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Time-varying term premia and the expectations hypothesis in Australia

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This article investigates whether the (rational) expectations hypothesis holds for Australian yield data (it does not), whether the hypothesis holds after adjusting for term premia estimated from an affine term structure model (it appears to) and whether the yield process implied by the term structure model can match the failure of the hypothesis on unadjusted yields (it can). These results suggest that the term structure model used in Finlay and Chambers (2009) does a reasonable job in capturing the risk-neutral and real-world dynamics of Australian interest rates, at least as measured through the prism of the expectations hypothesis.

I. Introduction

The expectations hypothesis states that the expected one-period holding return from a long-term bond should equal the yield available on a contemporaneous one-period bond. Arbitrage among traders justifies the hypothesis, with traders exploiting predictable differences between one-period yields and bond-holding returns until these differences are zero.

Let P_t^n be the price of a n-period zero-coupon bond at time t (therefore maturing at $t + n$). The continuously compounded yield on such a bond is given by $R_t^n = -\ln(P_t^n)/n$. If we set $r_t = R_t^1$ then the one-period excess return on a n-period bond at time t is given by $D_{t+1}^n = \ln(P_{t+1}^{n-1}/P_t^n) - r_t$ and one can show that the expected one-period excess return is given by

$$E_t\left[D_{t+1}^n\right] = -(n-1)E_t\left[R_{t+1}^{n-1} - R_t^n\right] + \left(R_t^n - r_t\right) \quad (1)$$

Under the expectations hypothesis this expectation should be zero. To empirically examine the hypothesis one typically rearranges Equation 1, imposes rational expectations and runs the regression

$$R_{t+1}^{n-1} - R_t^n = \alpha_n + \beta_n\left[\frac{R_t^n - r_t}{n-1}\right] + \varepsilon_t \quad (2)$$

The hypothesis implies $\beta_n = 1$ for each n, whereas α_n is included to allow for the presence of any constant term premia. Studies estimating Equation 2 consistently find negative $\hat{\beta}_n$ coefficients however, which become more negative with increasing maturity (Table 1).

The observed failure of the expectations hypothesis has been widely attributed to time-varying term premia, or time-varying excess returns, which compensate investors for taking long positions. If term premia exist and remain unaccounted for in Equation 2, the time-varying component of the term premia will be captured in the error, ε_t. Omitting the term premia from the regression will therefore result in biased and inconsistent estimators of β_n (Tzavalis and Wickens, 1997; Harris, 2001). However, this empirical failure of the expectations hypothesis provides us with a useful yardstick with which to judge models of the term structure of interest rates. As argued in Dai and Singleton (2002), if term premia estimates from a model can correct for the failure, then this suggests that the model captures the risk-neutral dynamics of interest rates well. Further, if the yield process implied by the term structure model can match the failure of the hypothesis on unadjusted data, this suggests that the model captures the real-world dynamics of interest rates well. Matching both these criteria is no easy

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Table 1. Regression coefficient $\hat{\beta}_n$: the literature

Study	Long-bond maturity n (months)					
	3	6	12	24	48	120
Fama and Bliss (1987)	−0.428 (0.481)	−0.883 (0.640)	−1.425 (0.825)	−1.705 (1.120)	−2.147 (1.418)	−4.173 (1.985)
Campbell and Shiller (1991)	−0.176 (0.362)	−1.029 (0.537)	−1.381 (0.683)	−1.815 (0.151)	−2.665 (1.634)	−5.024 (2.316)
McCulloch and Kwon (1993)	0.003 (0.282)	−0.145 (0.442)	−1.435 (0.599)	−1.448 (1.004)	−2.262 (1.458)	−4.226 (2.076)

Note: SE are given in parentheses.

task – Dai and Singleton (2002) examine a number of quite sophisticated term structure models estimated on US interest rates and find that only one manages to satisfy both requirements.

II. Data

We take our estimated term premia from Finlay and Chambers (2009). In that article an affine term structure model is fitted to Australian interest rates over the period 1993 to 2007. The model takes the cash rate as a constant plus the sum of three latent factors, where the latent factors follow a zero-mean Ornstein–Uhlenbeck process, the continuous time analogue of a vector autoregression. The model is essentially the '$A_0(3)$' model of Duffee (2002) and has been implemented, for example, by Kim and Orphanides (2005) and Kim and Wright (2005) among others.

The Australian zero-coupon yield curve is also estimated by Finlay and Chambers (2009) utilizing the Merrill Lynch Exponential Spline Model methodology (Li *et al.*, 2001). To adjust for term premia in interest rates, we simply subtract the estimated term premia from the zero-coupon yields. More details on the estimation of the zero-coupon yields and term premia are available in Finlay and Chambers (2009) and Dai and Singleton (2002).

III. Empirical Results

Table 2 presents the β_n estimates from regression 2 using unadjusted yields, as well as the β_n estimates using yields adjusted for term premia. Although the estimates are sensitive to the sample period (Campbell and Shiller, 1991, p. 503), the results presented here are calculated over the full range of available data, and further analysis indicates that the range of estimates produced by varying the sample period typically lie within the confidence bounds for the estimated parameters over the full sample.

As expected, the unadjusted yields are inconsistent with the expectations hypothesis, with the estimated β_n coefficients becoming increasingly negative for long-term bonds. After adjusting for term premia, however, the estimated β_n coefficients lie closer to their expectations hypothesis implied value of $\beta_n = 1$. This is consistent with Dai and Singleton (2002) and suggests that the term structure model employed produces results that are consistent with, and can correct for, time-varying term premia. That is, the model seems to capture the risk-neutral dynamics of interest rates well.

Figures 1 and 2 provide a visual representation of the estimated β_n and their 90% confidence intervals across the range of maturities up to 10 years. One can see the negative trend of the unadjusted $\hat{\beta}_n$ coefficients and how the adjusted coefficients fluctuate around their expectations hypothesis implied value of 1. Under a 90% confidence interval across the range of maturities, $\hat{\beta}_n$ is significantly different from one for the unadjusted data, whereas $\beta_n = 1$ cannot be rejected for the adjusted data. All $\hat{\alpha}_n$ are estimated as effectively 0.

Next we turn to the real world and investigate whether the model of Finlay and Chambers (2009) implies a population β_n pattern that matches the sample $\hat{\beta}_n$ extracted from regression 2 using unadjusted yields. That is, we look at whether the term structure model implies the same pattern of behaviour as that

Table 2. Regression coefficient $\hat{\beta}_n$

	Long-bond maturity n (months)						
	3	6	12	24	48	96	120
Unadjusted yields	0.319 (0.088)	0.403 (0.137)	0.423 (0.238)	0.140 (0.363)	−0.026 (0.548)	−0.700 (0.846)	−1.051 (1.032)
Adjusted yields	0.390 (0.088)	0.638 (0.137)	0.994 (0.238)	0.906 (0.363)	1.189 (0.548)	1.115 (0.846)	1.010 (1.032)

Note: SE are given in parentheses.

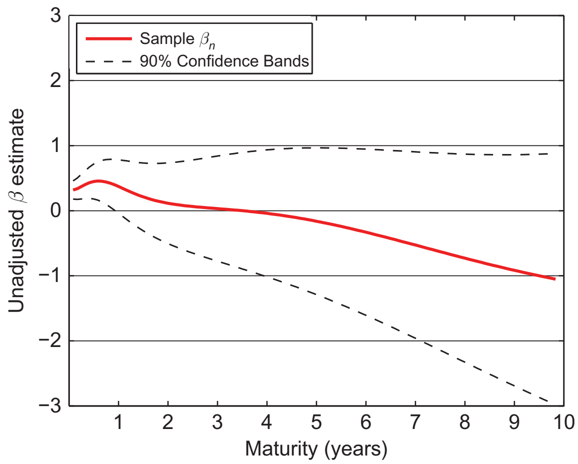

Fig. 1. β_n Estimates from unadjusted data

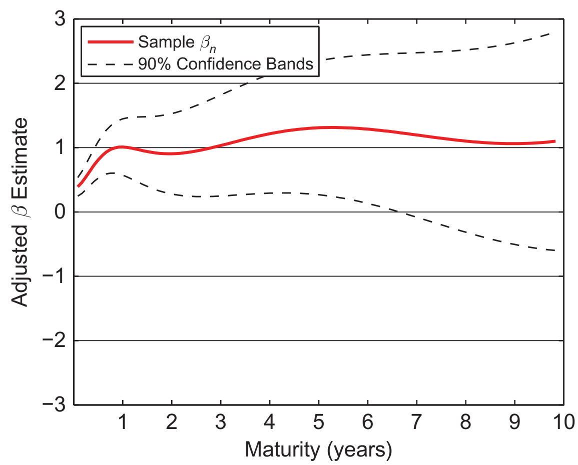

Fig. 2. β_n Estimates from adjusted data

seen in the data. The model-implied β_n coefficients are given by

$$\beta_n = \frac{\operatorname{cov}\left[R_{t+1}^{n-1} - R_t^n, \left(R_t^n - r_t\right)/(n-1)\right]}{\operatorname{var}\left[\left(R_t^n - r_t\right)/(n-1)\right]} \tag{3}$$

where the appendix provides explicit expressions for the components of Equation 3. Figure 3 shows the population β_n coefficients obtained from Equation 3 as well as the coefficients from regression 2 on unadjusted yield data.

As shown, the model-implied β_n coefficients track the downward slope of the $\hat{\beta}_n$ regression coefficients, particularly for maturities out to around 4 years. For estimates on bonds of maturity lengths greater than about 6 months, the model-implied estimates lie within the 90% confidence bounds of the estimated $\hat{\beta}_n$. These results suggest that the term structure model succeeds in capturing the real-world dynamics of interest rates as

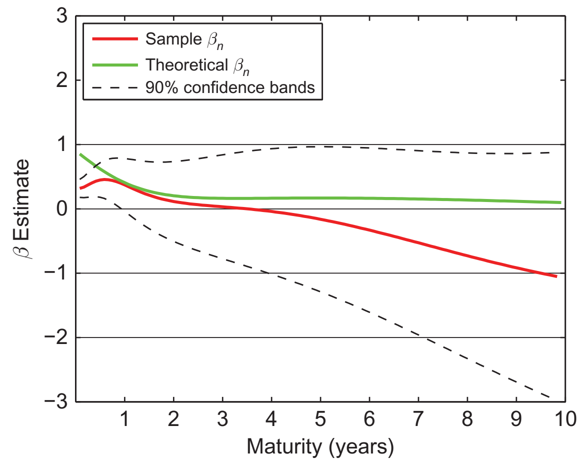

Fig. 3. Model-implied coefficients

well as the risk-neutral dynamics, at least as measured through the prism of the expectations hypothesis.

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Appendix

Here we detail the calculation of Equation 3. From Finlay and Chambers (2009)

$$R_t^n = -\tfrac{\ln P_t^n}{n} = k_n + \boldsymbol{p}_n' \boldsymbol{x}_t$$

where $\boldsymbol{x}_t = (x_{1,t}, x_{2,t}, x_{3,t})$ is a 3×1 vector of latent factors and k_n and $\boldsymbol{p}_n$ are functions of the estimated term structure model parameters that are taken as 'truth' (Dai and Singleton, 2002, p. 422). Note that $\boldsymbol{p}_n$ and k_n are referred to as $\boldsymbol{b}_{n/12}$ and $a_{n/12}$ in Finlay and Chambers (2009). Then

$$\begin{aligned}\operatorname{var}\left[\frac{R_t^n - r_t}{n-1}\right] &= \operatorname{var}\left[\frac{k_n + \boldsymbol{p}_n'\boldsymbol{x}_t - k_1 - \boldsymbol{p}_1'\boldsymbol{x}_t}{n-1}\right] \\ &= \operatorname{var}\left[\left(\frac{\boldsymbol{p}_n' - \boldsymbol{p}_1'}{n-1}\right)\boldsymbol{x}_t\right] = \boldsymbol{p}_n^* \operatorname{var} \boldsymbol{x}_t \left(\boldsymbol{p}_n^*\right)'\end{aligned}$$

for $\boldsymbol{p}_n^* = \left(\boldsymbol{p}_n' - \boldsymbol{p}_1'\right)/(n-1)$. Next, note that $\boldsymbol{x}_{t+1} = e^{-K\frac{1}{12}}\boldsymbol{x}_t + \boldsymbol{\varepsilon}_t$ with $\boldsymbol{x}_t$ and $\boldsymbol{\varepsilon}_t$ uncorrelated, we have

$$\begin{aligned}&\operatorname{cov}\left[R_{t+1}^{n-1} - R_t^n, \frac{R_t^n - r_t}{n-1}\right] \\ &= \operatorname{cov}\left[\left(\boldsymbol{p}_{n-1}' e^{-K\frac{1}{12}} - \boldsymbol{p}_n'\right)\boldsymbol{x}_t, \frac{\boldsymbol{p}_n' - \boldsymbol{p}_1'}{n-1}\boldsymbol{x}_t\right] \\ &= \bar{\boldsymbol{p}}_n \operatorname{var} \boldsymbol{x}_t \left(\boldsymbol{p}_n^*\right)'\end{aligned}$$

for $\bar{\boldsymbol{p}}_n = \boldsymbol{p}_{n-1}' e^{-K\frac{1}{12}} - \boldsymbol{p}_n'$, where K is a matrix of the underlying model parameters. The β_n from Equation 3 can then be calculated using $\operatorname{var} \boldsymbol{x}_t = vec^{-1}\left[(K \otimes I + I \otimes K)^{-1} vec\left(\sum \sum'\right)\right]$ where again $\sum$ is a matrix of underlying model parameters. Greater detail and estimated model parameters are available in Finlay and Chambers (2009).