} = 0$ and for Λ_{π^2} we have

$$
\begin{aligned}
\Lambda_{\pi^2} &= \frac{\theta}{\eta}(1-\bar{n})\left[\frac{\eta+\theta(1-\eta)}{\eta}\frac{1-(1-\bar{n})}{(1-(1-\bar{n}))^2}(1-\bar{n}) - (\theta-1)\frac{1-(1-\bar{n})}{1-(1-\bar{n})} + \frac{\theta}{\eta}\right] && (132) \\
&= \frac{\theta}{\eta}(1-\bar{n})\left[\frac{\eta+\theta(1-\eta)}{\eta}\frac{1}{\bar{n}}(1-\bar{n}) - (\theta-1) + \frac{\theta}{\eta}\right] && (133) \\
&= \frac{\theta}{\eta}(1-\bar{n})\frac{\eta+\theta(1-\eta)}{\eta}\left[\frac{1}{\bar{n}}(1-\bar{n}) + 1\right] && (134) \\
&= \frac{\theta}{\eta}\frac{\eta+\theta(1-\eta)}{\eta}\frac{(1-\bar{n})}{\bar{n}} && (135)
\end{aligned}
$$

The properties regarding the monotonicity of the coefficients for small inflation follow from straightforward but lengthy a first order Taylor around $(\pi, n) = (1, \bar{n})$ in the coefficients and are thus omitted. □

Lemma 5. *Define $\mathbb{W}(\pi) = -(V - V^F)(1-\beta)$ as the expected ergodic difference between the utility of the representative household in the economy with sticky prices and in the economy with flexible prices, where V and V^F are the utility of the representative household in the economy without and with flexible prices. Assume the approximations in Lemmas 3 and 4 without higher order terms. If we ignore terms $\hat{\Delta}_{t-1}\hat{z}_t$ with $\hat{z}_t \in \left\{\hat{\Delta}_{t-1}, \hat{y}_t\right\}$ for all t, then*

$$\mathbb{W}(\pi) = \Xi_0(\pi) + \Xi_1(\pi) + \frac{\rho_\Delta(1-\beta)}{1-\beta\rho_\Delta}\left(\eta + \tilde{y} + \tilde{\Delta}\eta\right)\hat{\Delta}_{-1} \tag{136}$$

$$\Xi_0(\pi) = \tilde{x} + \frac{\xi\bar{n}^2}{2}\tilde{n}^2 + \frac{(\tilde{y}+\tilde{x})^2}{2\eta} \tag{137}$$

$$\begin{aligned}
\Xi_1(\pi) = {} & (1-\beta)\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[\frac{1}{2\eta}\hat{y}_t^2 + \frac{\xi\bar{n}^2}{2}(\tilde{n}_t - \tilde{n})^2\right]\right] \\
& + (1-\beta)\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[\left(\frac{\eta+\tilde{y}+\tilde{x}}{(1-\beta\rho_\Delta)}\frac{\Lambda_{\pi^2}}{2} + \frac{\eta\Lambda_\pi^2}{2}\right)\hat{\pi}_t^2 - 2\left(\frac{\eta+\tilde{y}+\tilde{x}}{(1-\beta\rho_\Delta)}\frac{\Lambda_{\pi n}}{2} + \frac{\eta\Lambda_\pi\Lambda_n}{2}\right)\hat{\pi}_t\hat{n}_t\right]\right] \\
& + (1-\beta)\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[\frac{\tilde{y}+\tilde{x}}{\eta}\hat{y}_t + \frac{\eta+\tilde{y}+\tilde{\Delta}\eta}{(1-\beta\rho_\Delta)}(\Lambda_\pi\hat{\pi}_t - \Lambda_n\hat{n}_t) + \xi\bar{n}^2(\tilde{n}_t - \tilde{n})\tilde{n}\right]\right] \\
& + (1-\beta)\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[\Lambda_\pi\hat{\pi}_t\hat{y}_t - \Lambda_n\hat{n}_t\hat{y}_t + \left(\frac{\eta+\tilde{y}+\tilde{x}}{(1-\beta\rho_\Delta)}\frac{\Lambda_{n^2}}{2} + \frac{\eta\Lambda_n^2}{2}\right)\hat{n}_t^2\right]\right]
\end{aligned} \tag{138}$$

Proof. From Lemma 3, we have that

$$\begin{aligned}
V := {} & \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[log(y_t) - h_t - \frac{\xi}{2}(n_t - \bar{n})^2\right]\right] \\
= {} & \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[-\frac{1}{2}\left[\frac{1}{\eta}\tilde{y}_t^2 + 2\eta\tilde{\Delta}_t + 2\tilde{\Delta}_t\tilde{y}_t + \eta\tilde{\Delta}_t^2\right] - \frac{\xi\bar{n}^2}{2}\tilde{n}_t^2 + \eta\log(\eta) - \eta + O\left(||(\tilde{y}_t,\tilde{\Delta}_t)||^3\right)\right]\right] \\
= {} & \underbrace{\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[-\frac{1}{2\eta}\tilde{y}_t^2 - \frac{\xi\bar{n}^2}{2}\tilde{n}_t^2\right]\right]}_{=\mathcal{A}_1} + \underbrace{\eta\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[-\tilde{\Delta}_t\right]\right]}_{=\mathcal{A}_2} \\
& + \underbrace{\mathbb{E}_0\left[-\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left[2\tilde{\Delta}_t\tilde{y}_t + \eta\tilde{\Delta}_t^2\right]\right]}_{=\mathcal{A}_3} + \frac{\eta\log(\eta)-\eta}{1-\beta} + \sum_{t=0}^{\infty}\beta^t O\left(\mathbb{E}_0[||(\tilde{y}_t,\tilde{\Delta}_t)||^3]\right),
\end{aligned} \tag{139}$$

where in the last step, we use the fact that $\mathbb{E}_0\left[O\left(||(\tilde{y}_t,\tilde{\Delta}_t)||^3\right)\right] = O\left(\mathbb{E}_0[||(\tilde{y}_t,\tilde{\Delta}_t)||^3]\right)$.

First, we characterize $\mathcal{A}_1$. Since $\tilde{y}_t = \hat{y}_t + \tilde{y}$, we have that

$$\begin{aligned}
\mathcal{A}_1 &= \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[-\frac{1}{2\eta}(\hat{y}_t+\tilde{y})^2 - \frac{\xi\bar{n}^2}{2}(\tilde{n}_t - \tilde{n} + \tilde{n})^2\right]\right] \\
&= \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[-\frac{1}{2\eta}(\hat{y}_t^2 + 2\hat{y}_t\tilde{y} + \tilde{y}^2) - \frac{\xi\bar{n}^2}{2}[(\tilde{n}_t-\tilde{n})^2 + 2\tilde{n}(\tilde{n}_t-\tilde{n}) + \tilde{n}^2]\right]\right] \\
&= \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[-\frac{1}{2\eta}(\hat{y}_t^2 + 2\hat{y}_t\tilde{y}) - \frac{\xi\bar{n}^2}{2}((\tilde{n}_t-\tilde{n})^2 + 2(\tilde{n}_t-\tilde{n})\tilde{n})\right]\right] - \frac{\tilde{y}^2}{2\eta(1-\beta)} - \frac{\xi\bar{n}^2}{2(1-\beta)}\tilde{n}^2.
\end{aligned} \tag{140}$$

Now, we show that $\mathcal{A}_2$ depends on the history of observable inflation and frequency of price changes. Since $\tilde{\Delta}_t = \hat{\Delta}_t + \tilde{\Delta}$, we have that

$$\mathcal{A}_2 = -\frac{\tilde{\Delta}\eta}{1-\beta} - \eta\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\hat{\Delta}_t\right], \tag{141}$$

and from Lemma 4, we have that

$$\hat{\Delta}_t = \Lambda_\pi\hat{\pi}_t - \Lambda_n\hat{n}_t + \rho_\Delta\hat{\Delta}_{t-1} + \frac{1}{2}[\Lambda_{\pi^2}\hat{\pi}_t^2 - 2\Lambda_{n\pi}\hat{\pi}_t\hat{n}_t + \Lambda_{n^2}\hat{n}_t^2] \tag{142}$$

Define $\hat{q}_t := \Lambda_\pi\hat{\pi}_t - \Lambda_n\hat{n}_t + \frac{1}{2}[\Lambda_{\pi^2}\hat{\pi}_t^2 - 2\Lambda_{n\pi}\hat{\pi}_t\hat{n}_t + \Lambda_{n^2}\hat{n}_t^2]$. We need to solve the following convolution

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\hat{\Delta}_t\right] \text{ with } \hat{\Delta}_t = \hat{q}_t + \rho_\Delta\hat{\Delta}_{t-1}. \tag{143}$$

For solving this iteration, define

$$W_s = \mathbb{E}_s\left[\sum_{t=0}^{\infty}\beta^t\hat{\Delta}_{s+t}\right]. \tag{144}$$

with $\hat{\Delta}_t = \hat{q}_t + \rho_\Delta\hat{\Delta}_{t-1}$.

$$\begin{aligned}
W_s &= \mathbb{E}_s\left[\sum_{t=0}^{\infty}\beta^t\hat{\Delta}_{s+t}\right] \\
&= \mathbb{E}_s\left[\sum_{t=0}^{\infty}\beta^t\left(\hat{q}_{s+t} + \rho_\Delta\hat{\Delta}_{s+t-1}\right)\right] \\
&= \mathbb{E}_s\left[\sum_{t=0}^{\infty}\beta^t\hat{q}_{s+t}\right] + \rho_\Delta\mathbb{E}_s\left[\sum_{t=0}^{\infty}\beta^t\hat{\Delta}_{s+t-1}\right] \\
&= \mathbb{E}_s\left[\sum_{t=0}^{\infty}\beta^t\hat{q}_{s+t}\right] + \rho_\Delta\mathbb{E}_s\left[\hat{\Delta}_{s-1} + \beta\sum_{t=0}^{\infty}\beta^t\hat{\Delta}_{s+t}\right] \\
&= \mathbb{E}_s\left[\sum_{t=0}^{\infty}\beta^t\hat{q}_{s+t}\right] + \rho_\Delta[\hat{\Delta}_{s-1} + \beta W_s].
\end{aligned} \tag{145}$$

Therefore,

$$W_s = \frac{1}{1-\beta\rho_\Delta}\mathbb{E}_s\left[\sum_{t=0}^{\infty}\beta^t \hat{q}_{s+t}\right] + \frac{\rho_\Delta}{1-\beta\rho_\Delta}\hat{\Delta}_{s-1} \tag{146}$$

and

$$\mathcal{A}_2 = -\frac{\log(\Delta)\eta}{1-\beta} - \frac{\eta}{1-\beta\rho_\Delta}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(\Lambda_\pi\hat{\pi}_t - \Lambda_n\hat{n}_t + \frac{1}{2}[\Lambda_{\pi^2}\hat{\pi}_t^2 - 2\Lambda_{n\pi}\hat{\pi}_t\hat{n}_t + \Lambda_{n^2}\hat{n}_t^2]\right)\right] - \frac{\eta\rho_\Delta}{1-\beta\rho_\Delta}\hat{\Delta}_{-1}. \tag{147}$$

Now, we apply similar steps for $\mathcal{A}_3$. Observe that $\tilde{\Delta}_t = \hat{\Delta}_t + \tilde{\Delta}$ and $\tilde{y}_t = \hat{y}_t + \tilde{y}$. Using Lemma 4, we have that

$$\begin{aligned}
\tilde{\Delta}_t\tilde{y}_t &= \left(\hat{\Delta}_t + \tilde{\Delta}\right)(\hat{y}_t + \tilde{y}) \\
&= \tilde{\Delta}\tilde{y} + \tilde{\Delta}\hat{y}_t + \hat{\Delta}_t\tilde{y} + \hat{\Delta}_t\hat{y}_t \\
&= \tilde{\Delta}\tilde{y} + \tilde{\Delta}\hat{y}_t + \hat{\Delta}_t\tilde{y} + \Lambda_\pi\hat{\pi}_t\hat{y}_t - \Lambda_n\hat{n}_t\hat{y}_t + \rho_\Delta\hat{\Delta}_{t-1}\hat{y}_t + O(||(\hat{\pi}_t,\hat{y}_t,\hat{n}_t)||^3) && (148) \\
\tilde{\Delta}_t^2 &= \left(\hat{\Delta}_t + \tilde{\Delta}\right)^2 \\
&= \tilde{\Delta}^2 + 2\tilde{\Delta}\hat{\Delta}_t + \hat{\Delta}_t^2 \\
&= \tilde{\Delta}^2 + 2\tilde{\Delta}\hat{\Delta}_t + \Lambda_\pi^2\hat{\pi}_t^2 + \Lambda_n^2\hat{n}_t^2 - 2\Lambda_\pi\Lambda_n\hat{\pi}_t\hat{n}_t + \rho_\Delta^2\hat{\Delta}_{t-1}^2 + O(||(\hat{\Delta}_{t-1},\hat{\pi}_t,\hat{n}_t)||^3) && (149)
\end{aligned}$$

Ignoring the terms $\hat{\Delta}_{t-1}\hat{y}_t$ and $\hat{\Delta}_{t-1}^2$, we have that

$$\begin{aligned}
\mathcal{A}_3 &= -\frac{1}{2}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[2\tilde{\Delta}_t\tilde{y}_t + \eta\tilde{\Delta}_t^2\right]\right] \\
&= -\frac{1}{2}\frac{2\tilde{\Delta}\tilde{y}}{1-\beta} - \frac{1}{2}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t[2\tilde{\Delta}\hat{y}_t + 2\Lambda_\pi\hat{\pi}_t\hat{y}_t - 2\Lambda_n\hat{n}_t\hat{y}_t]\right] - \frac{2\tilde{y}}{2}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\hat{\Delta}_t\right] \\
&\quad - \frac{\tilde{\Delta}^2\eta}{2(1-\beta)} - \frac{\eta}{2}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[\Lambda_\pi^2\hat{\pi}_t^2 + \Lambda_n^2\hat{n}_t^2 - 2\Lambda_\pi\Lambda_n\hat{\pi}_t\hat{n}_t\right]\right] - \frac{2\tilde{\Delta}\eta}{2}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\hat{\Delta}_t\right] \\
&\quad + O(\mathbb{E}_0\left[||(\hat{\Delta}_{t-1},\hat{\pi}_t,\hat{y}_t,\hat{n}_t)||^3\right]) \\
&= -\frac{2\tilde{\Delta}\tilde{y} + \tilde{\Delta}^2\eta}{2(1-\beta)} - \frac{1}{2}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t[2\tilde{\Delta}\hat{y}_t + 2\Lambda_\pi\hat{\pi}_t\hat{y}_t - 2\Lambda_n\hat{n}_t\hat{y}_t + \eta\Lambda_\pi^2\hat{\pi}_t^2 + \eta\Lambda_n^2\hat{n}_t^2 - \eta 2\Lambda_\pi\Lambda_n\hat{\pi}_t\hat{n}_t]\right] \\
&\quad - \frac{2\tilde{y} + 2\tilde{\Delta}\eta}{2}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\hat{\Delta}_t\right] + O(\mathbb{E}_0\left[||(\hat{\Delta}_{t-1},\hat{\pi}_t,\hat{y}_t,\hat{n}_t)||^3\right]) && (150)
\end{aligned}$$

For simplicity, we will ignore the $O(\cdot)$ term. Using result (146)

$$\mathcal{A}_3 = -\frac{2\tilde{\Delta}\tilde{y} + \tilde{\Delta}^2\eta}{2(1-\beta)} - \frac{1}{2}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t[2\tilde{\Delta}\hat{y}_t + 2\Lambda_\pi\hat{\pi}_t\hat{y}_t - 2\Lambda_n\hat{n}_t\hat{y}_t + \eta\Lambda_\pi^2\hat{\pi}_t^2 + \eta\Lambda_n^2\hat{n}_t^2 - \eta 2\Lambda_\pi\Lambda_n\hat{\pi}_t\hat{n}_t]\right] \tag{151}$$

$$-\frac{\tilde{y}+\tilde{\Delta}\eta}{(1-\beta\rho_\Delta)}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(\Lambda_\pi\hat{\pi}_t - \Lambda_n\hat{n}_t + \frac{1}{2}[\Lambda_{\pi^2}\hat{\pi}_t^2 - 2\Lambda_{n\pi}\hat{\pi}_t\hat{n}_t + \Lambda_{n^2}\hat{n}_t^2]\right)\right] - \frac{(\tilde{y}+\tilde{\Delta}\eta)\rho_\Delta}{(1-\beta\rho_\Delta)}\hat{\Delta}_{-1} \tag{152}$$

Using equations (140), (147) and (152), we have that the welfare function up to second order is given by

$$\begin{aligned}
\mathbb{W}(\pi) &= -(1-\beta)(V - V^F) = -(1-\beta)(\mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3) \\
&= \frac{\tilde{y}^2}{2\eta} + \frac{\xi\bar{n}^2}{2}\tilde{n}^2 + \log(\Delta)\eta + \frac{2\eta\tilde{\Delta}\tilde{y} + \tilde{\Delta}^2\eta^2}{2\eta} + \frac{(1-\beta)\rho_\Delta}{1-\beta\rho_\Delta}\left(\eta + \tilde{y} + \tilde{\Delta}\eta\right)\hat{\Delta}_{-1} \\
&\quad + (1-\beta)\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[\frac{1}{2\eta}\hat{y}_t^2 - \frac{\xi\bar{n}^2}{2}(\tilde{n}_t - \tilde{n})^2\right]\right] \\
&\quad + (1-\beta)\frac{\eta + \tilde{y} + \tilde{\Delta}\eta}{(1-\beta\rho_\Delta)}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(\Lambda_\pi\hat{\pi}_t - \Lambda_n\hat{n}_t + \frac{1}{2}[\Lambda_{\pi^2}\hat{\pi}_t^2 - 2\Lambda_{n\pi}\hat{\pi}_t\hat{n}_t + \Lambda_{n^2}\hat{n}_t^2]\right)\right] \\
&\quad + (1-\beta)\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[\frac{1}{\eta}\hat{y}_t\tilde{y} + \xi\bar{n}^2(\tilde{n}_t - \tilde{n})\tilde{n}\right]\right] \\
&\quad + \frac{(1-\beta)}{2}\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t[2\tilde{\Delta}\hat{y}_t + 2\Lambda_\pi\hat{\pi}_t\hat{y}_t - 2\Lambda_n\hat{n}_t\hat{y}_t + \eta\Lambda_\pi^2\hat{\pi}_t^2 + \eta\Lambda_n^2\hat{n}_t^2 - \eta 2\Lambda_\pi\Lambda_n\hat{\pi}_t\hat{n}_t]\right]
\end{aligned} \tag{153}$$

Using the fact that $\tilde{x} = \log\left(x^{-\theta}/1\right) = \eta\log\left(x^{-\theta/\eta}/1\right) = \eta\log(\Delta/1) = \eta\tilde{\Delta}$

$$\begin{aligned}
\mathbb{W}(\pi) &= \underbrace{\tilde{x} + \frac{\xi\bar{n}^2}{2}\tilde{n}^2 + \frac{(\tilde{y}+\tilde{x})^2}{2\eta}}_{=\Xi_0(\pi)} + \frac{\rho_\Delta(1-\beta)}{1-\beta\rho_\Delta}\left(\eta + \tilde{y} + \tilde{\Delta}\eta\right)\hat{\Delta}_{-1} \\
&\quad + (1-\beta)\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[\frac{1}{2\eta}\hat{y}_t^2 + \frac{\xi\bar{n}^2}{2}(\tilde{n}_t - \tilde{n})^2\right]\right] \\
&\quad + (1-\beta)\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[\left(\frac{\eta + \tilde{y} + \tilde{x}}{(1-\beta\rho_\Delta)}\frac{\Lambda_{\pi^2}}{2} + \frac{\eta\Lambda_\pi^2}{2}\right)\hat{\pi}_t^2 - 2\left(\frac{\eta + \tilde{y} + \tilde{x}}{(1-\beta\rho_\Delta)}\frac{\Lambda_{\pi n}}{2} + \frac{\eta\Lambda_\pi\Lambda_n}{2}\right)\hat{\pi}_t\hat{n}_t\right]\right] \\
&\quad + (1-\beta)\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[\frac{\tilde{y}+\tilde{x}}{\eta}\hat{y}_t + \frac{\eta + \tilde{y} + \tilde{\Delta}\eta}{(1-\beta\rho_\Delta)}(\Lambda_\pi\hat{\pi}_t - \Lambda_n\hat{n}_t) + \xi\bar{n}^2(\tilde{n}_t - \tilde{n})\tilde{n}\right]\right] \\
&\quad + (1-\beta)\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[\Lambda_\pi\hat{\pi}_t\hat{y}_t - \Lambda_n\hat{n}_t\hat{y}_t + \left(\frac{\eta + \tilde{y} + \tilde{x}}{(1-\beta\rho_\Delta)}\frac{\Lambda_{n^2}}{2} + \frac{\eta\Lambda_n^2}{2}\right)\hat{n}_t^2\right]\right]
\end{aligned} \tag{154}$$

□

Proof of Proposition 1 in the main text. From Lemma 2, at zero trend inflation, we have that $\tilde{x} = \tilde{y} = 0$. Also, it is easy to check that $\tilde{n} = 0$, so $\Theta_0(1) = 0$. From Lemma 4, we have that $\Lambda_n = \Lambda_\pi = \Lambda_{n\pi} = \Lambda_{n^2} = 0$ and $\rho_\Delta = (1 - \bar{n})$. Ignoring predetermined components, from Lemma 5, we have that

$$\begin{aligned}\mathbb{W}(1) = (1-\beta)\mathbb{E}_0 &\left[\sum_{t=0}^{\infty}\beta^t \left[\frac{1}{2\eta}\hat{y}_t^2 + \frac{\xi\bar{n}^2}{2}\tilde{n}_t^2\right]\right] \\ &+ (1-\beta)\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[\left(\frac{2\eta}{2(1-\beta(1-\bar{n}))}\frac{\Lambda_{\pi^2}}{2}\right)\hat{\pi}_t^2\right]\right]. \end{aligned} \tag{155}$$

By the law of iterated expectations $\mathbb{E}[\mathbb{E}_0[\cdot]] = \mathbb{E}[\cdot]$, so

$$\begin{aligned}\mathbb{L}(\pi) &\approx \mathbb{E}[\mathbb{W}(1)] \\ &= \frac{1}{2\eta}\mathbb{E}[\hat{y}_t^2] + \frac{\xi\bar{n}^2}{2}\mathbb{E}[\tilde{n}_t^2] + \left(\frac{2\eta}{2(1-\beta(1-\bar{n}))}\frac{\Lambda_{\pi^2}}{2}\right)\mathbb{E}\left[\hat{\pi}^2\right]\end{aligned} \tag{156}$$

Since $\Lambda_{\pi^2} = \frac{\theta}{\eta}\frac{\eta+\theta(1-\eta)}{\eta}\frac{1-\bar{n}}{\bar{n}}$ at zero trend inflation from Lemma 4, we have that

$$\mathbb{L}(\pi) \approx \underbrace{\frac{1}{2\eta}}_{=\Gamma_y}\mathbb{E}[\hat{y}_t^2] + \frac{\xi\bar{n}^2}{2}\mathbb{E}[\tilde{n}_t^2] + \underbrace{\frac{1}{2}\frac{\theta}{1-\beta(1-\bar{n})}\left(1+\theta\left(\frac{1}{\eta}-1\right)\right)\frac{1-\bar{n}}{\bar{n}}}_{=\Gamma_\pi}\mathbb{E}\left[\hat{\pi}^2\right] \tag{157}$$

□

Proof of Proposition 2 in the main text. With business cycle shocks absent, from Lemma 5, we have that

$$\mathbb{W}(\pi) = \underbrace{\tilde{x} + \frac{\xi\bar{n}^2}{2}\tilde{n}^2 + \frac{(\tilde{y}+\tilde{x})^2}{2\eta}}_{=\Xi_0(\pi)} \tag{158}$$

Given that $\Theta_0(\pi)$ is up to second order equal to $\Xi_0(\pi)$, from Lemma 2, we have that

$$\tilde{y} = \eta\left[-\frac{1+\theta\left(\frac{1}{\eta}-1\right)}{\theta-1}\log\left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right) + \log\left(\frac{1-\beta(1-n)\pi^{\frac{\theta}{\eta}}}{1-\beta(1-n)\pi^{\theta-1}}\right)\right] \tag{159}$$

$$\tilde{x} = \frac{\theta}{\theta-1}\log\left(\frac{1-(1-n)\pi^{\theta-1}}{n}\right) + \eta\log\left(\frac{n}{1-(1-n)\pi^{\frac{\theta}{\eta}}}\right) \tag{160}$$

□

Proof of Proposition 3 in the main text. From Lemma 5, ignoring predetermined terms, and using the law of iterated expectations $\mathbb{E}[\mathbb{E}_0[\cdot]] = \mathbb{E}[\cdot]$, we have that

$$\mathbb{E}[\mathbb{W}(\pi)] = \tilde{x} + \frac{\xi\bar{n}^2}{2}\tilde{n}^2 + \frac{(\tilde{y}+\tilde{x})^2}{2\eta} + \mathbb{E}[\Xi_1(\pi)] \tag{161}$$

$$\begin{aligned}
\mathbb{E}[\Xi_1(\pi)] = & \underbrace{\frac{1}{2\eta}}_{=\Gamma_y} \mathbb{E}[\hat{y}_t^2] + \underbrace{\frac{\xi\bar{n}^2}{2}}_{=\Gamma_n} \mathbb{E}\left[(\tilde{n}_t - \tilde{n})^2\right] + \underbrace{\left(\frac{\eta+\tilde{y}+\tilde{x}}{(1-\beta\rho_\Delta)}\frac{\Lambda_{\pi^2}}{2} + \frac{\eta\Lambda_\pi^2}{2}\right)}_{=\Gamma_\pi} \mathbb{E}[\hat{\pi}_t^2] \\
& + \underbrace{(-1)2\left(\frac{\eta+\tilde{y}+\tilde{x}}{(1-\beta\rho_\Delta)}\frac{\Lambda_{\pi n}}{2} + \frac{\eta\Lambda_\pi\Lambda_n}{2}\right)}_{=\Gamma_{n\pi}} \mathbb{E}\left[\hat{\pi}_t\hat{n}_t\right] \\
& + \frac{\tilde{y}+\tilde{x}}{\eta}\mathbb{E}\left[\hat{y}_t\right] + \frac{\eta+\tilde{y}+\tilde{\Delta}\eta}{(1-\beta\rho_\Delta)}\left(\Lambda_\pi\mathbb{E}\left[\hat{\pi}_t\right] - \Lambda_n\mathbb{E}\left[\hat{n}_t\right]\right) + \xi\bar{n}^2\tilde{n}\mathbb{E}\left[\tilde{n}_t - \tilde{n}\right] \\
& + \Lambda_\pi\mathbb{E}\left[\hat{\pi}_t\hat{y}_t\right] - \Lambda_n\mathbb{E}\left[\hat{n}_t\hat{y}_t\right] + \left(\frac{\eta+\tilde{y}+\tilde{x}}{(1-\beta\rho_\Delta)}\frac{\Lambda_{n^2}}{2} + \frac{\eta\Lambda_n^2}{2}\right)\mathbb{E}\left[\hat{n}_t^2\right]
\end{aligned} \tag{162}$$

where Λ_π, Λ_{π^2}, $\Lambda_{\pi n}$, Λ_n and ρ_Δ are defined in Lemma 4 equations. □